# Behavior of DES

- $X$ : set of states ; $x \in X$ typical element /state
  $\Sigma$ : set of events ; $\sigma \in \Sigma$ typical element/event

- Behavior of DES may be described by sequences of triples :

  $(x_0, \sigma_1, t_1)\ (x_1, \sigma_2, t_2) \ldots$

  $x_0 \in X$ : initial state
  $x_i \in X$ : ith state ; $\sigma_i \in \Sigma$ : ith event
  $t_i \in \mathbb{R}$ : Instance of ith state transition

- Such behavioral model is called timed model (contains timing information)
- Timed model used for achieving quantitative goals :
  minimization of average delay in communication network.
- Untimed models ignore timing information ; contain information about orderly occurrence of states and events.
- Used for achieving qualitative goals :
  buffer in mfg. system must never overflow
  message sequence be received in the order it was sent

  Such properties do not depend on when events occurred ; rather in what order they occurred.

- We will only deal with qualitative or logical behaviors.

# Languages

- At qualitative or logical level behavior described by:

  $(x_0, \sigma_1)\ (x_1, \sigma_2)\ \ldots$

- DES deterministic if given a state and event occurring in that state, next state is uniquely known.
- For deterministic systems behavior may be described by:

  $\sigma_1\ \sigma_2\ \ldots$

  and initial state $x_0$.
- Sequence of events called trace / string; collection of strings : language
- $\Sigma^*$ : set of all finite length traces, including zero length trace "$\varepsilon$".
- language: subset of $\Sigma^*$ ; $H, K$ symbols used
- trace : member of $\Sigma^*$ ; $s, t$ symbols used
- $|s|$ : length of $s$ ; $s \leq t \Rightarrow s$ a prefix of $t$

  $s < t \Rightarrow s$ a proper prefix of $t$

- Example: Buffer with capacity one.

  states: empty and full ; events : arrival and departure.

  state transition from empty to full on arrival

  state transition from full to empty on departure

  initial state : empty

  Language of buffer : all sequences of the type:

  arrival. departure. arrival. departure ...

# Language Models

- $K \subseteq \Sigma^*$ ; $K \neq \phi$ be all traces that occur in a DES ; called <u>generated</u> lang.
  For a trace to occur all prefixes must occur first $\Rightarrow K = pr(K)$
- $K_m \subseteq K$ : traces whose execution imply completion of task ; called <u>marked</u> language
- Language model : $(K_m, K)$ with $K_m \subseteq K = pr(K) \neq \phi$
- <u>Example</u> : Buffer with capacity one; $a$ : arrival event ; $d$ : departure

  generated language = $pr((a.d)^*)$

  Suppose $s \in pr((a.d)^*)$ implies completion of task iff buffer is empty.

  marked language : $(a.d)^*$

Elevator moves between floors 1 & 2. Events = {up, down,

$$\overline{K} = pr(K) = \{ t \in \Sigma^* \mid t \leq s, \text{ where } s \in K \}$$

$$t \leq s \iff t \in \overline{s}$$

<u>HW:</u> ~~[illegible]~~ state machine & lang model of plant & spec for traffic control

# State Machines

- Alternative way of describing a language model
- SM is a 5-tuple: $G := (X, \Sigma, \alpha, x_0, X_m)$

$X =$ set of states

$\Sigma =$ finite set of events

$\alpha(x_1, \sigma_1) = \{x_2, x_3\}$ $\alpha: X \times (\Sigma \cup \{\varepsilon\}) \longrightarrow 2^X$ state transition function

$x_0 \in X$: initial state

$X_m \subseteq X$: marked states

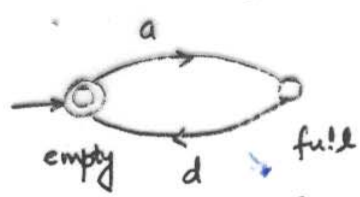
- SM is in general nondeterministic state machine with $\varepsilon$-moves

$\varepsilon$-NSM: $\alpha: X \times (\Sigma \cup \{\varepsilon\}) \longrightarrow 2^X$

NSM: $\alpha: X \times \Sigma \rightarrow 2^X$ (no $\varepsilon$-moves)

DSM: $\alpha: X \times \Sigma \rightarrow X$ (deterministic SM)

**Example:** Buffer of capacity one with language model $(pr((ad)^*), (ad)^*)$




DSM for buffer

$X = \{\text{empty}, \text{full}\}$ ; $\Sigma = \{a, d\}$ ; $x_0 = \text{empty}$ ; $X_m = \{\text{empty}\}$

$\alpha(\text{empty}, a) = \text{full}$ ; $\alpha(\text{full}, d) = \text{empty}$.

- State transition function is a partial map (defined on a subset of $X \times (\Sigma \cup \{\varepsilon\})$).

## Language model represented by SM:

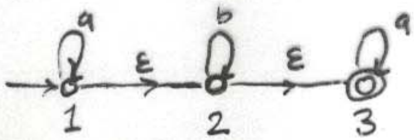
- epsilon-closure of $x$ : $\varepsilon_G^*(x)$ = set of states reachable from $x$ on zero or more $\varepsilon$-moves

- extension of $\alpha$ from events to traces:

$$\alpha^*(x,\varepsilon) = \varepsilon_G^*(x) \quad = \text{set of states reached on zero length string}$$

$$\alpha^*(x, s\sigma) = \varepsilon_G^*\big(\alpha(\underline{\alpha^*(x,s)}, \sigma)\big) = \text{set of states reached on } s\sigma.$$

- Example:



$\alpha^*(1,\varepsilon) = \varepsilon_G^*(1) = \{1,2,3\}$; $\alpha^*(2,\varepsilon) = \varepsilon_G^*(2) = \{2,3\}$ ; $\alpha^*(3,\varepsilon) = \varepsilon_G^*(3) = \{3\}$

$\alpha^*(1,a) = \varepsilon_G^*\big(\alpha(\alpha^*(1,\varepsilon),a)\big) = \varepsilon_G^*\big(\alpha(\{1,2,3\},a)\big) = \varepsilon_G^*(\{1,3\}) = \{1,2,3\}$

$\alpha^*(1,ab) = \varepsilon_G^*\big(\alpha(\alpha^*(1,a),b)\big) = \varepsilon_G^*\big(\alpha(\{1,2,3\},b)\big) = \varepsilon_G^*(\{2\}) = \{2,3\}$

HW: compute $\varepsilon^*(\cdot)$ and $\alpha^*(1, aba)$ for state machine in problem 2, Chapter 1.

$$L(G) = \{ s \in \Sigma^* \mid \alpha^*(x_0, s) \neq \phi \} \qquad \text{generated lang.}$$

In the example above, $\alpha^*(1, bab) = \phi \Rightarrow bab \notin L(G)$.

$$L_m(G) = \{ s \in L(G) \mid \alpha^*(x_0, s) \cap X_m \neq \phi \}. \qquad \text{marked lang.}$$

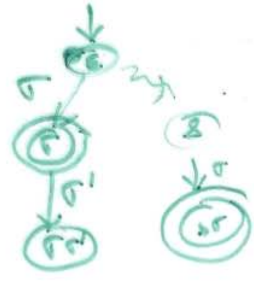
Then $(L_m(G), L(G))$ is a language-model. } HW

- $(L_m(G), L(G))$ language-model for any $G$
- Conversely, given a languge model $(K_m, K)$, exists DSM $G$ s.t.

$$(L_m(G), L(G)) = (K_m, K)$$

$X := \{ s \in K$[redacted]$\}$ ; $x_0 := \varepsilon$ ; $X_m := \{ s \in K_m$[redacted]$\}$

$$\forall s \in X, \sigma \in \Sigma: \quad \alpha(s,\sigma) := \begin{cases} s\sigma & \text{if } s\sigma \in X \\ \text{undefined} & \text{otherwise} \end{cases}$$




- Any deterministic DES can be represented as a DSM/ lang. model