Supervisor for discrete event plant

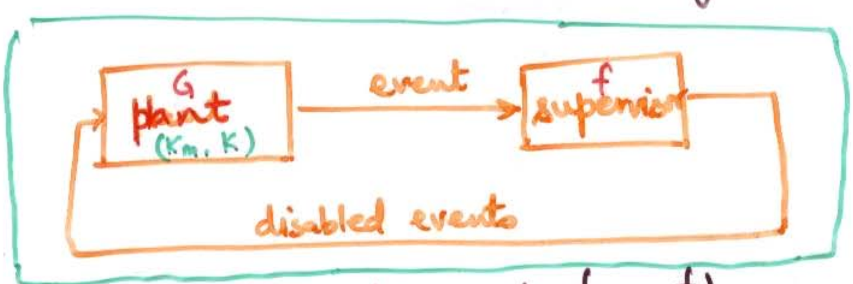
- Deterministic discrete event plant with model (K_m, K) or equivalently, $G := (X, \Sigma, \alpha, x_0, X_m)$

  <u>Example</u>: $G = M_1 \| M_2 \| TU$ (in previous example)

- For control purposes: $\Sigma = \underbrace{\Sigma_u}_{\text{uncontrollable}} \cup \underbrace{(\Sigma - \Sigma_u)}_{\text{controllable}}$

  supervisor: $\boxed{f : K \to 2^{\Sigma - \Sigma_u}}$

  $\forall s \in K$: $f(s) \subseteq \Sigma - \Sigma_u$ is set of controllable events disabled followed by execution of trace s



(K_m^f, K^f)

- Controlled language model $= (K_m^f, K^f)$
  - $\varepsilon \in K^f$; $[s \in K^f,\ s\sigma \in K,\ \sigma \notin f(s)] \Leftrightarrow [s\sigma \in K^f]$
  - $K_m^f := K^f \cap K_m$ (marked lang. that survives under contrl)

- (K_m^f, K^f) is a language model $\Leftrightarrow$ $K_m^f \subseteq K^f = pr(K^f) \neq \emptyset$

  $\Rightarrow pr(K_m^f) \subseteq K^f$ (may exist generated trace $s \in K^f$ which is not a prefix of marked trace $s \notin pr(K_m^f)$ $\Rightarrow$ system may "block")

- f called <u>nonblocking</u> if $pr(K_m^f) = K^f \Leftrightarrow K^f \subseteq pr(K_m^f)$

(each generated trace is a prefix of some marked trace).

Synchronous composition based supervisor

- Supervisor f "restricts" behavior of plant
  This can also be achieved by synchronous composition:
- Let $S := (Y, \Sigma, \beta, y_0, Y_m)$ be supervisor state machine
  $\Rightarrow \; L(G\|S) = L(G) \cap L(S) \quad ; \quad L_m(G\|S) = L_m(G) \cap L_m(S).$
- S restricts the behavior of G. Additional conditions:
  (i) S must not disable any uncontrollable event, i.e.,
  $s \in L(G\|S), \sigma \in \Sigma_u, s\sigma \in L(G) \Rightarrow s\sigma \in L(G\|S)$, i.e., $\boxed{L(G\|S)\Sigma_u \cap L(G) \subseteq L(G\|S)}$.
  (ii) S must also satisfy: $\boxed{L_m(G\|S) = L(G\|S) \cap L_m(G)}$

- S called <u>Σ_u-enabling</u> if $L(G\|S)\Sigma_u \cap L(G) \subseteq L(G\|S)$
  <u>non-marking</u> if $L_m(G\|S) = L(G\|S) \cap L_m(G) = L(S) \cap L_m(G)$
  <u>non-blocking</u> if $pr(L_m(G\|S)) = L(G\|S)$.

<u>Note:</u> S nonmarking if $L_m(S) = L(S)$, i.e, each state in S is marked.

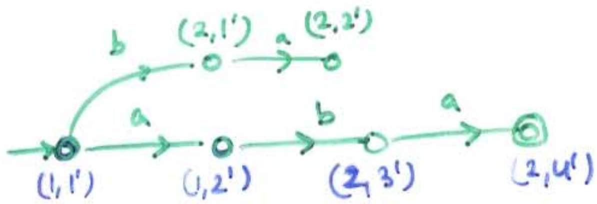
- <u>Example</u>:

G

$\Sigma_u = \{b\}$.

S

$$\begin{cases} f(\varepsilon) = f(b) = f(ab) = \phi \\ f(a) = f(aba) = f(ba) = \{a\} \end{cases}$$



G || S

$L(G\|S)\Sigma_u \cap L(G) \subseteq L(G\|S) \Rightarrow S$ Σ_u-enabling.
$a \in L(S) \cap L_m(G) - L_m(G\|S) \Rightarrow S$ not non-marking.
$pr(L_m(G\|S)) \subsetneq L(G\|S) \Rightarrow S$ not non-blocking.