EE 324 LAB 5

Approximating Continuous-time Systems with Discrete-time Systems

In this lab, you will learn how to derive a discrete-time system as an approximation of a continuous-time system.

We consider two popular methods for transforming a continuous-time system into discrete-time systems which approximate the behavior of the continuous-time system. In other words, we want to derive transformations form $H(s)$ to $H(z)$. Let T denote the sampling time.

Prelab:

Problem 1: The Euler Approximation

The mathematical basis behind this transformation is to approximate the derivative as follows:

$$\frac{dx}{dt}(kT) \cong \frac{x[(k+1)T] - x(kT)}{T}$$

or, using k instead of kT for convenience,

$$\dot{x}(k) \cong \frac{1}{T}[x(k+1) - x(k)]$$

and applying this approximation to the differential equation.

1. From the transfer function $H(s)$ (in Laplace domain) of the system, derive the differential equation that characterizes it.

2. Evaluate the equation at time kT.

3. Use Euler's approximation to derive a difference equation, typical of a discrete system.

4. From the difference equation, compute the transfer function $H_E(z)$ of the resulting discrete system (in Z domain).

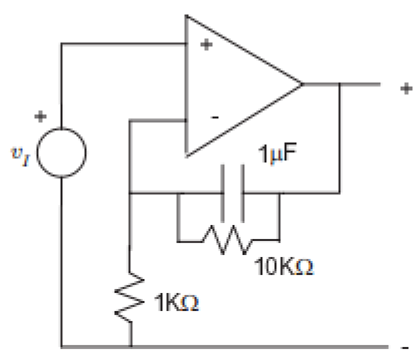

Figure 1 Op-Amp Circuit to Analyze

5. What are the relations between the poles and the zeros of the continuous-time system, and the poles and the zeros of the discrete-time system, respectively? Can the discrete system be unstable? Why?

6. Verify that H_E(z) can be directly obtained from $H(s)$ by taking the transformation $s = \frac{\text{z-1}}{T}$.

Problem 2: The Trapezoidal Approximation

This transformation is based on approximating the integral with the trapezoidal rule. Recall that the integral:

$$x(t) = \int_0^t \dot{x}(\tau)d\tau,$$

can be written as:

$$x[(k+1)T] = x(kT) + \int_{kT}^{(k+1)T} \dot{x}(\tau)d\tau \cong x(kT) + [\dot{x}[(k+1)T] + \dot{x}(kT)]\frac{T}{2},$$

where the area described by the integral is approximated by the area of a trapezoid with height T and the two bases $\dot{x}[(k+1)T]$ and $\dot{x}(kT)$ respectively. Using k instead of kT for convenience,

$$\frac{\dot{x}(k+1) + \dot{x}(k)}{2} \cong \frac{x(k+1) - x(k)}{T}$$

which amounts to a more precise, but implicit approximation of the derivative.

1. Evaluate the differential equation derived previously at times kT and $(k+1)T$.

2. Use the trapezoidal approximation to derive a difference equation, typical of a discrete system.

3. From the difference equation, compute the transfer function $H_T(z)$ of the resulting discrete system (in Z-domain).

4. What are the relations between the poles and the zeros of the continuous-time system and the poles and zeros of the discrete-time system, respectively? Can the discrete system be unstable? Why?

5. Verify that $H_T(z)$ can be directly obtained from $H(s)$ by taking the transformation $s = \frac{2}{T}\frac{z-1}{z+1}$.

Laboratory Assignment:

Use Simulink to simulate the continuous-time system, as well as the discrete-time systems from the two approximation methods for different sampling times T and different inputs, discuss your findings, and suggest a T which provides a satisfactory approximation for the two methods.