Examples of "post-to-pre" Analysis

①

$$[x+y = x+y]$$

P_1:

$$z = x\ ; \qquad \text{replace } z \text{ by } x.$$

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$$u = z\ ; \qquad \text{replace } u \text{ by } z$$

$$[u = x+y]$$

So, we obtain $T\ \{P_1\}\ [u = x+y]$.

②

$$[(x+1-1=0) \wedge (1 = x+1)] \vee [(x+1-1 \neq 0) \wedge (x+1 = x+1)]$$

P_2:

$$a = x+1\ ; \qquad \text{replace } a \text{ by } x+1$$

$$[(a-1=0) \wedge (1 = x+1)] \vee [(a-1 \neq 0) \wedge (a = x+1)]$$

$$\text{If } (a-1=0) \text{ then } y=1 \text{ else } y=a\ ; \qquad \text{replace } y \text{ by } 1 \qquad \text{replace } y \text{ by } a$$

$$[y = x+1]$$

precondition can be simplified to: $[(x=0) \wedge (x=0)] \vee [(x \neq 0) \wedge T]$

$$\equiv (x=0) \vee (x \neq 0) \equiv T.$$

Thus we obtain, $T\ \{P_2\}\ [y = x+1]$.

③ invariant for "If $(z \neq x)$ then $z := z+1;\ y := y * z$" ?

Guess: $y = z!$

Verify:

$$[(z \neq x) \wedge (y*(z+1) = z+1\,!)] \vee [(z=x) \wedge (y = z!)]$$

$$\text{If } (z \neq x) \text{ then } z := z+1\ ;\ y := y*z \qquad \text{replace } y \text{ by } y*z \text{ and } z \text{ by } z+1.$$

$$[y = z!]$$

precondition: $[(z \neq x) \wedge (y = z!)] \vee [(z=x) \wedge (y = z!)] \equiv [y = z!] \wedge \underbrace{[(z \neq x) \vee (z=x)]}_{T}$

$$\equiv [y = z!]$$

So following holds: $[y = z!]\ \{\text{while } (z \neq x) \text{ do } z := z+1;\ y := y*z\}\ [y = z!] \wedge [z = x]$.