Applying Induction (Examples)

① $f(n) \leftrightarrow \left(\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \right)$

1. $\sum_{i=0}^{0} i = 0 = \frac{0(0+1)}{2}$ — Proof of base step
2. $\left[\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \right]$ — ind. hyp.
3. $\sum_{i=0}^{n+1} i = \sum_{i=0}^{n} i + (n+1)$ — tautology
4. $\sum_{i=0}^{n+1} i = \frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2}$ — 2 & 3
5. $\left(\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \right) \rightarrow \left(\sum_{i=0}^{n+1} i = \frac{(n+1)(n+2)}{2} \right)$ — $\rightarrow$ introduction on 2 & 4
6. $\forall n \in \mathbb{N}: \sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ — induction on 1 & 5.

② $f(n) \leftrightarrow (\forall n \geq 5 : 2^n \geq n^2)$

1. $2^5 = 32 \geq 5^2 = 25$ — base-step proof
2. $[n \geq 5]$ — assume
3. $2^n \geq n^2$ — induction hyp.
4. $2^{n+1} = 2\, 2^n$ — tautology
5. $2^{n+1} \geq 2n^2 = 2((n+1)-1)^2$ — 3 & 4 (~~1 & 2~~ to get ≥ 0)

$$= 2(n+1)^2 - 4(n+1) + 2$$
$$= (n+1)^2 + \underbrace{(n+1)((n+1)-4) + 2}_{\geq 0}$$
$$\geq (n+1)^2$$

6. $\forall n \geq 5 : (2^n \geq n^2) \rightarrow (2^{n+1} \geq (n+1)^2)$ — $\rightarrow$ introduction on 2,3,5
7. $\forall n \geq 5 : 2^n \geq n^2$ — induction on 1 & 6.