Predicate Logic Definition Cond.

- **Formulae of logic algebra:**

  $t_1, \ldots, t_n$ terms, $r \in R$ n-ary relation $\Rightarrow r(t_1, \ldots, t_n)$: atomic formula

  α, β formulae $\Rightarrow \neg\alpha, \alpha \vee \beta, \alpha \wedge \beta, \alpha \rightarrow \beta, \alpha \leftrightarrow \beta$ formulae

  $\alpha(x)$ formula, x variable $\Rightarrow \forall x\, \alpha(x)$, $\exists x\, \alpha(x)$ formulae

  ($\forall$ and $\exists$ are called quantifiers)

- **Sentence of logic algebra:**

  Every formula whose variables have been quantified is sentence.

  (quantified var. also called bound, and otherwise free)

- **Example $(\mathbb{N}, 0, 1, +, \cdot, <)$:**

  Terms: $0, 1$

  $x, y, z, \ldots$

  $t_1 + t_2,\ t_1 t_2$ $(x+y,\ xz+1,\ (xy+z)(x+1))$

  Formulae: $\alpha = (t_1 < t_2)$ $(x+y+y < z+1)$

  $\neg\alpha,\ \alpha_1 \wedge \alpha_2,\ \alpha_1 \vee \alpha_2$ $\neg(x+y < z) \vee (x+1 < y+x)$

  $\exists x \alpha,\ \forall x \alpha$ $\exists x \forall y\ (x < y),\ y \cdot z < x$

- **Given 1^{st} order formula $F(x)$ over numbers, examples of 1^{st} order sentences**

  Exist at least n numbers satisfying $F(x)$: $\exists x_1 \ldots \exists x_n \left(\bigwedge_{i,j} \neg(x_i = x_j) \bigwedge_i F(x_i) \right)$

  Exist at most n numbers satisfying $F(x)$: $\forall x_1 \ldots \forall x_{n+1} \left(\bigwedge_i F(x_i) \rightarrow \bigvee_{i,j} (x_i = x_j) \right)$

  Exist infinitely numbers satisfying $F(x)$: $\forall x \exists y\ ((x < y) \wedge F(y))$

  Exist finitely many numbers satisfying $F(x)$: $\exists x \forall y\ (F(y) \rightarrow (y < x))$