Examples involving quantified assertions

- Definition of prime:

$\forall x \in \mathbb{N} : \text{prime}(x) \Longleftrightarrow \neg(\exists y_1, y_2 \in \mathbb{N} : y_1 y_2 = x \wedge y_1 \neq x)$

- Consider sequences $A = a_1 a_2 \cdots a_m$, $B = b_1 b_2 \cdots b_n$ (a_i, b_i : symbol)

(i) $(A = B) \Leftrightarrow (\forall i \leq n : a_i = b_i) \wedge (m = n)$

(ii) Exists a position in seq. where both A and B seqs have same symbol

$\exists i \leq \min(m, n) : a_i = b_i$

(iii) A is a prefix of B

$(m \leq n) \wedge (\forall i \leq m : a_i = b_i)$

(iv) Notation: $a_i < a_j$ denotes symbol "a_i preceeds a_j"

Now we define a predicate SORTED : Sequences $\rightarrow \mathbb{B}$

$\text{SORTED}(A) \Leftrightarrow \forall i, j \leq n : i < j \Rightarrow a_i < a_j$.

- Consider graph $G = (V, E)$, V: vertex set, E: edge set

Predicate EDGE : $V \times V \rightarrow \mathbb{B}$ defines existence of edge between vertex pairs

Suppose $\left\{ \begin{array}{l} V = \{a, b, c, d\} \\ \text{EDGE}(a,b) \wedge \text{EDGE}(b,c) \wedge \text{EDGE}(a,d) \end{array} \right\}$ is the graph

Consider sentences,

(i) $\forall x \in V, \exists y \in V : \text{EDGE}(x, y)$

(ii) $\exists x \in V, \forall y \in V : \text{EDGE}(x, y) \vee (x = y)$

Then first is TRUE and second is FALSE.