

# Using Assumptions for Deduction

• Prove	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$	
1.	$[p \rightarrow q]$	assumption
2.	$p \vee \neg p$	tautology
3.		$[p]$ assumption from 2
4.		$q$ MP from 1, 3
5.		$\neg p \vee q$ $\vee$ introduction in 4
6.	$p \rightarrow (\neg p \vee q)$	$\rightarrow$ introduction using 3 & 5
7.		$[\neg p]$ assumption from 2
8.		$\neg p \vee q$ $\vee$ introduction using 7
9.	$\neg p \rightarrow (\neg p \vee q)$	$\rightarrow$ introduction using 7 & 8
10.	$\neg p \vee q$	case analysis using 2, 6 & 9
11.	$(p \rightarrow q) \rightarrow (\neg p \vee q)$	$\rightarrow$ introduction using 1 & 10
12.	$[\neg p \vee q]$	assumption
13.		$[\neg p]$ assumption from 12
14.		$p \rightarrow q$ vacuous proof from 13
15.	$\neg p \rightarrow (p \rightarrow q)$	$\rightarrow$ introduction from 13 & 14
16.		$[q]$ assumption from 12
17.		$\neg q \rightarrow \neg p$ vacuous proof on 16
18.		$p \rightarrow q$ contrapositive from 17
19.	$q \rightarrow (p \rightarrow q)$	$\rightarrow$ introduction from 16 & 18
20.	$(p \rightarrow q)$	case analysis from 12, 15 & 19
21.	$(\neg p \vee q) \rightarrow (p \rightarrow q)$	$\rightarrow$ introduction from 12 & 20
22.	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$	$\leftrightarrow$ introduction from 11 & 21

• Note the nesting of assumptions, and associated indentation.

• Can proofs be automated?  $(f \rightarrow g)$  iff  $\neg(f \rightarrow g)$  contradiction/  
not satisfiable

• Some proof strategies discussed in Section 3.6

To prove  $p \rightarrow q$  :

- (i) start with  $p$ , establish  $q$  (direct)
- (ii) start with  $\neg q$ , establish  $\neg p$  (contrapositive)
- (iii) start with  $\neg(p \rightarrow q)$ , establish FALSE (contradiction)