Deduction/Inferencing by rules

- **Example**: Prove $(p \lor q) \land \neg p \rightarrow q$

  1. $(p \lor q) \land \neg p$ hypothesis

  2. $p \lor q$ $\land$ elimination in 1

  3. $\neg p$ $\land$ elimination in 1

  4. $p \rightarrow q$ vacuous proof from 3

  5. $q \rightarrow q$ tautology

  6. q case analysis using 2, 4, 5

- Now we have a new rule of inference:
$$\frac{p \lor q \qquad \neg p}{q}$$
Alternative-elimination

- **Example**: Prove $(p \land \neg p) \rightarrow q$
  (starting from contradiction anything can be proved!)

  1. $p \land \neg p$ hypothesis

  2. $\neg p$ $\land$ elimination in 1

  3. $p \rightarrow q$ vacuous proof from 2

  4. p $\land$ elimination in 1

  5. q modus ponens using 3, 4