

Rules of inference

• A rule of inference is a tautology of the form:

$$\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n \rightarrow \beta$$

that is written as

each α_i is a premise conclusion

$$\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_n \\ \hline \beta \end{array}$$

Here α_i 's and β are all prop. formulae.

① Modus ponens
(way to put)

$$\begin{array}{c} p \rightarrow q \\ p \\ \hline q \end{array}$$

② Modus tollens
(way to remove)

$$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \neg p \end{array}$$

③ \leftrightarrow introduction

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow p \\ \hline p \leftrightarrow q \end{array}$$

④ Contrapositive

$$\begin{array}{c} p \rightarrow q \\ \hline \neg q \rightarrow \neg p \end{array}$$

⑤ Case analysis

$$\begin{array}{c} p \rightarrow r \\ q \rightarrow r \\ p \vee q \\ \hline r \end{array}$$

⑥ Vacuous proof

$$\begin{array}{c} \neg p \\ \hline p \rightarrow q \end{array}$$

⑦ \wedge introduction

$$\begin{array}{c} p \\ q \\ \hline p \wedge q \end{array}$$

⑧ \wedge elimination

$$\begin{array}{c} p \wedge q \\ \hline p \end{array}$$

⑨ \vee introduction

$$\begin{array}{c} p \\ \hline p \vee q \end{array}$$

⑩ Contradiction

$$\begin{array}{c} p \\ \neg p \\ \hline \text{FALSE} \end{array}$$

⑪ Substitution
($\alpha \equiv \beta$)

$$\begin{array}{c} \alpha \\ \hline \beta \end{array}$$

⑫ Tautology
($\alpha \equiv \text{TRUE}$)

$$\begin{array}{c} \hline \alpha \end{array}$$

• Inference rules can be verified via truth table:
 $\alpha_1 \wedge \dots \wedge \alpha_n \rightarrow \beta$ should evaluate to a tautology