Recursive and Causal Reconstruction of Sparse Signal Sequences

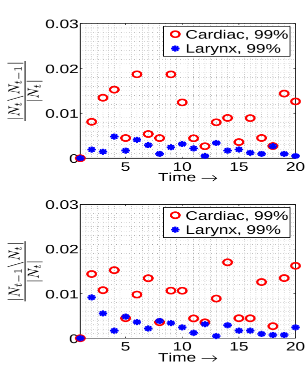
Namrata Vaswani, ECE Dept, Iowa State University, Ames, IA

This work involves the design and analysis of recursive algorithms for causally reconstructing a time sequence of (approximately) sparse signals from a greatly reduced number of linear projection measurements [1], [2], [3], [4], [5], [6], [7]. By "recursive", we mean use only the previous estimate and the current measurements to get the current estimate. The signals are sparse in some transform domain referred to as the sparsity basis and their sparsity patterns (support set of the sparsity basis coefficients) can change with time. The most important example of the above problem occurs in dynamic magnetic resonance imaging (MRI) for real-time medical applications such as interventional radiology, MR image guided surgery, or functional MRI to track brain activation changes. MRI is a technique for cross-sectional imaging that sequentially captures the 2D Fourier projections of the cross-section to be reconstructed. Cross-sectional images of the brain, heart, larynx or other human organ images are usually piecewise smooth, e.g. see the first row of Fig. 1(b) or 1(c), and thus approximately sparse in the wavelet domain. *In a time sequence, the sparsity pattern changes with time, but slowly.* Slow sparsity pattern change is empirically verified for medical image sequences in Fig. 1(a) and in [2] and for video in [7].

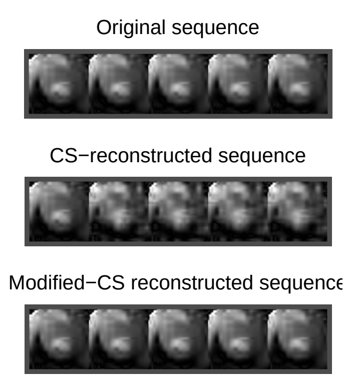
Since MR data acquisition is sequential, the ability to accurately reconstruct with fewer measurements directly translates to reduced scan times. Shorter scan times along with online (causal) and fast (recursive) reconstruction allow the possibility of real-time imaging of fast changing physiological phenomena.

Since the recent introduction of compressive sensing (CS) [8], [9], the static sparse reconstruction problem has been thoroughly studied. But most existing algorithms for the dynamic problem just use CS to jointly reconstruct the entire time sequence in one go [10], [11], [12]. This is a batch solution with very high complexity. The alternative - doing CS at each time separately (simple CS) - is online and fast but requires many more measurements. To the best of our knowledge, *our work [1] was the first to address the problem of causally and recursively reconstructing sparse signal sequences using fewer measurements* than those needed for simple CS. The *computational complexity of our proposed algorithms is only as much as that of simple CS*, and this is much lower than that of batch CS.

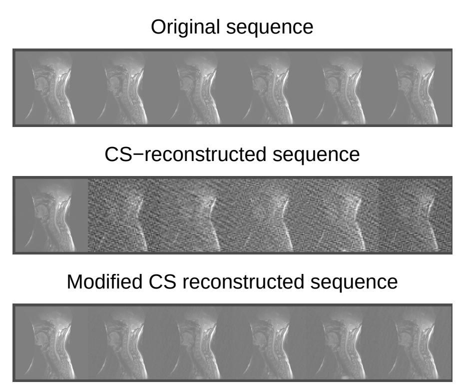
We summarize our contributions in the next subsection and after that we discuss the related work.



(a) Verifying slow support change



(b) Cardiac sequence and its recon



(c) Larynx (vocal tract) sequence and its recon.

Fig. 1. In Fig. 1(a), N_t refers to the 99% energy support of the 2D discrete wavelet transform of the cardiac sequence of Fig. 1(b) and of the larynx sequence (as a person speaks a vowel) of Fig. 1(c). Its size, $|N_t|$, varied between 4121-4183 ($\approx 0.07m$) for the larynx sequence and between 1108-1127 ($\approx 0.06m$) for cardiac, i.e. both are wavelet sparse. Here m is the image size (number of pixels). We plot the number of additions (top) and the number of removals (bottom) as a fraction of $|N_t|$. *Notice that all support change sizes are less than 2% of the support size.* In Figs. 1(b) and 1(c), we compare the reconstruction quality from only 16% MRI measurements at $t > 0$ (and 50% at $t = 0$) using simple compressive sensing (CS) with that using our proposed approach (modified-CS). Fig. 1(b) is for a sparsified cardiac sequence: modified-CS achieved exact reconstruction while clearly CS did not. Fig. 1(c) is for an actual larynx sequence: modified-CS error was less than 2%, CS error was 15-20%.

A. Our Contributions

All of our work described below uses one or both of the following easily verifiable observations.

1) The sparsity patterns of natural signal/image sequences usually change "slowly" over time [see Fig. 1(a)].
2) In most cases, the values of the nonzero coefficients also change gradually over time.

When using only fact 1 above, *the recursive sparse reconstruction problem can be reformulated as one of sparse reconstruction with partially "known" support*. The support estimate from the previous time serves as the "known" part. We can further improve the proposed algorithm by also using fact 2.

- The key idea of *our first approach (LS-CS-residual or LS-CS)* is to replace CS on the current observation by CS on the least squares (LS) observation residual computed using the "known" part of the support [1], [2]. The LS residual measures a signal that has much fewer large components compared to the original signal (it is what can be called a "sparse-compressible" signal). As a result, when fewer measurements are available, the LS-CS reconstruction error is much lower than that of simple CS.
  - By also using fact 2, we can replace the LS residual by the *Kalman filtering residual (KF-CS)* [1]. This improves the reconstruction particularly when the number of measurements is too few even for LS-CS.
- Even though LS-CS and KF-CS improve reconstruction accuracy over simple CS, but they cannot be used for "exact" reconstruction from fewer noise-free measurements. This led to *our second and more powerful approach* - *modified-CS* [3], [4]. Denote the "known" part of the support by T. Modified-CS tries to find the signal that is sparsest outside of T and that satisfies the data constraint. If T has small error (few extras and misses), modified-CS can achieve *exact* reconstruction from very few measurements, e.g. see Figs. 1(b), 1(c).
  - By also using fact 2 (gradual change of nonzero coefficient values), one can design *regularized modified-CS* which also constrains the change of the nonzero coefficient values along T [4].
- We have been able to show *very promising proof-of-concept applications of the above ideas in high fidelity real-time dynamic MR imaging of various human organs* [7], [4]. See Fig. 1 for some examples, and also see the PI's webpage, `http://www.ece.iastate.edu/~namrata/research/SequentialCS.html`.

Under the practically valid assumption of slowly changing support (fact 1), we have also been able to prove all of the following.

- Modified-CS *achieves exact reconstruction under much weaker sufficient conditions* (i.e. using much fewer noise-free measurements) than those needed to provide the same guarantee for simple CS [4].
- For both LS-CS and modified-CS (noisy), under fairly mild assumptions (bounded noise, high enough SNR, and weaker requirements on the number of measurements than what is needed for bounding simple CS error),
  - the error bounds are much smaller than those for simple CS [2], [13], and
  - *the support change errors, and hence the reconstruction errors, are "stable"*, i.e. they remain bounded by small time-invariant values at all times [2], [5].
- Since the above analysis only compares sufficient conditions or upper bounds, all of the above conclusions have been backed up by exhaustive simulation comparisons [2], [4]. We have also compared the above four approaches with each other under different conditions and discussed which is better when and why [6].

It is important to mention that *the proof of stability is one of the most challenging parts* of our work since (i) it requires carefully bounding the "detection delay" (the delay within which a set of newly added coefficients to the support get detected) and (ii) it requires a deletion scheme that successfully deletes the falsely added and removed coefficients from the support estimate either at every time or every-so-often. To the best of our knowledge, this is the *first stability result* for any recursive and causal sparse reconstruction approach. Proving stability of KF-CS or reg-modified-CS is even more difficult (because of dependence on past reconstructed values) and is being studied in ongoing work. *Stability is critical for any recursive algorithm since it ensures that the error does not blow up over time*. For example, for LS-CS, it ensures that the extras in the support estimate get deleted either at each time or every-so-often and the undetected support size does not keep increasing over time. Without the former, eventually the estimated support size will exceed the available number of measurements, thus making LS estimation impossible, while without the latter, the effective noise seen by the LS estimator will keep increasing.

The work discussed above is being supported by a 2009 NSF grant to the PI, CCF-0917015 (Recursive Reconstruction of Sparse Signal Sequences). The motivation for this work (particularly for KF-CS [1]) came from trying to answer the question of how to detect and estimate "effective dimension" and "effective subspace" change on-the-fly while tracking signal sequences and this was supported by a 2007 NSF grant to the PI, ECCS-0725849 (Change Detection in Nonlinear Systems and Applications in Shape Analysis).

B. Related Work

Our first paper on the topic was [1]. There has been some recent work on recursive sparse reconstruction in [14] but in it the authors mostly focus on the time-invariant sparsity pattern case. The related problem of sparse reconstruction with partial knowledge of the support was simultaneously addressed in our work [3] and in [15]. Recently (in Feb 2010), we learnt about the older work of von Borries et al [16] which also suggests an approach similar to modified-CS.

We would like to point out that our goals are quite different from (although have sometimes been confused with) (a) work that uses the previous estimate and homotopy to speed up the current optimization, but not to reduce the number of measurements required, e.g. [17], and also from (b) work that recursively improves the reconstruction of a single signal from sequentially arriving measurements, e.g. [18].

References

[1] N. Vaswani, "Kalman filtered compressed sensing," in *IEEE Intl. Conf. Image Proc. (ICIP)*, 2008.

[2] N. Vaswani, "LS-CS-residual (LS-CS): Compressive Sensing on Least Squares residual," *IEEE Trans. Signal Processing*, vol. 58, 2010, to appear (shorter versions in ICIP'08, ICASSP'09).

[3] N. Vaswani and W. Lu, "Modified-cs: Modifying compressive sensing for problems with partially known support," in *IEEE Intl. Symp. Info. Th. (ISIT)*, 2009.

[4] N. Vaswani and W. Lu, "Modified-cs: Modifying compressive sensing for problems with partially known support," *Accepted (with mandatory revisions) to IEEE Trans. Signal Processing, Arxiv preprint arXiv:0903.5066v3*, 2010 (shorter version in ISIT'09).

[5] N. Vaswani, "Stability of ls-cs-residual and modified-cs for signal sequence reconstruction," in *IEEE Intl. Symp. Info. Th. (ISIT)*, 2010, submitted.

[6] W. Lu and N. Vaswani, "Regularized modified bpdn for compressive sensing with partially known support," in *IEEE Intl. Symp. Info. Th. (ISIT)*, 2010, submitted.

[7] C. Qiu and N. Vaswani, "Compressive sensing on the least squares and kalman filtering residual for real-time dynamic mri and video reconstruction," *IEEE Trans. Image Proc.*, 2009, submitted (shorter version in ICASSP'09).

[8] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Info. Th.*, vol. 52(2), pp. 489–509, February 2006.

[9] D. Donoho, "Compressed sensing," *IEEE Trans. Info. Th.*, vol. 52(4), pp. 1289–1306, April 2006.

[10] U. Gamper, P. Boesiger, and S. Kozerke, "Compressed sensing in dynamic mri," *Magnetic Resonance in Medicine*, vol. 59(2), pp. 365–373, January 2008.

[11] M. Wakin, J. Laska, M. Duarte, D. Baron, S. Sarvotham, D. Takhar, K. Kelly, and R. Baraniuk, "Compressive imaging for video representation and coding," in *Proc. Picture Coding Symposium (PCS), Beijing, China*, April 2006.

[12] H. Jung, K. H. Sung, K. S. Nayak, E. Y. Kim, and J. C. Ye, "k-t focuss: a general compressed sensing framework for high resolution dynamic mri," *Magnetic Resonance in Medicine*, 2009.

[13] W. Lu and N. Vaswani, "Modified bpdn for noisy compressive sensing with partially known support," in *IEEE Intl. Conf. Acoustics, Speech, Sig. Proc. (ICASSP)*, 2010.

[14] D. Angelosante and G.B. Giannakis, "Rls-weighted lasso for adaptive estimation of sparse signals," in *IEEE Intl. Conf. Acoustics, Speech, Sig. Proc. (ICASSP)*, 2009.

[15] A. Khajehnejad, W. Xu, A. Avestimehr, and B. Hassibi, "Weighted l1 minimization for sparse recovery with prior information," in *IEEE Intl. Symp. Info. Th. (ISIT)*, 2009.

[16] R. von Borries, C. J. Miosso, and C. Potes, "Compressed sensing using prior information," in *IEEE Intl. Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, 2007.

[17] M. S. Asif and J. Romberg, "Dynamic updating for sparse time varying signals," in *CISS*, 2009.

[18] D.M. Malioutov, S. Sanghavi, and A. S. Willsky, "Compressed sensing with sequential observations," in *IEEE Intl. Conf. Acoustics, Speech, Sig. Proc. (ICASSP)*, 2008.