Detection and Estimation Theory

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What is Estimation and Detection

- Goal: extract useful information from noisy signals.
- Parameter Estimation: Given a set of "observations" and given an assumed probabilistic "model", get the "best" estimate of the parameters of the model's parameters.
  - Examples: DC signal in white noise, LPC coefficients of speech, image registration parameters, channel model
- State Estimation: Given "observations" which are noise-corrupted functions of the "state" (again assume a "model"), and given a "prior" model of the state's pdf (with known parameters), get the "best" estimate of the "state".
  - Equivalent to parameter estimation with a prior (Bayesian estimation)

- Requires parameter estimation to estimate the prior model parameters and some of the likelihood model parameters.
- Includes causal and non-causal estimation of a time-sequence of "states"
- Examples: all of the above with a prior on the parameter, Signal/Image denoising/restoration, Kalman filtering for tracking moving objects

- Detection (or binary estimation): Estimation among two (or a small number of) possible hypothesis, choose the "best" of two possible hypothesis.
  - Examples: bit or bit sequence detection at the receiver, radar or image based object detection, face recognition
- In most of this class, we will study the most commonly used optimality criteria to quantify "best", how to find these best

estimates or detection decisions, and limits on performance (e.g. CR-LB).

Types of Estimation Problems

- Model-based (parametric) v/s Non-parametric. We will only study parametric.
- Bayesian v/s Classical
- Online v/s Batch (offline) Estimators

Syllabus Topics and Examples

- Classical Estimation: Min. Variance Unbiased (MVU) and ML.
  - Meaning, sufficient statistics, factorization thorem, R-B theorem, Cramer Rao Lower Bound, ML estimation and its connection to CRLB and efficient estimators
  - Exact and approximate ML techniques (gradient descent, EM algorithm and alternating minimization).
  - Least squares estimation: MVUE for linear, Gaussian models
  - Applications: DC signal in noise, frequency and phase of a sine wave in noise, more generally: signal denoising (e.g. speech, image), speech AR model parameter estimation, image registration, communications channel parameter estimation
- Bayesian Estimation

- Regularized LS, Recursive LS (online estimator), adaptive techniques
  - State sequences: non-causal Wiener (batch), Kalman filter (online)
  - Applications: Restoration/denoising with a prior model on the signal (Wiener filtering), optical flow with a prior (to disallow very large motion), object tracking (Kalman filtering)
- Hidden Markov Models (part of Bayesian estimation)
  - Definition, Three problems (parameter estimation, state estimation, likelihood computation)
  - ML state estimation (Viterbi algorithm)
  - Forward backward algorithm and its use in parameter estimation and likelihood computation
  - Applications: Speech recognition, video-based gesture

recognition, Decoding convolutional codes

- Detection Theory
  - LRT, Generalized LRT and Marginalized-LRT for complex hypothesis,
  - MAP (Bayesian) detection
  - Sequence detection
  - Applications: bit sequence decoding, radar-based object detection, face/object/activity classification/recognition, change detection in sequences
- Monte Carlo methods
  - Importance sampling, Bayesian IS, Sequential IS.
    If time permits: Applications in optimization or in computation expectations (or other integrals), e.g. “optimal” state estimation.

Notation and Some Problem Formulations

- Notation:
  θ: parameter,
  $p(\underline{x};\theta)$: PDF of $\underline{x}$ with parameter(s) θ (correct: $f_{\underline{X}}(\underline{x};\theta)$),
  Conditional: $p(\underline{x}|\theta)$: PDF of $\underline{x}$ given θ (correct: $f_{\underline{X}|\Theta}(\underline{x}|\theta)$)
  $E_{\underline{x}}[.]$: expectation w.r.t. the PDF of $\underline{x}$
- Mean Squared Error:

$$\arg\min_{\hat{\theta}} E_x[(\hat{\theta}(x)-\theta)^2]$$

- Min Variance Unbiased Estimator:

$$\arg\min_{\hat{\theta}:E[\hat{\theta}(x)]=\theta} E_x[(\hat{\theta}(x)-\theta)^2]$$

- Bayesian MSE:

$$\arg\min_{\hat{\theta}} E_\theta[E_x[(\hat{\theta}(x) - \theta)^2]]$$

(typically MMSE estimation refers to Min Bayesian MSE)

- ML:

$$\arg\min_{\theta}[-\log p(\underline{x};\theta)]$$

- MAP:

$$\arg\min_{\theta}[-\log p(\theta|\underline{x};)] = \arg\min_{\theta}[-\log(p(\underline{x}|\theta)p(\theta))]$$

Background Required

- Basic undergraduate probability (at the level of EE 322)
  - PMF, PDF, Mean, Variance, Conditioning, Independence, PMF of functions of r.v.'s, Common distributions (particularly multivariate Gaussian), for vector random variables.
  - Will do a quick recap today or in next class.
- Vector calculus (Will recap wherever needed).
- Basic linear algebra
- Classes that use Estimation and Detection: Advanced Communications (521), Digital Image Processing (528)

Books

- S.M. Kay, Statistical Signal Processing, Volumes I and II
  - You should have a copy of Volume 1 (Estimation Theory)
- Vincent Poor, An Introduction to Signal Detection and Estimation
- Kailath, Hassibi and Sayed, Linear Estimation
- Other references will be posted.

Disability Accommodation

If you have a documented disability and anticipate needing accommodations in this course, please make arrangements to meet with me soon. You will need to provide documentation of your disability to Disability Resources (DR) office, located on the main floor of the Student Services Building, Room 1076 or call 515-294-7220.