Registration — Hough transform — To Do.
— Deformation

— Affine Registration

3D
X: configura. $K\times3$ matrix

$\|X\| = \sqrt{\mathrm{trace}(X^T X)}$

→ Points $\{x_j, y_j\}_{j=1}^{K} \longleftrightarrow \{x'_j, y'_j\}_{j=1}^{K}$

$I \qquad\qquad I'$

$$\begin{bmatrix} x'_j \\ y'_j \end{bmatrix} = \begin{bmatrix} x_j & y_j & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_j & y_j & 1 \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \end{bmatrix}$$

→ use LMSE to solve

→ can also use weighted LS or Regularized LS

→ Sequence of image: keep updating estimate
— use Recursive LS.

→ paraperspective camera motion

— Scaled Euclidean

→ weak perspective camera motion

→ use complex notation

$$c = \begin{bmatrix} x_1 + j y_1 \\ x_2 + j y_2 \\ \vdots \\ x_k + j y_k \end{bmatrix} \qquad c' = \begin{bmatrix} x'_1 + j y'_1 \\ \vdots \\ x'_k + j y'_k \end{bmatrix}$$

$$\hat{\Delta}, \hat{s}, \hat{\theta} = \arg\min_{\Delta, s, \theta} \| c' - (c - \Delta 1_k) s e^{j\theta} \|^2$$

$\Delta = \Delta_x + j\Delta_y$: translation

s = scale, θ : rotation

$\frac{(c' - 1_k c')}{k}$ -

$$\Delta = \frac{1_k^T}{k}(c - c'), \quad s = \frac{| c_{trans}^{*} c_{trans} |}{\| c_{trans} \|^2}$$

$$\theta = \arg(c_{trans}^{*} c'_{trans})$$

→ Set of frames: $c_1 \ c_2 \ \ldots \ c_N$.

- get mean shape & alignment parameters

∴ Generalized Procrustes Analysis.

→ Preshape, shape : Define

→ Tangent coordinates w.r.t. a $\underline{u}$

Pre-shapes $w_1 \ w_2 \ \ldots \ w_N$.

✓ Partial Proc tangent — $v_1 = [I - uu^*] \underbrace{w_1 e^{j\theta_1}}_{z_1(u)}$ $\quad \theta_1 = \arg(w_1^* u)$

✓ Partial Proc — Shape-1. $\arg\min_{\theta} \| w_i - w_j e^{j\theta} \|^2$.

Full Proc. — Shape-2 $\quad \arg\min_{s,\theta} \| w_i - w_j s e^{j\theta} \|^2$.

My Research: We do shape-1 ∴ then $(w_j e^{j\theta})$ is also a "pre-shape" vector - which can be used as the pole for next frame

— Euclidean matching ↔ Orthographic camera.
(Rigid in plane motion)

→ GPA: $u^* = \arg\min_{u,s,\theta} \sum_{i=1}^{n} \| w_i - u^* s e^{j\theta} \|^2$

Shape 2

$u^* = \arg\max_u \left[u^* \sum w_i w_i^* u \right]$

- largest eigen vector

→ Same for 3D: no closed form: iter. min.