Receiver-Based Channel Allocation for Wireless Cognitive Radio Mesh Networks

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Abstract—**Empowered by the cognitive radio technology and motivated by the sporadic channel utilization, both spatially and temporally, dynamic spectrum access networks, also referred to as cognitive radio networks, have emerged as a solution to improve spectrum utilization and provide more flexibility to wireless communication. In this paper, we study the channel allocation problem in wireless cognitive mesh networks. For the allocation to be feasible, served mesh clients must establish connectivity with a backbone network in both the upstream and the downstream directions, and must have the SINR (signal-to-interference and noise-ratio) of the uplink and the downlink with their parent mesh routers within a predetermined threshold. We propose a receiver-based channel allocation strategy and show that this strategy outperforms other strategies in terms of the number of mesh clients served, and the fact that no common control channel is needed for coordinating the communication process. Furthermore, we formulate the receiver-based channel allocation problem in wireless cognitive mesh network as a mixed integer linear program (MILP) and propose a heuristic solution.**

I. INTRODUCTION

Motivated by the significant variance in channel utilization both spatially and temporally, a new dynamic spectrum access/allocation communication paradigm has recently been devised [1]. Empowered by the emerging technology of cognitive radios [2], the new paradigm allows unlicensed wireless users (usually referred to as secondary users (SUs)) to opportunistically access portions of the spectrum that are licensed to some other users (usually referred to as primary users (PUs)) but are currently unused (vacant). However, whenever a PU is back to use its licensed channel(s), all SUs that are currently using this channel must vacate to avoid interference with the primary network. This new paradigm leads to better channel utilization and higher throughput and service reliability for SUs.

Although its potential appears promising, cognitive radio networking entails several challenges that are not present in traditional wireless networks. Such challenges include spectrum sensing, allocation, management, and sharing in addition to network coordination and legacy protocol compatibility.

The cognitive radio technology allows SUs to change their communication parameters like power, operating frequency, and modulation dynamically. Although it gives SUs more flexibility and adaptability, it makes the coordination of the communication process much more complicated. This complexity arises from the fact that SUs might be operating on different frequency channels at different times. This requires the communicating pair of SUs to negotiate their channel availability and decide on one channel for communication. But, the negotiation itself must take place over a common channel that is known to the communicating pair apriori; this channel is usually referred to as the *common control channel (CCC)*. At the network level, this *CCC* has to be common network wide to guarantee network operation. However, relying on a CCC has several drawbacks, like:

(1) Depending on the network size (area and number of nodes), the number and distribution of primary stations, and the pattern of primary channels usage, the probability of having a CCC might be very low [3].
(2) From a security point of view, a denial of service (DoS) attack that jams the CCC will break the network operation.
(3) Sharing one control channel between all SUs will lead to congestion on this channel which will consequently cause performance degradation for the overall network.

Recently, alternative solutions to the *CCC* approach have been proposed in literature, some of them are reviewed next.

a) Local control channels: Instead of a single control channel common to all SUs [4], Zhao et al. [3] proposed the use of local control channels (LCCs) each of which is common to only a group of SUs. Using this approach, SUs at group boundaries, i.e., those that have neighbors in two different groups, might have to use (listen to and transmit on) more than one control channel. Although this approach is better than the CCC approach, it has its own drawbacks. First, the jamming problem is not alleviated although the scale of its effect is reduced to a group-level rather than a network-level. Second, SUs at group boundaries need to be either equipped with multiple transceivers or keep switching a single transceiver to listen to multiple LCCs, as well as the data channel. This will result in an increase in inter-group communication delay and degradation in network throughput.

b) Selective broadcasting: another alternative solution was to broadcast control information either over all available channels [5] or over a small subset of channels which covers all the neighbours of a node [6]. In [6], each node transmits the control information on a selected group of channels instead of a single control channel and this is why the approach is called selective broadcasting. Similar to the previous solution, a node might have to listen to more than one control channel. This requires the channel activity operations (listening/transmitting) to be synchronized in order for a communicating pair of nodes to successfully exchange control information.

In this paper, we study the problem of channel allocation with QoS guarantees in cognitive wireless mesh networks [7] assuming that both mesh clients (MCs) and mesh routers (MRs) are cognitive nodes (i.e., employ cognitive radios) that have to use unutilized licensed spectrum. The objective of this study is to devise a channel allocation strategy that simplifies the coordination function of cognitive wireless mesh networks, i.e., alleviates the need of a CCC or LCCs, and optimizes the revenue in terms of number of MCs served. We show, in this paper, that the receiver-based channel allocation strategy (defined in Section III) is the best choice.

The rest of this paper is organized as follows. In Section II, we present the network model and layout the assumptions of this work. The receiver-based channel allocation strategy is presented in Section III. In Section IV, we study the complexity of the receiver-based channel allocation problem and propose a mixed integer linear program (MILP) formulations for different channel allocation strategies. In Section V, we propose a heuristic algorithm for the receiver-based channel allocation problem in wireless cognitive mesh networks. The performance of the proposed heuristic algorithm is evaluated in Section VI. We also evaluate the optimal performance of the proposed receiver-based channel allocation strategy versus other strategies in Section VI. We conclude in Section VII.

II. SYSTEM MODEL

In this section, we present the system model and assumptions and state the objectives of this work.

A. Assumptions

The general network structure consists of a number of MRs, some of them are directly connected to a backbone network, each of which manages all the MCs in its cell. This structure has the following properties:

- Each MC is associated with exactly one MR, and communication within the cell takes place in one hop.
- All MCs and MRs are equipped with cognitive radios, and they communicate with each other over unused licensed channels to reach the backbone network.
- We assume that a subset of MRs, that we call *gateways*, are directly connected to the backbone network. Therefore, each non-gateway MR should be able to reach at least one of the gateways in multiple hops of MRs in order to establish connectivity with the backbone network. From now on, we use the abbreviation *MR* to refer to a mesh router regardless of whether it is a gateway or not, and use the word *node* to refer to an SU (MR or MC).
- For each served MC, the reliability of its uplink ($MC \rightarrow MR$) and that of its downlink ($MR \rightarrow MC$) must meet a given QoS requirement (a threshold reliability).

Throughout this paper, we assume the following:

- The channel availability at a node (MC or MR) is quasi-static, i.e., the channel status does not change in a short period of time. Therefore, this work is more suited to spatial spectrum underutilization than temporal underutilization. However, it can still be used for the case of temporal underutilization if the PU activity is not very dynamic.
- A simple path loss model for channel attenuation.
- The reception and transmission circuits of the cognitive radio transceiver work on the same channel at any time.

Spectrum Sensing: In this work, we assume that the cognitive mesh network rely on an infrastructure sensor network for spectrum sensing. The sensor network provides unlicensed users in its range of operation with information about spectrum occupancy. This approach of spectrum sensing has been receiving an increasing attention in the last few years [8], [9], [10]. An example is the "SEnsor network for Dynamic cOgnitive Radio Access" (SENDORA); a project carried out by multiple institutions in Europe to develop a new approach to support coexistence between primary (licensed) and secondary (unlicensed) wireless users in the same area with the help of a sensor networks [11]. This approach frees the secondary network from the spectrum sensing task, and makes it the responsibility of a cooperative sensor network which can be specially designed to achieve high sensing accuracy. Therefore, SUs do not need to have a CCC for the sake of cooperative channel sensing, yet it is sill needed for communication coordination. Moreover, a central processing unit in the secondary network can exploit the sensor network to acquire information about channel occupancy at particular locations allowing for centralized solutions to be considered.

The solutions proposed in this paper are centralized, and are carried out by one of the gateway MRs with the help of the infrastructure sensor network for collecting the input information and disseminating the outcome.

B. Objective

Our objective now is to evaluate the performance of different channel allocation strategies for cognitive radio mesh networks. For any allocation strategy to be feasible, the following two conditions must be satisfied for all served MCs.

(1) A path from the MC to at least one gateway must exist; we call this the *upstream connectivity constraint*. Also, a path from at least one gateway to the MC must exist; we call this the *downstream connectivity constraint*. The two paths may be disjoint, and the gateways may be different.

(2) Potential interference caused by intra-cell communication from cells other than the parent cell of an MC must be bounded to achieve a predetermined SINR to guarantee a BER (bit error rate) QoS requirement.

We aim at finding the best channel allocation strategy that satisfies these conditions for the maximum number of MCs.

III. RECEIVER-BASED CHANNEL ALLOCATION

Based on the joint temporal and spatial distribution of the availability of the licensed spectrum, different SUs might observe different sets of available channels. Therefore, four modes of operation can be defined for each node's transceiver:

1) *Tunable Transmitter - Tunable Receiver (TT-TR)*: an SU can transmit/receive on any of the available channels.

2) *Tunable Transmitter - Fixed Receiver (TT-FR)*: an SU can transmit on any of the available channels, but receives on a fixed channel.
3) *Fixed Transmitter - Tunable Receiver (FT-TR)*: an SU can receive on any of the available channels, but transmits on a fixed channel.
4) *Fixed Transmitter - Fixed Receiver (FT-FR)*: an SU transmits/receives on a fixed channel.

TT-TR is the most commonly assumed communication paradigm in multihop cognitive radio networks. It allows an SU to use any of its available channels for transmission and/or reception. Therefore, the channel allocation problem under this paradigm will be to assign channels to links. This means that a node might use different channels for its incoming and outgoing communication links with its neighbors. The drawback of this paradigm, which is shared with the *FT-TR* paradigm, is that a CCC is needed for channel negotiation. In other words, these two communication paradigms cannot be used without a CCC because the transmitter needs to inform the receiver about its intention to transmit so that the receiver can tune to the transmitter's channel. Because of the problems of the CCC approach discussed earlier in Section I, and the fact that the probability that a CCC exists could be low [3], it is a necessity to devise a new communication paradigm in which the requirement of a CCC is avoided. In this paper, we propose a channel allocation strategy based on the *TT-FR* paradigm, we call this strategy a *receiver-based channel allocation (RBA)*. Based on this allocation strategy, each node (MC or MR) is allocated a fixed channel to receive on, but it is allowed to transmit on different channels. Therefore, if each node knows the channels allocated to its neighbors, no channel negotiation is needed, and consequently no CCC is needed as well for this purpose. *To be specific, if node A wants to communicate with node B, where B is assigned channel f_B, then node A must have f_B among its list of available channels. If so, node A tunes its transceiver to f_B and initiates communication with B according to the MAC mechanism used. Then, communication takes place on f_B.*

We do not consider the $FT-FR$ mode in this study because it is a special case of the $TT-FR$ that has advantage of not requiring a CCC, but the disadvantage of limited connectivity.

A. Issues and challenges

In this subsection, we would like to emphasize on some issues and challenges related to the proposed RBA approach.

a) Degree of connectivity: it is expected that the assignment of one channel for each node to receive on will result in a decreased level of connectivity. One might think that the *TT-TR* approach will result in the highest degree of connectivity, because transmission and reception are allowed on any channel, which intuitively implies that the number of served MCs will be higher compared to *TT-FR*. However, this is not necessarily true because of the following:

- The *TT-TR* approach requires the existence of a CCC, while *TT-FR* does not, and the probability that the network is connected depends on the probability that a CCC exists.
- The channel used as a CCC cannot be used for data communication. This gives the *TT-FR* one more channel to use for data communication.
- The effect on connectivity depends on the channel availability distribution at a node and its neighbors. It is more likely for a channel available at a particular node to be available at its neighbors, which lowers the likelihood that a node gets disconnected from its neighbors when it is assigned a fixed channel to receive on.

b) Deafness Problem: the deafness problem is recognized in wireless networks with directional antennas [12]. Deafness is caused when node A wants to communicate with node B while B is currently communicating with node C. Node A translates the absence of a reply from B (caused by the fact that B's antenna is tuned to the direction of node C) as a collision at B, and consequently backs-off (this is based on CSMA/CA medium access). The problem becomes worse if B has multiple packets to transmit, which will cause A to unnecessarily back-off several times. The same problem may occur under the proposed RBA approach. Let f_A, f_B, and f_C be the frequency channels assigned to nodes A, B, and C for reception respectively . Assume that node A wants to communicate with node B which is currently communicating with node C on channel f_C. Then, A will fail to reach B on f_B because B is currently tuned to f_C, and unnecessarily backs-off, i.e., node A, which results in the same deafness problem recognized in the directional antennas case. This problem is not present in the *TT-TR* and *FT-TR* approaches because the neighbors of the transmitter will know about the ongoing communication by overhearing the control information transmitted on the CCC.

c) Multi-channel hidden node problem: the last challenge is the hidden node problem. This problem is well known in wireless communication, however, it is a bit different using the *TT-FR* approach. In traditional single-channel wireless networks, under an IEEE 802.11 based MAC protocol, an RTS/CTS handshake between the transmitter and the receiver solves the hidden node problem. However, following the *TT-FR* mode, hidden nodes may exist on multiple channels. Thus, RTS/CTS handshake must be cloned on multiple channels, resulting in significant delay with single radio interface.

B. Medium access mechanism

Although designing a MAC protocol is beyond the scope of this paper, we sketch a simple hybrid TDMA-CSMA/CA mechanism (not necessarily the most efficient one) for the sake of showing the viability of the proposed RBA approach. The basic idea is to assign each SU a time slot (the length of which should be carefully designed to guarantee certain success probability) during which it acts as a receiver only. At the beginning of the slot assigned to a particular SU, say i, all other SUs that want to communicate with i must tune to the channel assigned to i. Then, they contend to gain access to that channel using the traditional CSMA/CA with RTS/CTS handshake. This will overcome both the multi-channel hidden node and the deafness problems.

IV. MILP FORMULATIONS FOR THE CHANNEL ALLOCATION PROBLEM

Before giving a formal definition for the receiver-based allocation, we present some notations and terminology.

- $\mathcal{B}$ is the set of non-gateway MRs .
- $\mathcal{G}$ is the set of gateways.
- $\mathcal{A}_i$ is the set of MCs that belong to the cell administrated by MR i. $\mathcal{A} = \bigcup_{i\in\mathcal{B}\cup\mathcal{G}} \mathcal{A}_i$.
- $\mathcal{L}_i$ is the set of available channels at node i obtained from the sensor network. Let $\mathcal{L}$ be the set of all available (orthogonal) channels in the system such that $|\mathcal{L}|{=}K$. Moreover, $\mathcal{L}_j{\subseteq}\mathcal{L}_i\ \forall j{\in}\mathcal{A}_i$ because an MC cannot use a channel that is not available to its parent MR.
- P_i^k is the transmission power of node i on channel k such that $P_i^k \leq P_r^{max}\ \forall i \in \mathcal{B}\cup\mathcal{G}$, and $P_i^k \leq P_c^{max}\ \forall i \in \mathcal{A}$. P_r^{max} and P_c^{max} are the maximum transmission powers of an MR and an MC respectively.
- Ψ_{ij}^k is the channel power gain from i to j on channel k.
- ζ_{ij}^k is the maximum interference that the cell managed by MR i may produce at the location of node j on channel k.

$$\zeta_{ij}^k = \max\{\max_{w\in\mathcal{A}_i} P_w^k\Psi_{wj}^k, P_i^k\Psi_{ij}^k\} \tag{1}$$

- ζ_{ij}^{max} is the maximum interference that the cell managed by MR i produces at the location of node j at maximum transmission power, i.e.,

$$\zeta_{ij}^{max} = \max\{\max_{w\in\mathcal{A}_i}\max_{k\in\mathcal{L}_w} P_c^{max}\Psi_{wj}^k, \max_{k\in\mathcal{L}_i} P_r^{max}\Psi_{ij}^k\} \tag{2}$$

- N_0 is the channel noise power, and it is assumed to be the same at all locations on all channels.
- γ is the minimum SINR value required to guarantee a certain BER at a node (reliability threshold).
- c_j^k is a binary variable that is set to 1 if channel k is assigned to node j, and 0 otherwise.

A. Receiver-based channel allocation (RBA) problem

The receiver-based channel allocation (RBA) problem in wireless cognitive mesh networks is defined as follows:

***Definition** 4.1: RBA problem*: given a wireless cognitive mesh network of $\mathcal{G}$ gateway MRs, $\mathcal{B}$ non-gateway MRs, and $\mathcal{A}_i$ MCs managed by MR i for all $i \in \mathcal{B}\cup\mathcal{G}$. Also, for all $j \in \mathcal{B}\cup\mathcal{G}\cup\mathcal{A}$, the geographic location of j and its channel availability $\mathcal{L}_j$ are given. Find a $TT{-}FR$ channel allocation that maximizes the number of served MCs such that for each served MC, the following conditions are satisfied: (1) A path from each MC (through its parent MR) to at least one MR in $\mathcal{G}$ exists. (2) A path from at least one MR in $\mathcal{G}$ to each MC (through its parent MR) exists. (3) The SINR of the uplinks ($MC \rightarrow MR$) and the downlinks ($MR \rightarrow MC$) is at least γ.

Note that the upstream and downstream paths for an MC must go through its parent MR. Therefore, the RBA problem be decomposed into two subproblems: (1) *channel allocation to MRs such that the upstream/downstream connectivity constrain is satisfied for MRs.* (2) *channel allocation to MCs such that reliable uplinks/downlinks with MRs are established for the maximum number of MCs.* The first subproblem can be represented as a network flow formulation as we show throughout this subsection. By adding few more constraints to jointly model the second subproblem, the whole RBA problem can then be formulated as an MILP. To show the complexity of the RBA problem, let us just consider the upstream/downstream connectivity subproblem.

***Definition** 4.2: Upstream/downstream connectivity problem (UDCP)*: given the network of MRs as a graph $G(\mathcal{B}\cup\mathcal{G}, E)$, where E is the set of connectivity edges between MRs, and the channel availability at each MR ($\mathcal{L}_i \forall i \in \mathcal{B}\cup\mathcal{G}$). Find a receiver-based channel assignment that maximizes the number of non-gateway MRs which are upstream and downstream connected with the gateway MR.

The *UDCP* problem can be shown to be NP-hard by a reduction from the *MAXIMUM k-SATISFIABILITY* problem. As the *UDCP* is NP-hard, the RBA problem is also NP-hard because the former is a special case of the latter.

Let us start with the network flow formulation for the first subproblem, i.e., upstream/downstream connectivity. Define a graph $G{=}(V, E{\cup}\overline{E})$ of a set of vertices $V{=}\mathcal{B}{\cup}\mathcal{G}{\cup}\{s, \overline{s}, d, \overline{d}\}$ and a set of edges $E{\cup}\overline{E}$. The vertices s and d represent a hypothetical source and hypothetical sink for the upstream flow respectively. On the other hand, $\overline{s}$ and $\overline{d}$ represent a hypothetical source and a hypothetical sink for the downstream flow respectively. E and $\overline{E}$ are the sets of upstream and downstream edges respectively. The set E is defined as follows:

- A directed edge $e = (s, j)$ exists for each vertex $j \in \mathcal{B}{\cup}\mathcal{G}$. The flow on such an edge is equal to the number of served MCs that belong to $\mathcal{A}_j$, i.e., $\sum_{i\in\mathcal{A}_j}\sum_{k\in\mathcal{L}_i} c_i^k$.
- A directed edge $e = (i, d)$ exists for each vertex $i \in \mathcal{G}$.
- A directed edge $e{=}(i, j)$ exists for any pair of MRs $i, j{\in}\mathcal{B}{\cup}\mathcal{G}$ if condition (3) is satisfied for at least one channel $k{\in}\mathcal{L}_i{\cap}\mathcal{L}_j$, where P_{th} is a threshold received signal strength requirement to detect the transmission.

$$\Psi_{ij}^k P_r^{max} \geq P_{th}, \tag{3}$$

The capacity of such an edge is given as $|\mathcal{A}|\cdot\sum_{k\in\mathcal{L}_i\cap\mathcal{L}_j} c_j^k$, i.e., the end-node j must be assigned a channel that belongs to the list of available channels at the start-node i in order for a flow upper-bounded by $|\mathcal{A}|$ to pass through e.

On the other hand, the set $\overline{E}$ is defined as follows:

- A directed edge $e = (\overline{s}, j)$ exists for each vertex $j \in \mathcal{G}$.
- A directed edge $e = (i, \overline{d})$ exists for each vertex $i \in \mathcal{B}{\cup}\mathcal{G}$. The flow on such an edge is equal to the number of served MCs that belong to $\mathcal{A}_i$, i.e., $\sum_{j\in\mathcal{A}_i}\sum_{k\in\mathcal{L}_j} c_j^k$.
- A directed edge $e{=}(i, j)$ exists for any pair of MRs $i, j{\in}\mathcal{B}{\cup}\mathcal{G}$ if condition (3) is satisfied for at least one channel $k{\in}\mathcal{L}_i{\cap}\mathcal{L}_j$. The capacity of e is $|\mathcal{A}|\cdot\sum_{k\in\mathcal{L}_i\cap\mathcal{L}_j} c_j^k$.

We define two flow commodities; *upstream flow* and *downstream flow* passing through edges in E and $\bar{E}$) respectively. Let f_{ij} denote the *upstream* flow on edge $(i, j){\in}E$, and g_{ij} denote the *downstream* flow on edge $(i, j){\in}\bar{E}$. The network flow representation is shown in Figure 1.

Let us now consider the second subproblem we mentioned earlier, i.e., channel allocation to MCs. First of all, it is of

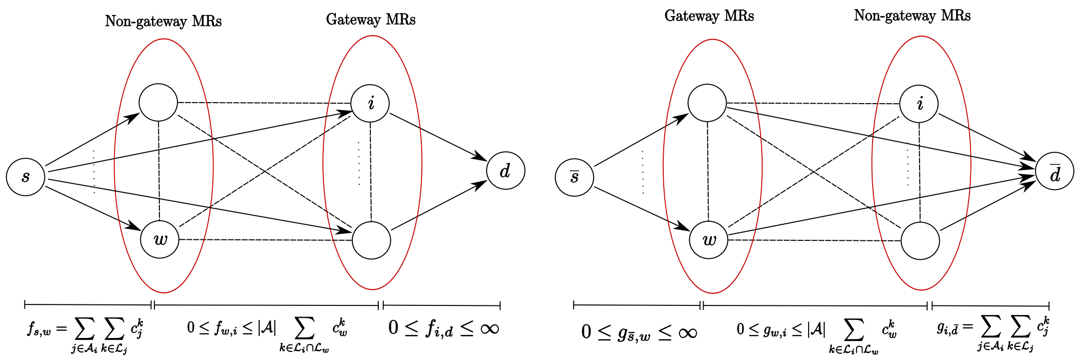

Fig. 1. The network flow representation of the upstream (left) and the downstream (right) connectivity. Bounds on the flow on the three groups of edges (from source to MRs, between MRs, and from MRs to destination) are shown below the graph drawing.

no benefit to assign a channel to an MC j unless its parent MR, say i, is assigned one that is common between the two, otherwise, MC j will not be able to access its parent MR i which means that j cannot be served. Therefore,

$$\sum_{k\in\mathcal{L}_j} c_j^k \le \sum_{k\in\mathcal{L}_i\cap\mathcal{L}_j} c_i^k,\ \forall\ i \in \mathcal{B}\cup\mathcal{G},\ j \in \mathcal{A}_i$$

Then, the downlink (from an MR to an MC) reliability is achieved if the following inequality is satisfied:

$$\begin{aligned}&\Psi_{ij}^k P_i^k - \gamma\left(N_0 + \textstyle\sum_{m\in\mathcal{B}\cup\mathcal{G}\setminus\{i\}} \zeta_{mj}^k\right) \ge\\ &\gamma(c_j^k - 1)\left(N_0 + \textstyle\sum_{m\in\mathcal{B}\cup\mathcal{G}\setminus\{i\}} \zeta_{mj}^{max}\right),\\ &\forall i \in \mathcal{B}\cup\mathcal{G}, j \in \mathcal{A}_i, k \in \mathcal{L}_i\cap\mathcal{L}_j\end{aligned} \tag{4}$$

Note that if MC j is assigned channel k, then the right hand side is equal to 0 and P_i^k must be set to a value that satisfies the required SINR threshold γ at MC j. On the other hand, if MC j is not assigned channel k, i.e., $c_j^k = 0$, then the inequality is satisfied for any positive (≥ 0) value of P_i^k because the term between parentheses in the right-hand side is an upper bound on the term between parentheses in the left-hand side.

Similarly, the uplink (from an MC to an MR) reliability is achieved if the following inequality is satisfied:

$$\begin{aligned}&\Psi_{ji}^k P_j^k - \gamma\left(N_0 + \textstyle\sum_{m\in\mathcal{B}\cup\mathcal{G}\setminus\{i\}} \zeta_{mi}^k\right) \ge\\ &\gamma(c_i^k + \textstyle\sum_{w\in\mathcal{L}_j} c_j^w - 2)\left(N_0 + \textstyle\sum_{m\in\mathcal{B}\cup\mathcal{G}\setminus\{i\}} \zeta_{mi}^{max}\right),\\ &\forall i \in \mathcal{B}\cup\mathcal{G}, j \in \mathcal{A}_i, k \in \mathcal{L}_i\cap\mathcal{L}_j\end{aligned} \tag{5}$$

The right hand side is equal to 0 if MR i assigned channel k and the MC is assigned a channel. In this case, P_j^k must be set to a value that satisfies the required SINR threshold γ at MR i. On the other hand, if channel k is not assigned to MR i or MC j is not assigned a channel, then the inequality is satisfied for any positive (≥ 0) value of P_j^k using the same argument as before. Finally, the *RBA* problem can be formulated as an MILP as follows:

Maximize $\sum_{i\in\mathcal{A}}\sum_{k\in\mathcal{L}_i} c_i^k$, subject to:

(a) Channel assignment:

$$\sum_{k\in\mathcal{L}_j} c_j^k \le 1,\ \forall\ j \in \mathcal{B}\cup\mathcal{G}\cup\mathcal{A}. \tag{6}$$

$$\sum_{k\in\mathcal{L}_j} c_j^k \le \sum_{k\in\mathcal{L}_i} c_i^k,\ \forall\ i \in \mathcal{B}\cup\mathcal{G},\ j \in \mathcal{A}_i \tag{7}$$

(b) Upstream connectivity constraints:

$$\sum_{j:(i,j)\in E} f_{ij} - \sum_{j:(j,i)\in E} f_{ji} = 0,\ i \in \mathcal{B}\cup\mathcal{G}. \tag{8}$$

$$\sum_{j:(s,j)\in E} f_{sj} = \sum_{j:(j,d)\in E} f_{jd} \tag{9}$$

$$f_{sj} = \sum_{i\in\mathcal{A}_j}\sum_{k\in\mathcal{L}_i} c_i^k,\ j \in \mathcal{B}\cup\mathcal{G}. \tag{10}$$

$$f_{ij} \le |\mathcal{A}| \cdot \sum_{k\in\mathcal{L}_i\cap\mathcal{L}_j} c_j^k,\quad (i,j) \in E. \tag{11}$$

(c) Downstream connectivity constraints:

$$\sum_{j:(i,j)\in\overline{E}} g_{ij} - \sum_{j:(j,i)\in\overline{E}} g_{ji} = 0,\ i \in \mathcal{B}\cup\mathcal{G}. \tag{12}$$

$$\sum_{j:(\overline{s},j)\in\overline{E}} g_{\overline{s}j} = \sum_{j:(j,\overline{d})\in\overline{E}} g_{j\overline{d}} \tag{13}$$

$$g_{j\overline{d}} = \sum_{i\in\mathcal{A}_j}\sum_{k\in\mathcal{L}_i} c_i^k,\ j \in \mathcal{B}\cup\mathcal{G}. \tag{14}$$

$$g_{ij} \le |\mathcal{A}| \cdot \sum_{k\in\mathcal{L}_i\cap\mathcal{L}_j} c_j^k,\quad (i,j) \in \overline{E}. \tag{15}$$

(d) Power control constraints:

$$P_i^k \le P_r^{max} \cdot \sum_{j:\{j\in\mathcal{A}_i, k\in\mathcal{L}_j\}} c_j^k,\ \forall\ i\in\mathcal{B}\cup\mathcal{G}, k\in\mathcal{L}_i \tag{16}$$

$$P_i^k \le P_r^{max},\ \forall\ i \in \mathcal{B}\cup\mathcal{G}, k \in \mathcal{L}_i \tag{17}$$

$$P_j^k \le P_c^{max} \cdot c_i^k,\ \forall\ i\in\mathcal{B}\cup\mathcal{G}, j\in\mathcal{A}_i,\ k\in\mathcal{L}_i\cap\mathcal{L}_j \tag{18}$$

(e) Inter-cell interference:

$$\zeta_{ij}^k \ge P_i^k \Psi_{ij}^k,\quad \forall i\in\mathcal{B}\cup\mathcal{G},\ j\in\mathcal{A}\cup\mathcal{B}\cup\mathcal{G}\setminus(\mathcal{A}_i\cup\{i\}), k\in\mathcal{L}_i\cap\mathcal{L}_j. \tag{19}$$

$$\zeta_{ij}^k \ge P_m^k \Psi_{mj}^k,\quad \forall i\in\mathcal{B}\cup\mathcal{G},\ j\in\mathcal{A}\cup\mathcal{B}\cup\mathcal{G}\setminus(\mathcal{A}_i\cup\{i\}),\ m\in\mathcal{A}_i,\ k\in\mathcal{L}_m\cap\mathcal{L}_j. \tag{20}$$

(f) Link reliability constraints:

$$\Psi_{ij}^{k} P_i^k - \gamma \left(N_0 + \sum_{m \in \mathcal{B} \cup \mathcal{G} \backslash \{i\}} \zeta_{mj}^{k} \right) \geq \gamma (c_j^k - 1) \left(N_0 + \sum_{m \in \mathcal{B} \cup \mathcal{G} \backslash \{i\}} \zeta_{mj}^{max} \right), \quad (21)$$

$$\forall i \in \mathcal{B} \cup \mathcal{G}, j \in \mathcal{A}_i, k \in \mathcal{L}_i \cap \mathcal{L}_j$$

$$\Psi_{ji}^{k} P_j^k - \gamma \left(N_0 + \sum_{m \in \mathcal{B} \cup \mathcal{G} \backslash \{i\}} \zeta_{mi}^{k} \right) \geq \gamma (c_i^k + \sum_{w \in \mathcal{L}_j} c_j^w - 2) \left(N_0 + \sum_{m \in \mathcal{B} \cup \mathcal{G} \backslash \{i\}} \zeta_{mi}^{max} \right), \quad (22)$$

$$\forall i \in \mathcal{B} \cup \mathcal{G}, j \in \mathcal{A}_i, k \in \mathcal{L}_i \cap \mathcal{L}_j$$

The above two equations are the same as equations (4) and (5), and are repeated here for clarity.

B. Transmitter-based channel allocation (TBA)

We refer to the channel allocation strategy that follows the *FT-TR* mode as the *transmitter-based channel allocation* (TBA). In TBA, a node (MC or MR) is assigned a fixed channel to transmit on, while it can receive on any of the channels available to it. As explained earlier, this strategy requires the existence of a CCC so that the transmitter node can make the receiver node tune to its channel. Therefore, we study this allocation strategy under two assumptions: first, a preassumed CCC exists; second, the existence of a CCC depends on channel availability and it is not preassumed. The *TBA problem* is defined similar to the *RBA problem* except that the $FT-TR$ is used instead of the $TT-FR$ approach. The MILP formulation of this problem is similar to that of the RBA problem, but with a little change to capture the allocation policy (i.e., transmitter-based instead of receiver-based). Therefore, we do not present the MILP formulation of the TBA problem in this paper.

C. All-tunable channel allocation (ATA)

Following the *TT-TR* mode, we propose the *All-tunable channel allocation* (ATA) strategy, under which channels are assigned to links rather than nodes. Therefore, an MR might have to listen/transmit on different channels. As for the MC, it will have to receive on one channel (the one assigned to the downlink from the MR to the MC), and transmit on one channel (the one assigned to the uplink from the MC to the MR). We also study this allocation strategy under two assumptions: first, a preassumed CCC exists; second, the existence of a CCC depends on channel availability and it is not preassumed. The *ATA problem* is defined similar to the *RBA problem* except that *TT-TR* mode is used. The MILP formulation of this problem is also similar to that of the RBA problem, but with some changes to capture the all-tunable-allocation policy instead of the receiver-based policy. Thus, the MILP formulation of the ATA problem is not presented in this paper as well. Table I summarizes the differences between the three strategies.

TABLE I
COMPARISON BETWEEN THE RBA, TBA, AND ATA STRATEGIES.

	RBA		TBA		ATA	
	Rx Ch.	Tx Ch.	Rx Ch.	Tx Ch.	Rx Ch.	Tx Ch.
MR	fixed	tunable	tunable	fixed	tunable	tunable
MC	fixed	fixed	fixed	fixed	fixed	fixed
CCC	No		Yes		Yes	

V. HEURISTIC SOLUTION FOR RBA

As our results (presented in Section VI) imply the superiority of RBA strategy over other allocation strategies, we propose in this section a heuristic solution for the RBA problem. We solve the problem in three phases:

(1) *Channel assignment to MRs*: in this phase, MRs are assigned channels such that their upstream and downstream connectivity with the gateway(s) is maintained.

(2) *Finding the maximum number of reliable uplinks*: based on the channel assignment made in the first phase, and the channel availability at each MC, we assign transmission powers to MCs such that the number of reliable uplinks is maximized. This power assignment is achieved using the power control algorithm proposed in Section V-B.

(3) *Channel assignment to MCs*: to the MCs that have reliable uplinks after phase (2), we allocate channels and transmission powers such that the number of reliably served MCs is maximized. An MC is reliably served if the reliability of its uplink and downlink is at least γ.

A. Phase 1: Channel assignment to MRs

The first phase in our solution to the RBA problem is to allocate channels to MRs such that the upstream and downstream connectivity with the gateway is established for the maximum number of MRs. Let us start with some definitions:

- $\mathbf{L}_{(\mathbf{t})}$ is an $N_r \times K$ matrix where N_r is the total number of MRs, and K is the total number of channels available in the system. This matrix represents channels that an MR can transmit on, hence the $(\mathbf{t})$ is the subscript of $\mathbf{L}$. Thus, the $(i,k)^{th}$ element of $\mathbf{L}_{(\mathbf{t})}$ is defined as,

$$\mathbf{L}_{(\mathbf{t})}[i,k] = \begin{cases} 1 & \text{if } k \in \mathcal{L}_i \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

- $\mathbf{L}_{(\mathbf{r})}$ is an $N_r \times K$ matrix that represents the channels that an MR can receive on. Although this matrix is initially the same as $\mathbf{L}_{(\mathbf{t})}$, it will become different when MRs are assigned channels to receive on as we will see later. The $(i,k)^{th}$ element of $\mathbf{L}_{(\mathbf{r})}$ is initially defined as,

$$\mathbf{L}_{(\mathbf{r})}[i,k] = \begin{cases} 1 & \text{if } k \in \mathcal{L}_i \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

- $\mathbf{I}$ is an $N_r \times N_r$ matrix that represents the accessibility between MRs. In other words,

$$\mathbf{I}[m,n]=\begin{cases} 1 & \text{if } \exists k\in\mathcal{L}_m\cap\mathcal{L}_n:\Psi^k_{mn}P_r^{max}\geq P_{th} \\ 0 & \text{otherwise} \end{cases} \qquad (25)$$

- $\mathbf{W}$ is an $N_r \times K$ matrix that represents the weights of assigning channels to MRs. The element $\mathbf{W}[i,k]$ is the weight of assigning channel k to MR i defined as the number of MCs in $\mathcal{A}_i$ that can access i on channel k.
- $\mathbf{C}_{(u)}$ is a row-vector of length N_r such that $\mathbf{C}_{(u)}[i] = 1$ if there exists a directed path from MR i to the gateway, and equals 0 otherwise. This connectivity is evaluated under the assumption that node i is assigned all the channels that have their values in the row-vector $\mathbf{L}_{(r)}[i,*]$ set to 1. In this work, we assume that a single gateway exists in the system. However, the proposed algorithm can be easily extended to the case of multiple gateways by assuming a hypothetical gateway that has all the channels available and is connected to all the actual gateways.
- $\mathbf{C}_{(d)}$ is a row-vector of length N_r such that $\mathbf{C}_{(d)}[i] = 1$ if there exists a directed path from the gateway to MR i, and equals 0 otherwise. Similar to $\mathbf{C}_{(u)}$, this connectivity is evaluated under the assumption that node i is assigned all the channels that have their values in the row-vector $\mathbf{L}_{(r)}[i,*]$ set to 1.
- Define:

$$\underset{x}{\operatorname{argmax}}\ (f(x), g(x)) := \quad \{x \mid \forall y : f(y) \leq f(x), \text{ and if } f(y) = f(x) \text{ then } g(y) \leq g(x)\}$$

The *Routers Channel Allocation* (RCA) algorithm is outlined in Algorithm 1. First, the matrices $\mathbf{L}_{(t)}$ and $\mathbf{L}_{(r)}$ are calculated using equations (23) and (24) respectively. These two matrices are then used to evaluate the upstream and downstream connectivity vectors $\mathbf{C}_{(u)}$ and $\mathbf{C}_{(d)}$ respectively (line 2). The algorithm, then, operates iteratively and selects one MR at each iteration for processing. The gateway is selected first, then MRs are selected in breadth first manner based on their connectivity with already processed MRs. Let MR i be the one selected at the current iteration. If there exists a subset of channels $\mathcal{L}_i^* \subseteq \mathcal{L}_i$ such that each channel of which preserves (if assigned alone to MR i) the connectivity in $\mathbf{C}_{(u)}$ and $\mathbf{C}_{(d)}$ (lines 6-11), then MR i must be assigned a channel from $\mathcal{L}_i^*$. From $\mathcal{L}_i^*$, channels that were not assigned to adjacent cells are preferred over other channels. The channel $\hat{k}$ with the maximum weight, i.e., $\mathbf{W}[i,\hat{k}]$, is selected, and ties are broken based on the number of MRs that can access MR i on $\hat{k}$.

If subset $\mathcal{L}_i^*$ is empty (which means that there is no channel that if assigned to MR i, the connectivity will be preserved), then the channel that allows the maximum number of neighboring MRs to access MR i is selected (lines 12-13).

After allocating a channel to MR i, the connectivity vectors $\mathbf{C}_{(u)}$ and $\mathbf{C}_{(d)}$ are updated. Also, all the MCs that belong to $\mathcal{A}_i$ and cannot access MR i on the selected channel $\hat{k}$ must be removed. Moreover, all MCs that have their parent MRs disconnected from the backbone network (either in the upstream or the downstream direction) must be removed.

Algorithm 1: Routers Channel Allocation (RCA)

```
   input  : $\mathcal{T} = \mathcal{B} \cup \mathcal{G}$, $\mathcal{L}_i\ \forall\ i \in \mathcal{B} \cup \mathcal{G}$
   output: channel assignment matrix $\mathbf{L}_{(\mathbf{r})}$.
1  Calculate $\mathbf{L}_{(\mathbf{t})}$ and $\mathbf{L}_{(\mathbf{r})}$ using (23) and (24) respectively;
2  Evaluate $\mathbf{C}_{(u)}$ and $\mathbf{C}_{(d)}$;
3  repeat
4      Pick up an MR $i$ from $\mathcal{T}$;
5      Let $\mathcal{L}_i^* \subseteq \mathcal{L}_i$ be the subset of channels from $\mathcal{L}_i$ such
       that if any of these channels is assigned to MR $i$, the
       connectivity in $\mathbf{C}_{(u)}$ and $\mathbf{C}_{(d)}$ will be preserved;
6      if $\mathcal{L}_i^* \neq \emptyset$ then
7          Let $\mathcal{S} \subseteq \mathcal{L}_i^*$ be the subset of channels from $\mathcal{L}_i^*$
           that are not assigned to any MR of the cells
           adjacent to cell $i$;
8          if $\mathcal{S} \neq \emptyset$ then
9              $\hat{k} = \underset{k\in\mathcal{S}}{\operatorname{argmax}}\ (\mathbf{W}[i,k], \sum_{j=0}^{N_r} \mathbf{I}[i,j] \cdot \mathbf{L}_{(\mathbf{t})}[j,k])$
10         else
11             $\hat{k} = \underset{k\in\mathcal{L}_i^*}{\operatorname{argmax}}\ (\mathbf{W}[i,k], \sum_{j=0}^{N_r} \mathbf{I}[i,j] \cdot \mathbf{L}_{(\mathbf{t})}[j,k])$;
12     else
13         $\hat{k} = \underset{k\in\mathcal{L}_i}{\operatorname{argmax}}\ (\mathbf{I}[i,*] \times \mathbf{L}_{(\mathbf{t})}[*,k])$;
14     $\mathbf{L}_{(r)}[i,*] = \overrightarrow{1}_{\hat{k}}$;
15     Update $\mathbf{C}_{(u)}$ and $\mathbf{C}_{(d)}$;
16     forall $w \in \mathcal{B}$ do
17         if $\mathbf{C}_{(u)}[w] = 0$ or $\mathbf{C}_{(d)}[w] = 0$ then
18             $\mathcal{A}_w = \emptyset$;
19     forall $j \in \mathcal{A}_i$ do
20         if $\hat{k} \notin \mathcal{L}_j$ then
21             $\mathcal{A}_i = \mathcal{A}_i \backslash \{j\}$;
22     $\mathcal{T} = \mathcal{T} \backslash \{i\}$;
23 until $\mathcal{T} = \emptyset$ ;
24 return $\mathbf{L}_{(\mathbf{r})}$;
```

B. Phase 2: Finding the maximum number of reliable uplinks

Before we move into the second phase of our solution strategy, we propose a power control algorithm (PCA). The PCA algorithm takes as an input two sets of links: a set of uplinks $\mathcal{Q}_u(k)$ and a set of downlinks $\mathcal{Q}_d(k)$, as well as the channel k on which those links operate. If there exists a power allocation for all links' transmitters such that the SINR at all links' receivers is at least γ, then the algorithm returns 1, otherwise it returns 0. To test the existence of a feasible power allocation (one that achieves the reliability of all links), we propose a simple linear programming (LP) formulation that aims at finding any feasible solution, i.e., no optimization objective. The LP is outlined in Algorithm 2. The first and the second constraints correspond to the interference caused by active cells at the receivers in other active cells (similar to

Algorithm 2: Power control Algorithm (PCA)

```
input : $\mathcal{Q}_u(k)$, $\mathcal{Q}_d(k)$, channel $k$
output: An integer in $\{0, 1\}$.
// Find the set of active, being an uplink receiver or a downlink transmitter for at least one link in $\mathcal{Q}_u(k) \cup \mathcal{Q}_d(k)$, MRs $\bar{\mathcal{B}}$.
$\bar{\mathcal{B}} := \{i : \exists e = (i, j) \in \mathcal{Q}_d(k) \mid e = (j, i) \in \mathcal{Q}_u(k)\}$;
// For each MR i, find the subset $\bar{\mathcal{A}}_i \subset \mathcal{Q}_u(k) \cup \mathcal{Q}_d(k)$ of links that do not belong to the cell managed by i.
$\bar{\mathcal{A}}_i := \{e \in \mathcal{Q}_u(k) : r(e) \neq i\} \cup \{e \in \mathcal{Q}_d(k) : t(e) \neq i\}$;
Solve the following LP:
    Maximize 0, subject to:
    $\Psi^k_{t(e_1),r(e_2)} P^k_{t(e_1)} \leq \zeta^k_{r(e_1),r(e_2)},\ \forall e_1 \in \mathcal{Q}_u(k), e_2 \in \bar{\mathcal{A}}_{r(e_1)}$;
    $\Psi^k_{t(e_1),r(e_2)} P^k_{t(e_1)} \leq \zeta^k_{t(e_1),r(e_2)},\ \forall e_1 \in \mathcal{Q}_d(k), e_2 \in \bar{\mathcal{A}}_{t(e_1)}$;
    $\Psi^k_{t(e),r(e)} P^k_{t(e)} - \gamma N_0 - \gamma \sum_{m \in \bar{\mathcal{B}} \setminus \{r(e)\}} \zeta^k_{m,r(e)} \geq 0, \forall e \in \mathcal{Q}_u(k)$;
    $\Psi^k_{t(e),r(e)} P^k_{t(e)} - \gamma N_0 - \gamma \sum_{m \in \bar{\mathcal{B}} \setminus \{t(e)\}} \zeta^k_{m,r(e)} \geq 0, \forall e \in \mathcal{Q}_d(k)$;
if the above LP has a feasible solution then
    return 1;
else
    return 0;
```

the inter-cell constraints (19) and (20) in Section IV. An active cell is a cell that has at least one link in $\mathcal{Q}_u(k) \cup \mathcal{Q}_d(k)$. The third and the fourth constraints, on the other hand, correspond to the reliability requirement of the uplinks and downlinks respectively. $r(e)$ and $t(e)$ denote the receiver node and the transmitter node of link e respectively.

Now, we can explain our solution for the second phase, i.e., maximizing the number of reliable uplinks. This phase is outlined in lines [6-13] in Algorithm 3. The output of the first phase is the allocation of exactly one channel $k \in \mathcal{L}_i$ for each MR $i \in \mathcal{B} \cup \mathcal{G}$. The idea is to go over the channels in $\mathcal{L}$ one by one. For each channel k, we find the set of potential uplinks on channel k, denoted as $\mathcal{Q}_u(k)$, as shown in line 7. If $\mathcal{Q}_u(k)$ is not empty, then for each uplink e, we find the *maximum channel gain*, λ_e, between $t(e)$ and all the receiving MRs in $\mathcal{Q}_u(k)$ except $r(e)$ as follows:

$$\lambda_e = \max_{i : \exists (j,i) \in \mathcal{Q}_u(k), i \neq r(e)} \Psi^k_{t(e),i} \tag{26}$$

Then, we process the uplinks in $\mathcal{Q}_u(k)$ in ascending order of their λ_e values. For each uplink, we use the PCA algorithm to find out whether it can be supported, i.e., reliably served, without affecting, i.e., breaking the reliability of, already reliably served uplinks. If so, the uplink is added to the set of reliable uplinks $\mathcal{Q}^r_u(k)$, otherwise it will not be added.

C. Phase 3: Channel allocation to MCs

The last phase is channel allocation to MCs, i.e., downlinks. First of all, the MCs to be considered in the phase are only the ones that have reliable uplinks with their parent MRs after the second phase. Therefore, in lines [15-16] of Algorithm 3, we set A_i $\forall i \in \mathcal{B} \cup \mathcal{G}$ to those MCs that have reliable uplinks with MR i. Similar to what we did with uplinks, we need to process potential downlinks in ascending order of their maximum channel gains. However, the case now is different. Each MC may have several channels available, i.e., $\mathcal{L}_j > 1$ for $j \in \mathcal{A}$. This provides us with multiple choices for each downlink, in contrary to the uplink case where each uplink has only one choice, i.e., the channel assigned to the MR of that uplink. Therefore, for each MC j, we will find $|\mathcal{L}_j|$ maximum channel gains each on one of the channels in $\mathcal{L}_j$. Let $\mathcal{P}$ be the set of all possible $(MC,\ channel)$ pairs defined as follows:

$$\mathcal{P} = \{(i, k) : i \in \bigcup_{j \in \mathcal{B} \cup \mathcal{G}} \mathcal{A}_j,\ k \in \mathcal{L}_i\}. \tag{27}$$

Recall that this set is evaluated after removing MCs that cannot be served on the uplink. Therefore, all MCs represented by at least one pair in $\mathcal{P}$ have passed the second phase, i.e., can be served reliably on the uplink. Let $p(i)$ denote the parent MR of MC i. Then for each pair $(i, k) \in \mathcal{P}$, the maximum channel gain $\lambda_{(i,k)}$ is calculated as follows:

$$\begin{aligned} \lambda_{(i,k)} = \ & \max\{ \max_{j : (j,k) \in \mathcal{P}, p(i) \neq p(j)} \Psi^k_{ij}, \\ & \max_{j \in \mathcal{B} \cup \mathcal{G} \setminus \{p(i)\} : \exists (m,j) \in \mathcal{Q}^r_u(k)} \Psi^k_{ij} \} \end{aligned} \tag{28}$$

The above equation finds the *maximum channel gain* $\lambda_{(i,k)}$ on channel k between MC i and any other MC that has channel k available or a MR that was assigned channel k in the first phase. Then, we process the pairs in $\mathcal{P}$ in ascending order of their *maximum channel gains*. For each pair (i, k), we add the downlink $(p(i), i)$ to the current set of reliable downlinks on channel k, $\mathcal{Q}^r_d(k)$ (initially empty), and the uplink $(i, p(i))$ to the current set of reliable uplinks on channel k', $\mathcal{Q}^r_u(k')$ (which is initially empty) where k' is the channel assigned to $p(i)$, i.e, $\mathbf{L}_{(r)}[i, k'] = 1$. Using the PCA algorithm, if both the uplink and the downlink can be served reliably without breaking the reliability of any link in $\mathcal{Q}^r_d(k)$ and $\mathcal{Q}^r_u(k')$, then this MC is added to the set of reliable MCs $\mathcal{A}^r$ and the downlink and the uplink are admitted to the set $\mathcal{Q}^r_d(k)$ and $\mathcal{Q}^r_u(k')$ respectively. Otherwise, the two links will be removed from $\mathcal{Q}^r_d(k)$ and $\mathcal{Q}^r_u(k')$ and the MC will not be added to $\mathcal{A}^r$. Once an MC is added to $\mathcal{A}^r$ by one of its pairs, other pairs of this MC in $\mathcal{P}$ will be ignored. This process is presented in lines [21-34] in Algorithm 3. Finally, to find a final power allocation for MRs and MCs, we run the PCA algorithm once for each channel over the set of reliable uplinks and downlinks on that channel.

The algorithm that combines all the three phases together is presented in Algorithm 3, which we call the *heuristic receiver-based channel allocation* (HRBA) algorithm.

VI. PERFORMANCE EVALUATION

In this section, we compare the optimal performance of all the three allocation strategies proposed in Section IV in terms of the number of MCs served, and evaluate the performance of the proposed HRBA algorithm. To vary the channel availability

Algorithm 3: Heuristic Receiver Based Allocation (HRBA)

```
input : $\mathcal{L}$; $\mathcal{B}$; $\mathcal{G}$; $\mathcal{A}_i \forall i \in \mathcal{B} \cup \mathcal{G}$; $\mathcal{L}_i \forall i \in \mathcal{B} \cup \mathcal{G} \cup \mathcal{A}$.
output: Set of reliable MCs $\mathcal{A}^r$; transmission powers;
        channel allocation to MRs $\mathbf{L}_{(r)}$; channels
        allocation to MCs $\bar{\mathbf{L}}_{(r)}$.
//Phase 1: allocate channels to MRs.
$\mathbf{L}_{(r)} = GRA(\mathcal{B} \cup \mathcal{G}, \mathcal{L}_i \forall i \in \mathcal{B} \cup \mathcal{G})$;
$\mathcal{R} = \emptyset$;
$P_i^k = 0, \ \forall i \in \mathcal{B} \cup \mathcal{G} \cup \mathcal{A}, \ k \in \mathcal{L}_i$;
//Phase 2: find the set of reliable uplinks.
forall $k \in \mathcal{L}$ do
    $\mathcal{Q}_u(k) = \{e : r(e) \in \mathcal{B} \cup \mathcal{G}, t(e) \in \mathcal{A}_{r(e)}, k \in \mathcal{L}_{t(e)}, \mathbf{L}_{(r)}[r(e), k] = 1\}$;
    if $\mathcal{Q}_u(k) \neq \emptyset$ then
        For $e \in \mathcal{Q}_u(k)$ find $\lambda_e$ using equation (26);
        $\mathcal{Q}_u^r(k) = \emptyset$;
        forall $e \in \mathcal{Q}_u(k)$ in ascending order of $\lambda_e$ do
            if $PCA(\mathcal{Q}_u^r(k) \cup \{e\}, \emptyset, k) = 1$ then
                $\mathcal{Q}_u^r(k) = \mathcal{Q}_u^r(k) \cup \{e\}$;
//Phase 3: allocate channels to MCs.
$\mathcal{A}_i = \emptyset, \ \forall i \in \mathcal{B} \cup \mathcal{G}$;
$\mathcal{A}_i = \{j : (j, i) \in \bigcup_{k \in \mathcal{L}} \mathcal{Q}_u^r(k)\}, \ \forall i \in \mathcal{B} \cup \mathcal{G}$;
Find the set $\mathcal{P}$ using equation (27);
For each pair $(i, k)$ in $\mathcal{P}$ find $\lambda_{(i,k)}$ using equation (28);
$\mathcal{Q}_d^r(k) = \emptyset, \ \mathcal{Q}_u^r(k) = \emptyset$;
Let $\bar{\mathbf{L}}_{(r)}$ be an $|\mathcal{A}| \times K$ matrix initially set to 0;
forall $(i, k) \in \mathcal{P}$ in ascending order of $\lambda_{(i,k)}$ do
    if $i \in \mathcal{A}^r$ then
        continue;
    $k' := \{k : \mathbf{L}_{(r)}[p(i), k] = 1\}$;
    $\mathcal{Q}_d^r(k) = \mathcal{Q}_d^r(k) \cup \{(p(i), i)\}$;
    $\mathcal{Q}_u^r(k') = \mathcal{Q}_u^r(k') \cup \{(i, p(i))\}$;
    $x = PCA(\mathcal{Q}_u^r(k), \mathcal{Q}_d^r(k), k)$;
    $y = PCA(\mathcal{Q}_u^r(k'), \mathcal{Q}_d^r(k'), k')$;
    if $x=1$ and $y=1$ then
        $\mathcal{A}^r = \mathcal{A}^r \cup \{i\}$;
        $\bar{\mathbf{L}}_{(r)}[i, k] = 1$;
    else
        $\mathcal{Q}_d^r(k) = \mathcal{Q}_d^r(k) \backslash \{(p(i), i)\}$;
        $\mathcal{Q}_u^r(k') = \mathcal{Q}_u^r(k') \backslash \{(i, p(i))\}$;
//Find final power allocation
$P_i^k = 0, \ \forall i \in \mathcal{B} \cup \mathcal{G} \cup \mathcal{A}, \ k \in \mathcal{L}_i$;
forall $k \in \mathcal{L}$ do
    $PCA(\mathcal{Q}_u^r(k), \mathcal{Q}_d^r(k), k)$;
$\mathcal{A}_i = \mathcal{A}_i \cap \mathcal{A}^r, \ \forall i \in \mathcal{B} \cup \mathcal{G}$;
return $\mathcal{A}^r$; $P_i^k \ \forall i \in \mathcal{B} \cup \mathcal{G} \cup \mathcal{A}, \ k \in \mathcal{L}_i$; $\mathbf{L}_{(r)}$; $\bar{\mathbf{L}}_{(r)}$;
```

distribution at different SUs, we varying the number and locations of PUs. The network is deployed in an $A \times A$ square area. The area is divided into N_r cells, such that $N_r=|\mathcal{B}|+|\mathcal{G}|$. We obtained the optimal solutions for $N_r=4$, and 9 MRs and 100 MCs. In all scenarios, we assume a single gateway located at the right-bottom cell. The number of PUs is varied to achieve different channel availability distributions. Each PU is randomly assigned one of the K orthogonal channels available in the system. A node cannot use channel k if it is less than R_p apart from a PU that is assigned channel k. R_p is set equal to the cell radius, i.e., $R_p=\frac{A}{2\sqrt{N_r}}$. The maximum transmission power of an MR is calculated as $\frac{2.5AN_0\gamma}{2\sqrt{N_r}}$ and of an MC as $\frac{AN_0\gamma}{\sqrt{N_r}}$, where $\gamma=15dB$ and $N_0=10^{-11}$ Watt. These values guarantee each MR to reach the four adjacent (up, down, left, and right) MRs, and each MC to reach its parent MR. For all the experiments in this section, the path-loss exponent $\alpha=3.76$.

A. Without a preassumed CCC

We first study the optimal performance of the three allocation strategies without presuming the existence of a CCC in the network. Figures 2 and 4 show the optimal performance of the three strategies for the case of 4 MRs and 9 MRs respectively. The number of active PUs is varied from 15 to 40 for the case of 4 MRs, and from 30 to 55 for the case of 9 MRs. Each point on the curves is the average of 100 randomly generated topologies. As the figures imply, the RBA approach outperforms the other two approaches. Notice that the difference in performance between RBA and ATA is higher for fewer PUs (i.e., the fewer the PUs the higher the channel availability). For instance, the number of served MCs using the RBA approach, in Figure 4, is on average 1.5 times that using the ATA approach for 30 PUs, however, this number jumps to 3.5 for the case of 55 PUs. The TBA approach, on the other hand, is always outperformed by the ATA approach, which is expected because they both require a CCC, but the ATA approach can use all the channels for transmission while the TBA approach is restricted to one channel only.

B. With a preassumed CCC

In this subsection, we evaluate the performance of the three allocation strategies with the presumption that a CCC exists. We add one more channel and make it available to all nodes, i.e., no PU can use this extra channel. Figures 3 and 5 show the optimal performance of the three strategies for the case of 4 MRs and 9 MRs respectively. The number of PUs is varied from 15 to 40 for the case of 4 MRs, and from 30 to 55 for the case of 9 MRs. Each point in the two figures is the average performance of 100 randomly generated topologies. As the figures indicate, the RBA still outperforms the other two strategies even though a CCC is preassumed. However, the difference between RBA and the other approaches is less in this case than the case when no CCC is preassumed. The figures also show that for fewer PUs (which means high channel availability), the performance of the RBA strategy is very close to that of the ATA strategy.

C. Performance of the HRBA algorithm

In this subsection, we compare the performance of the HRBA algorithm to the optimal performance obtained using

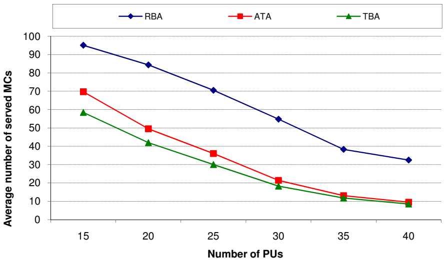

Fig. 2. The performance of the three allocation strategies without a preassumed CCC. $|\mathcal{B}| = 3$, $|\mathcal{G}| = 1$, $|\mathcal{A}| = 100$, and $K = 6$.

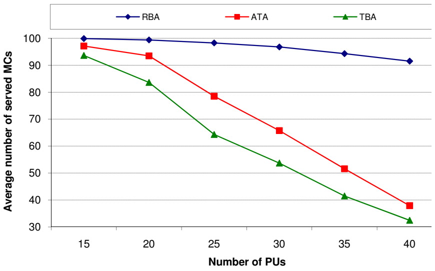

Fig. 3. The performance of the three allocation strategies with a preassumed CCC. $|\mathcal{B}| = 3$, $|\mathcal{G}| = 1$, $|\mathcal{A}| = 100$, and $K = 7$.

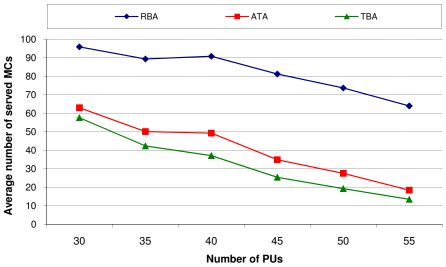

Fig. 4. The performance of the three allocation strategies without a preassumed CCC. $|\mathcal{B}| = 8$, $|\mathcal{G}| = 1$, $|\mathcal{A}| = 100$, and $K = 6$.

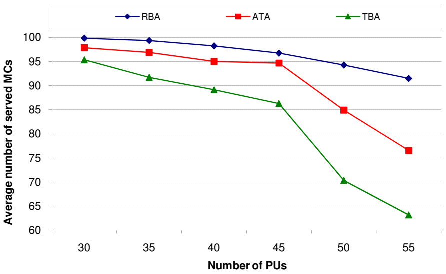

Fig. 5. The performance of the three allocation strategies with a preassumed CCC. $|\mathcal{B}| = 8$, $|\mathcal{G}| = 1$, $|\mathcal{A}| = 100$, and $K = 7$.

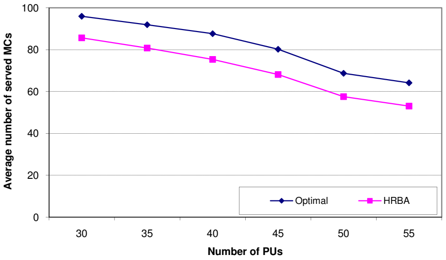

Fig. 6. The performance of the HRBA algorithm compared to the optimal solution. $|\mathcal{B}| = 8$, $|\mathcal{G}| = 1$, $|\mathcal{A}| = 100$, and $K = 6$.

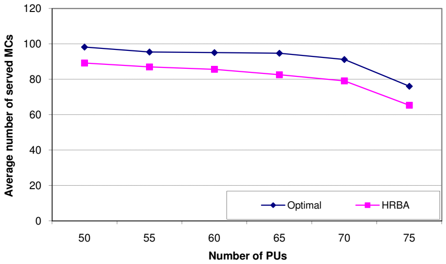

Fig. 7. The performance of the HRBA algorithm compared to the optimal solution. $|\mathcal{B}| = 15$, $|\mathcal{G}| = 1$, $|\mathcal{A}| = 100$, and $K = 6$.

the MILP formulation in Section IV. In Figure 6, we show the number of served MCs (the average over a 100 randomly generated topologies) obtained using the HRBA algorithm and the MILP formulation for $|\mathcal{B}|$=8, $|\mathcal{G}|$=1, $|\mathcal{A}|$=100 and C=6. As the figure shows, the performance of the HRBA algorithm is close to the optimal solution, within with $\approx$14.7% of the optimal (on average). Figure 7 shows the same results for $|\mathcal{B}|$=15, $|\mathcal{G}|$=1, $|\mathcal{A}|$=100 and C=6. Again, the HRBA algorithm is, on average, within $\approx$12% of the optimal solution.

VII. CONCLUSIONS

We have studied, in this work, the channel allocation problem in wireless cognitive mesh networks. By controlling the tunability of the transmission and reception parts of the cognitive radio, four different modes of operation were defined for cognitive transceivers. Three channel allocation strategies based on the aforementioned modes were defined, namely receiver-based allocation *RBA*, transmitter-based allocation *TBA*, and all-tunable allocation *ATA*. MILP formulations were proposed RBA and ATA strategies with the objective of maximizing the number of served MCs with a reliability guarantees on the uplink and downlink for each MC. However, the MILP for the TBA case was omitted for lack of space. Results show that the proposed RBA strategy outperforms the TBA and the ATA strategies even when a CCC is preassumed.

We also proposed a heuristic solution for the RBA problem. Results show that the accuracy of the proposed algorithm is, on average, within 28% of the optimal solution. As a future direction, we plan to propose a MAC protocol specially designed to work with the RBA strategy.

VIII. ACKNOWLEDGEMENTS

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