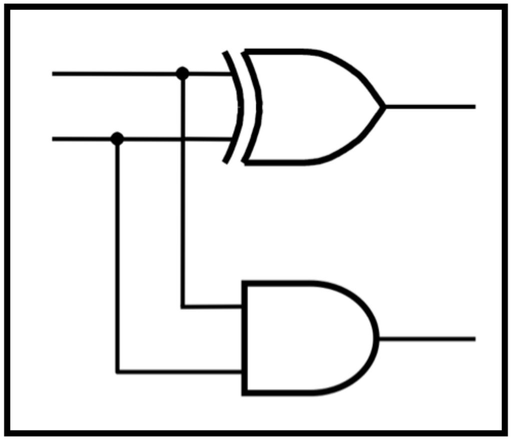


CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Analysis of Synchronous Sequential Circuits

CprE 2810: Digital Logic
Iowa State University, Ames, IA

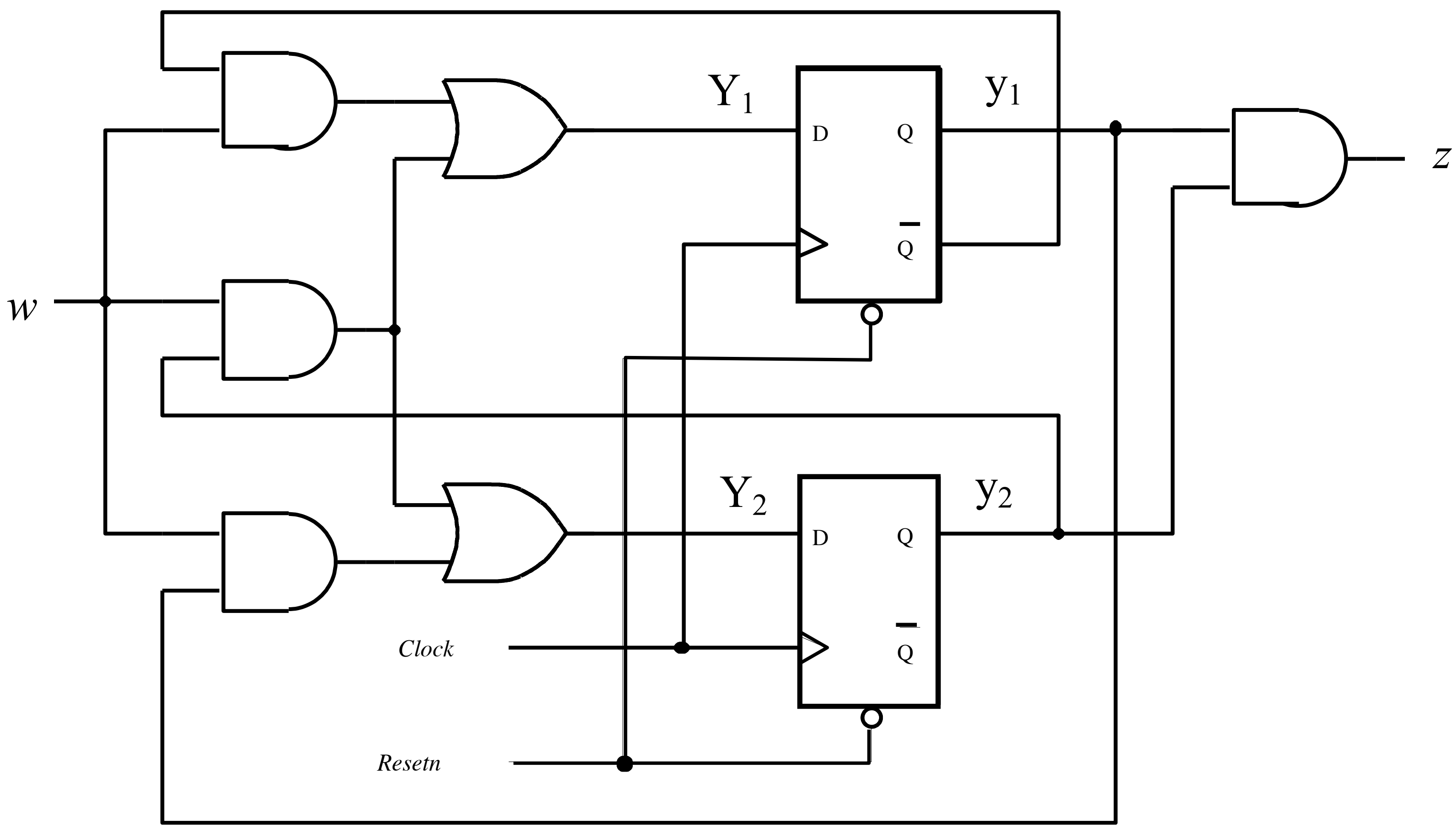
Goal for Today's Lecture

- Given a circuit diagram for a synchronous sequential circuit, the goal is to figure out the FSM that it implements.

- Steps: determine the present state variables, the next state variables, the state-assigned table, the state table, and finally the state diagram.

- In other words, the goal is to reverse engineer the circuit.

What does this circuit do?

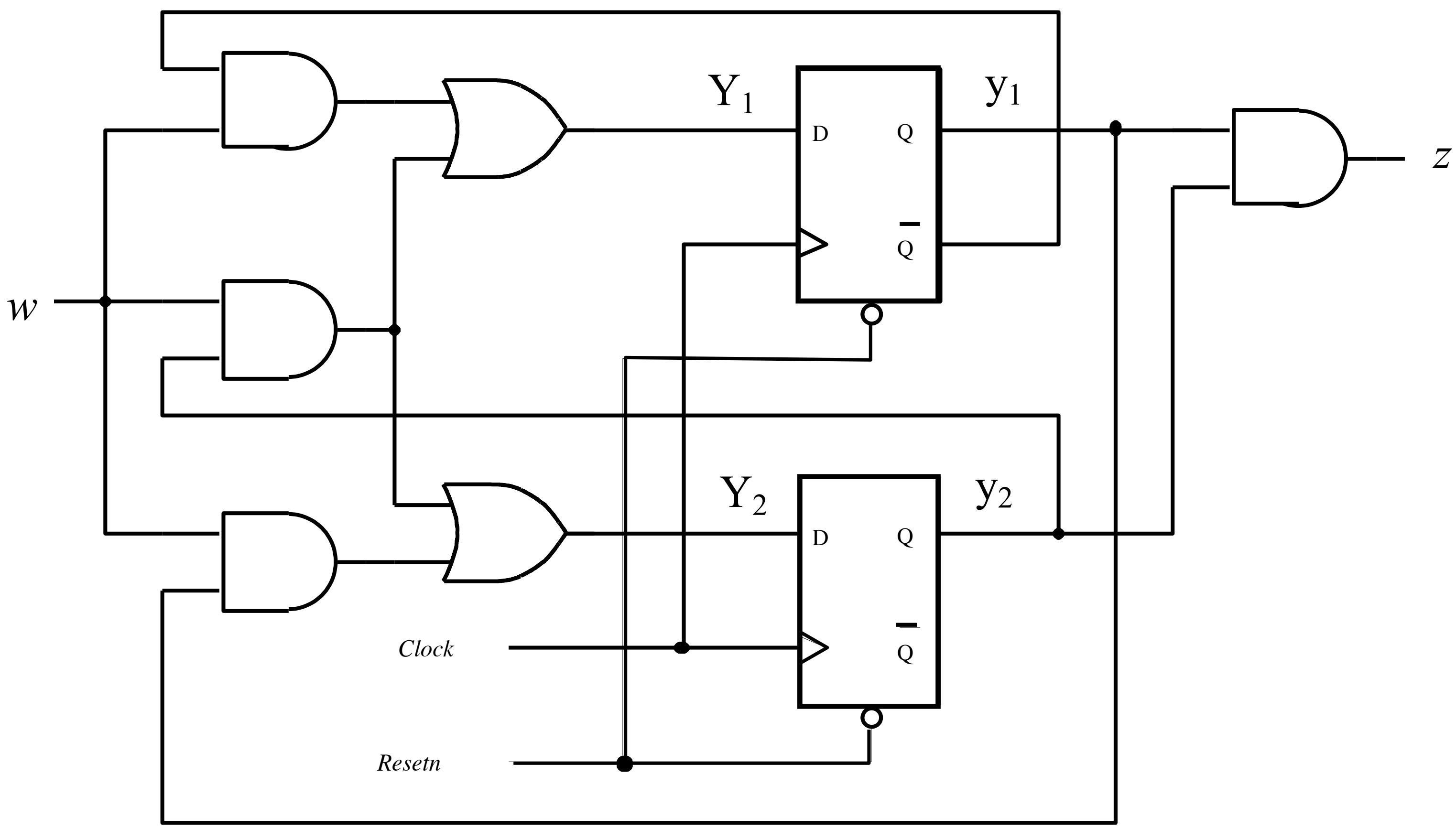


[Figure 6.75 from the textbook]

Approach

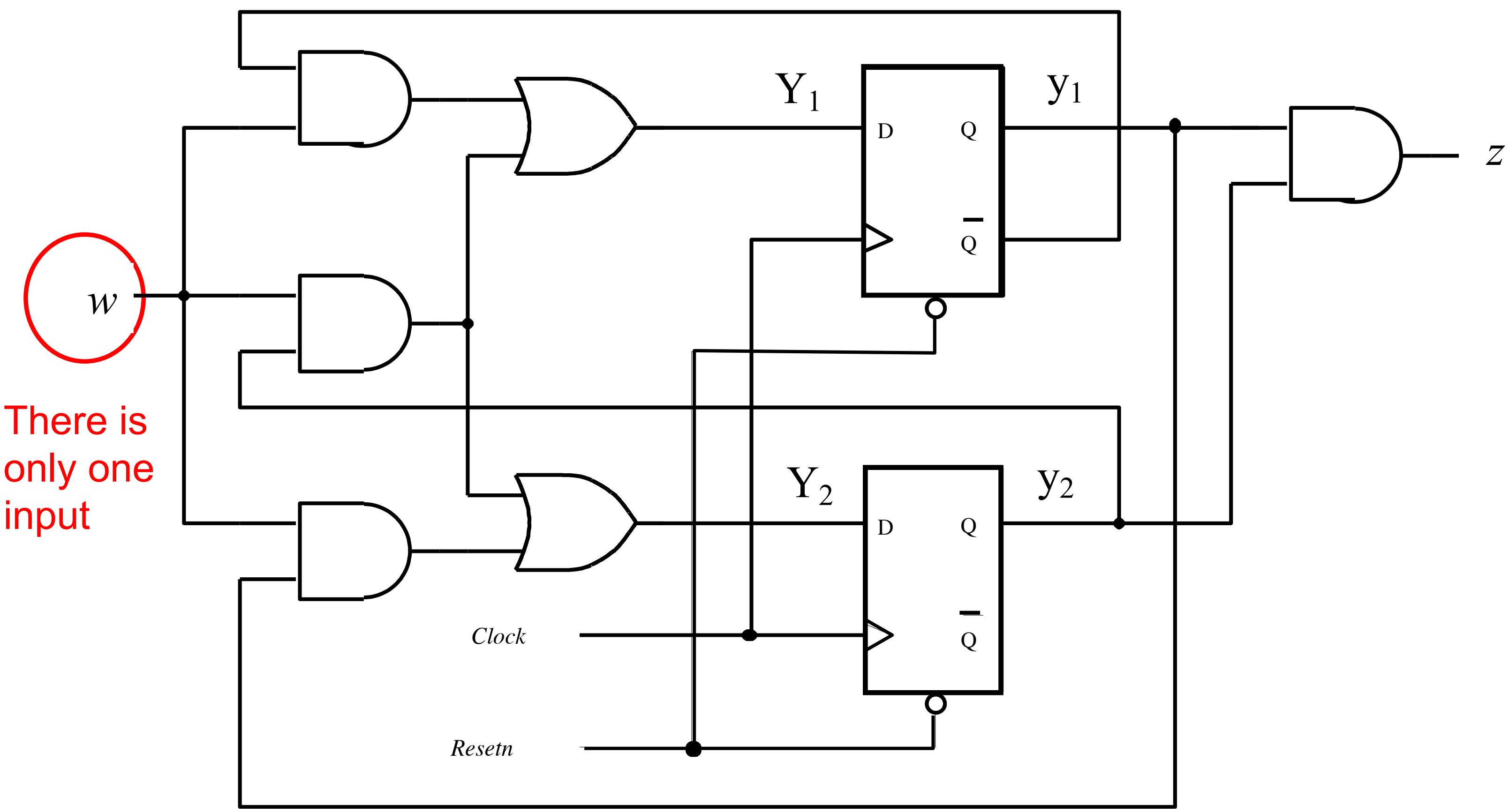
- Find the flip-flops
- Outputs of the flip-flops = present state variables
- Inputs of the flip-flops determine the next state variables
- Determine the logical expressions for the outputs
- Given this info it is easy to do the state-assigned table
- Next do the state table
- Finally, draw the state diagram.

Where are the inputs?



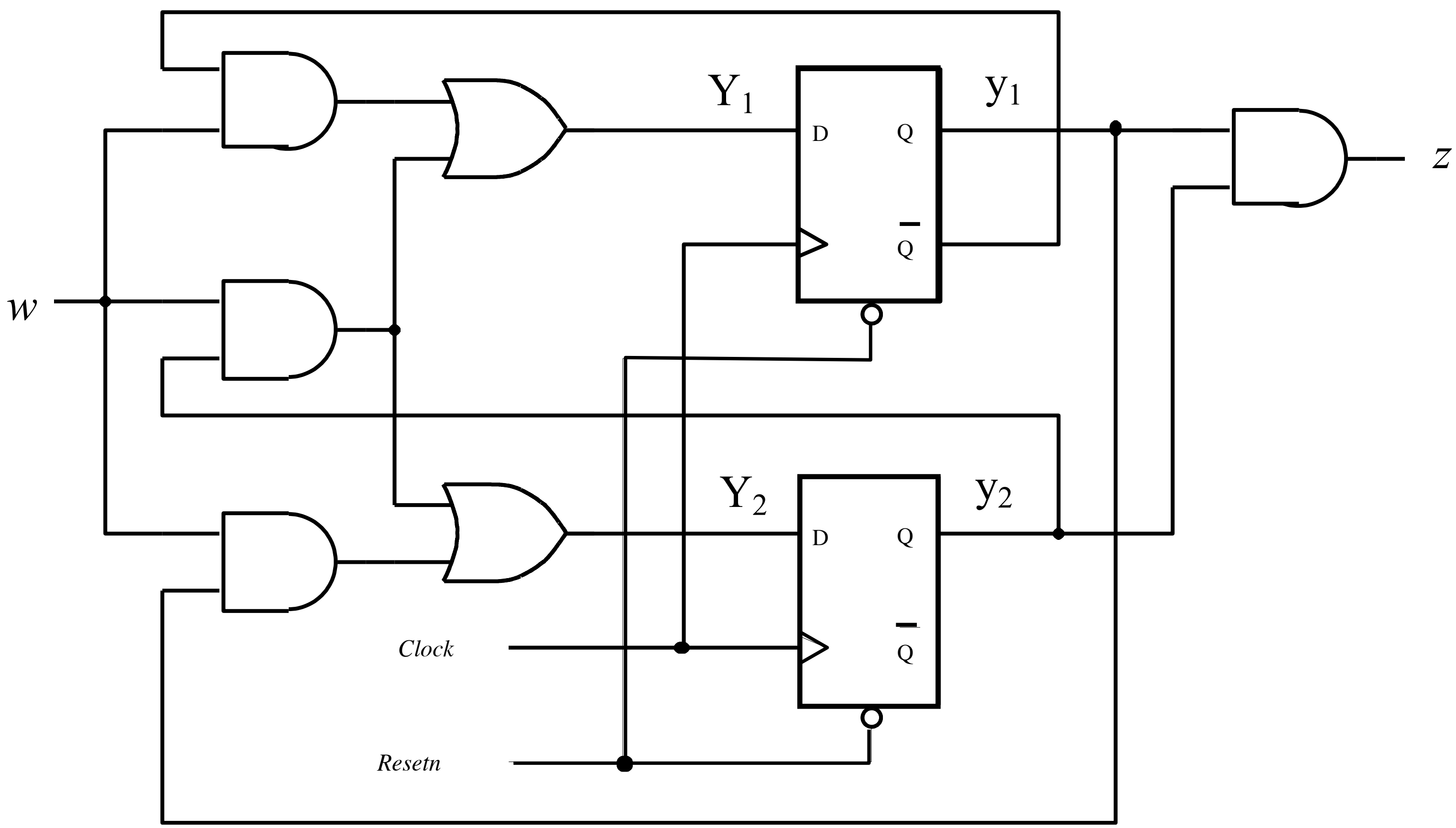
[Figure 6.75 from the textbook]

Where are the inputs?



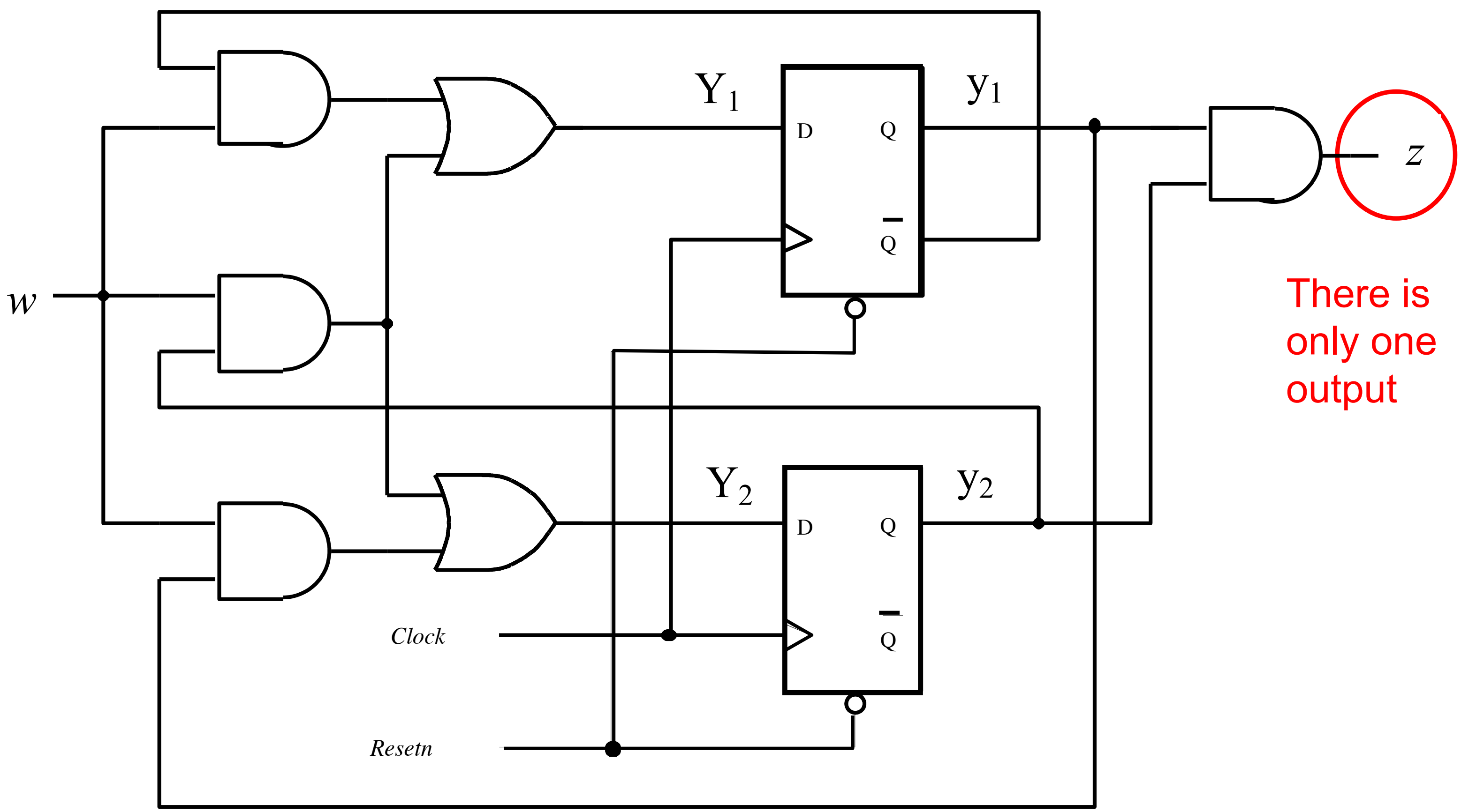
[Figure 6.75 from the textbook]

Where are the outputs?



[Figure 6.75 from the textbook]

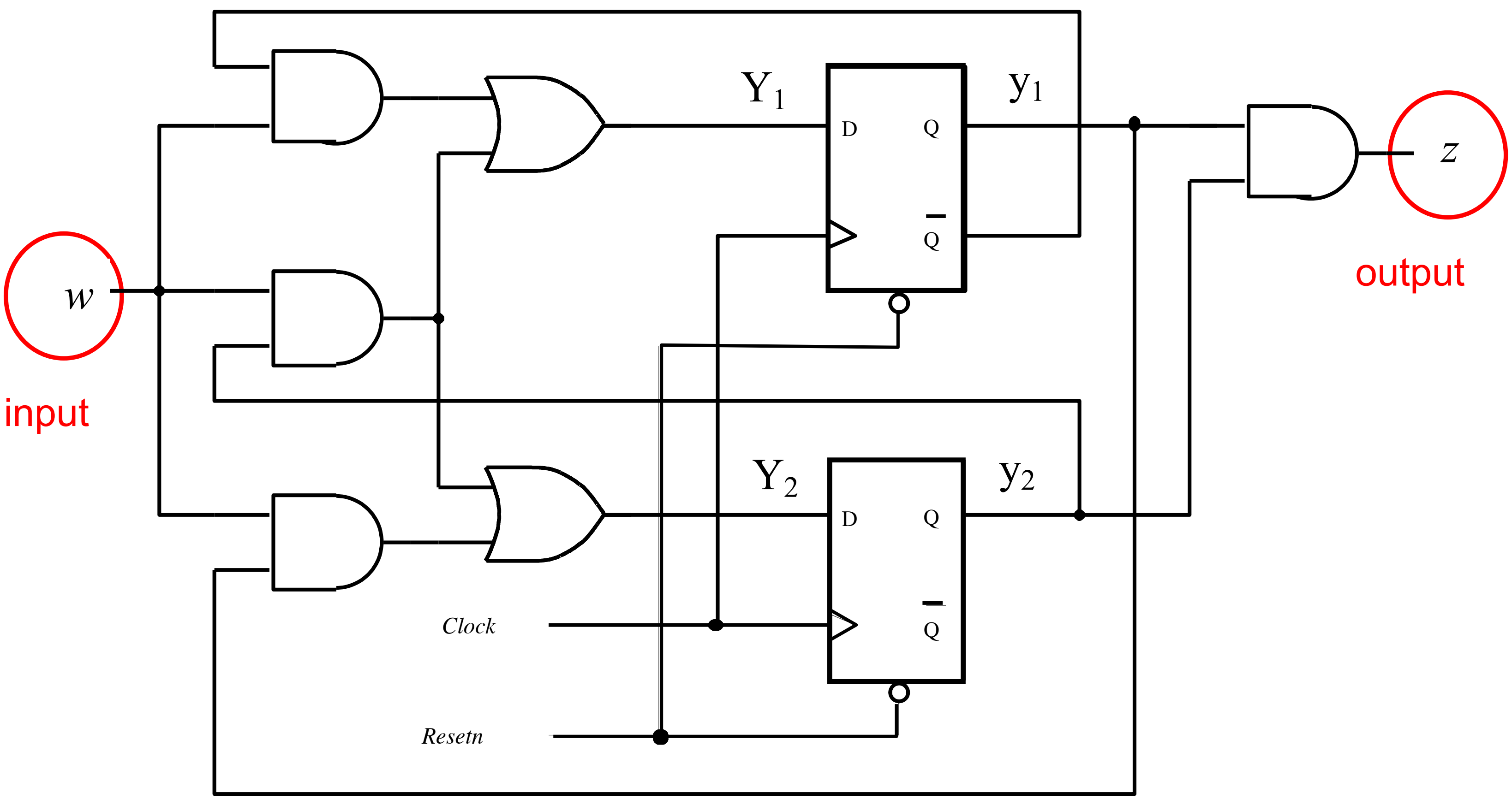
Where are the outputs?



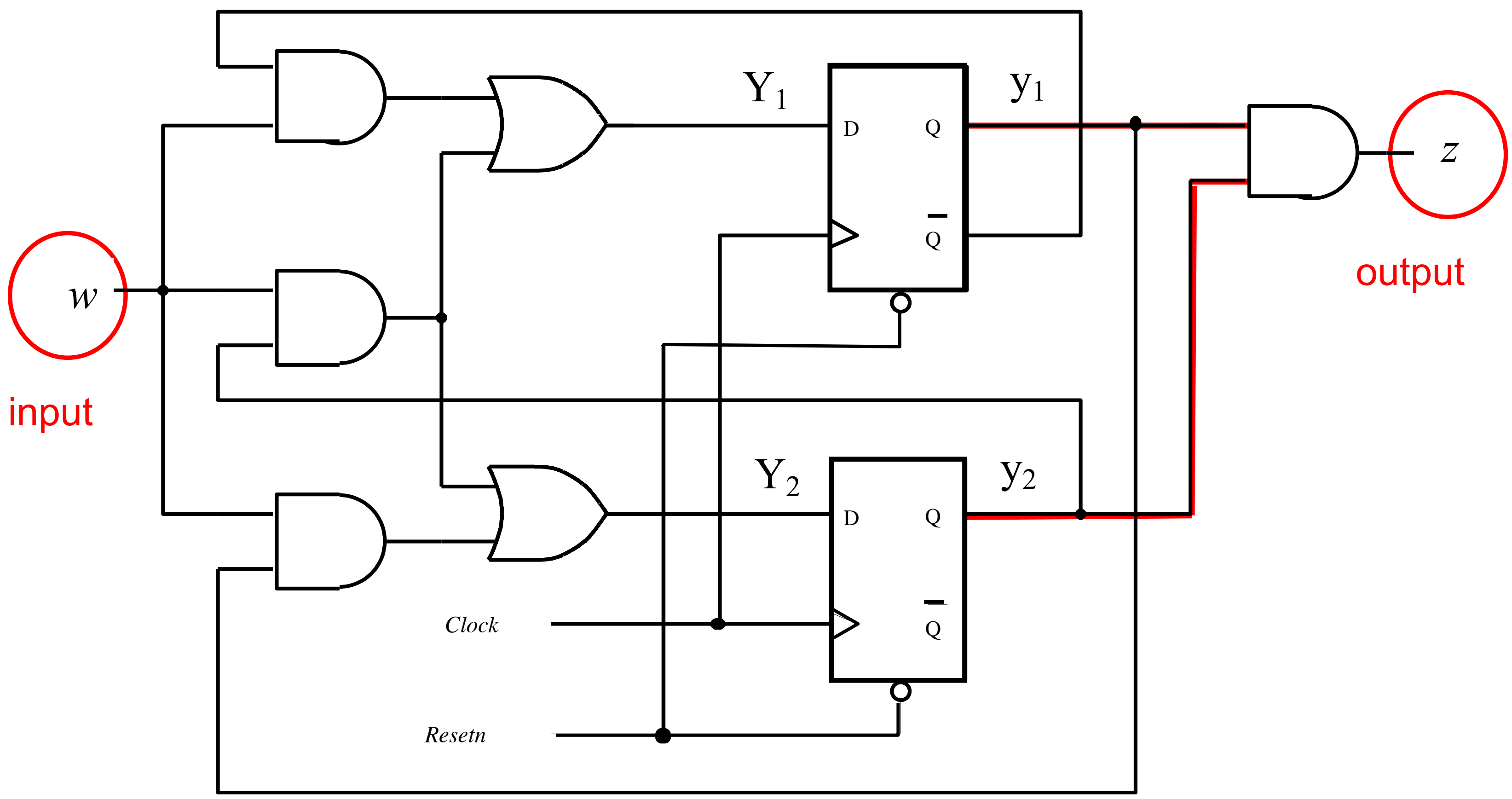
[Figure 6.75 from the textbook]

Where kind of machine is this?

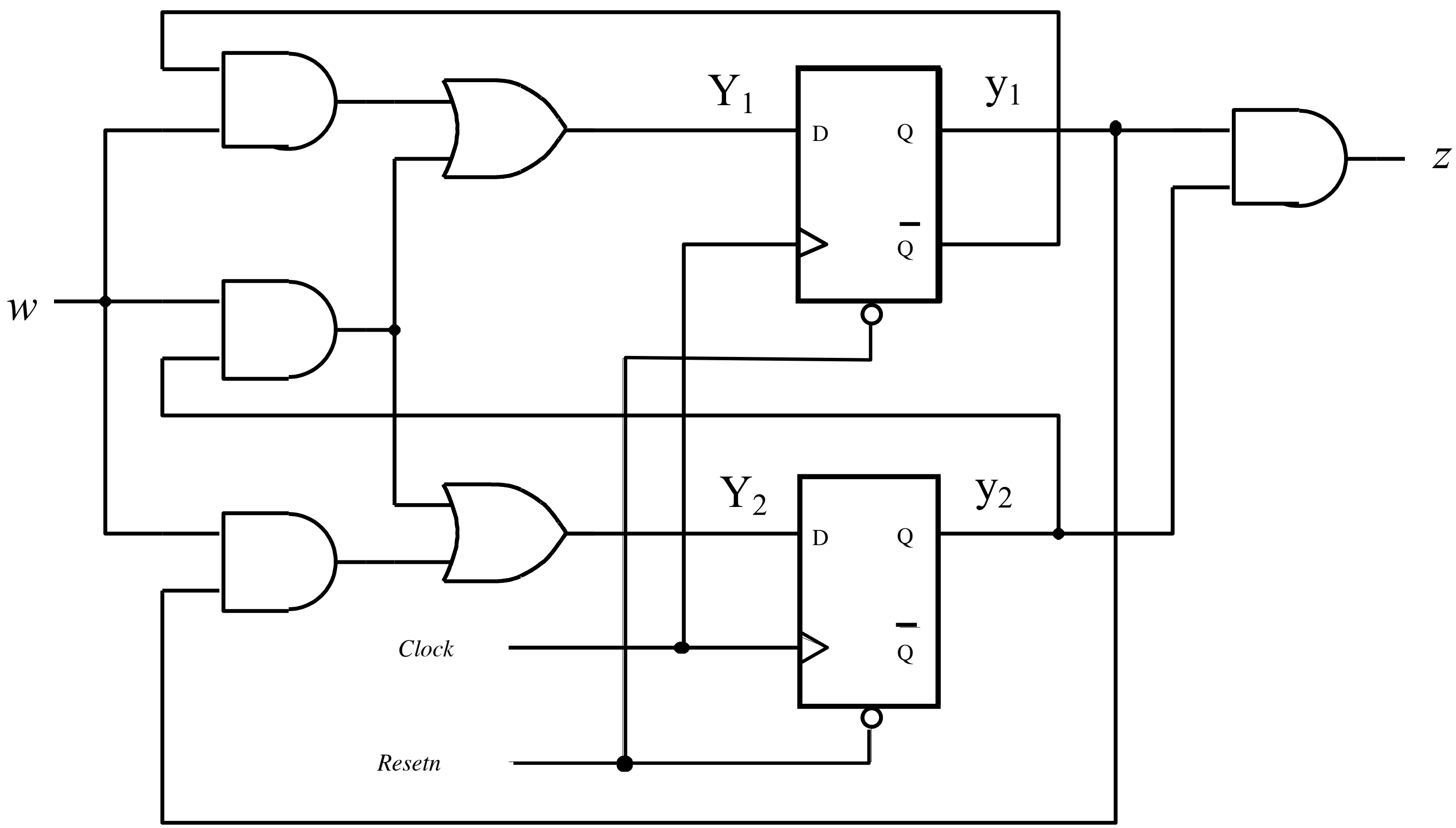
Moore or Mealy?



Moore: because the output does not depend directly on the primary input



Where are the memory elements?



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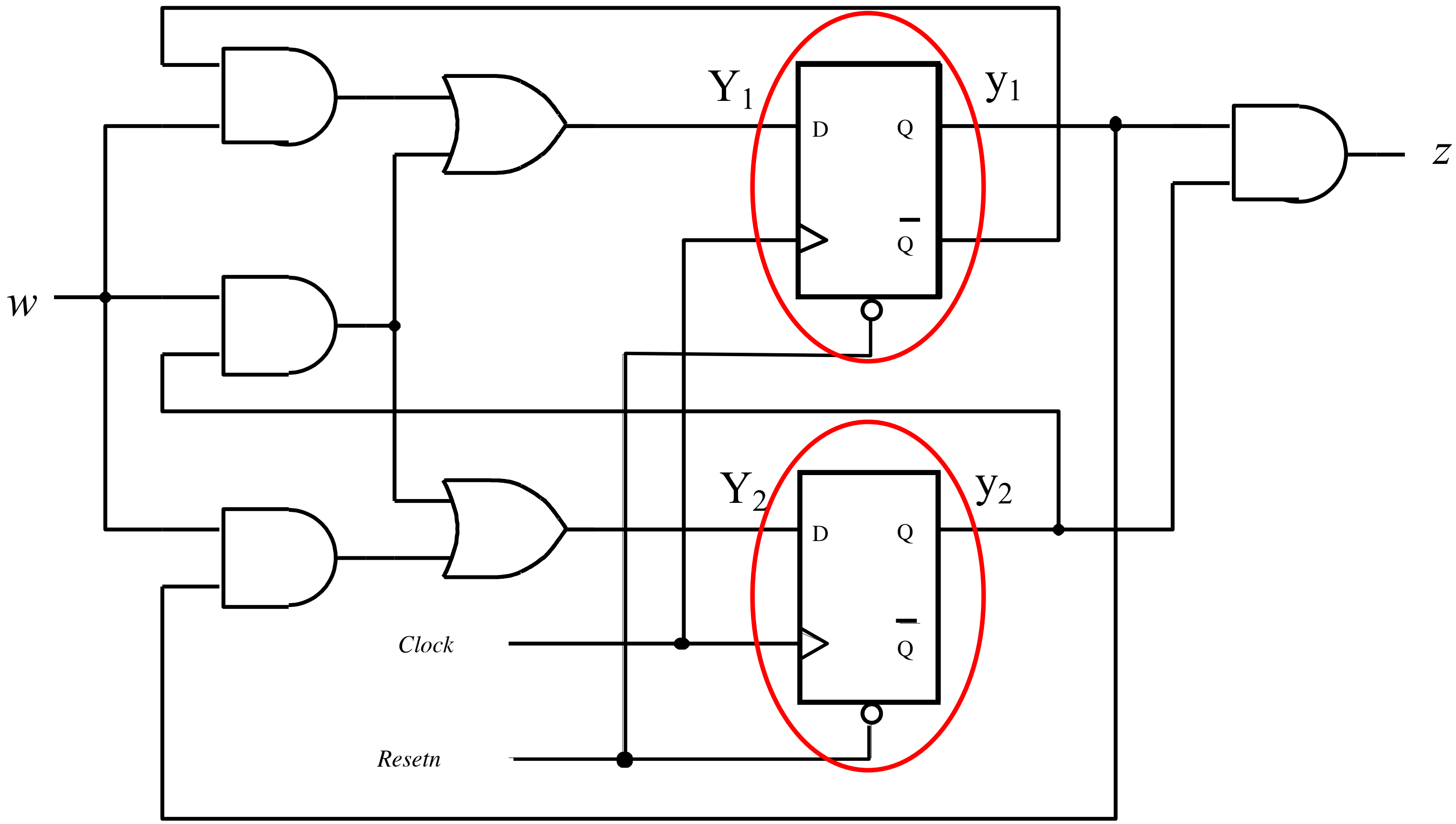
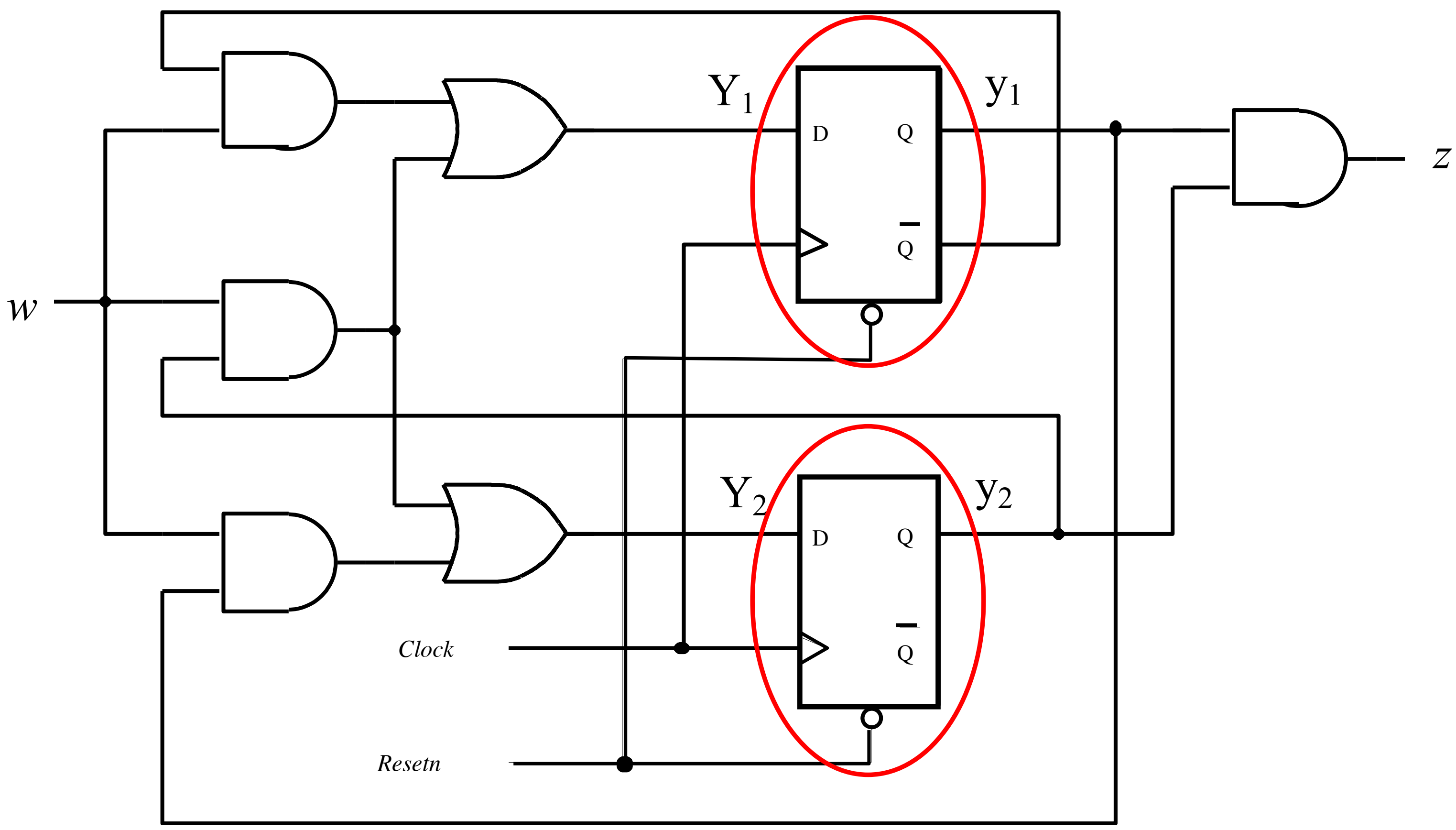
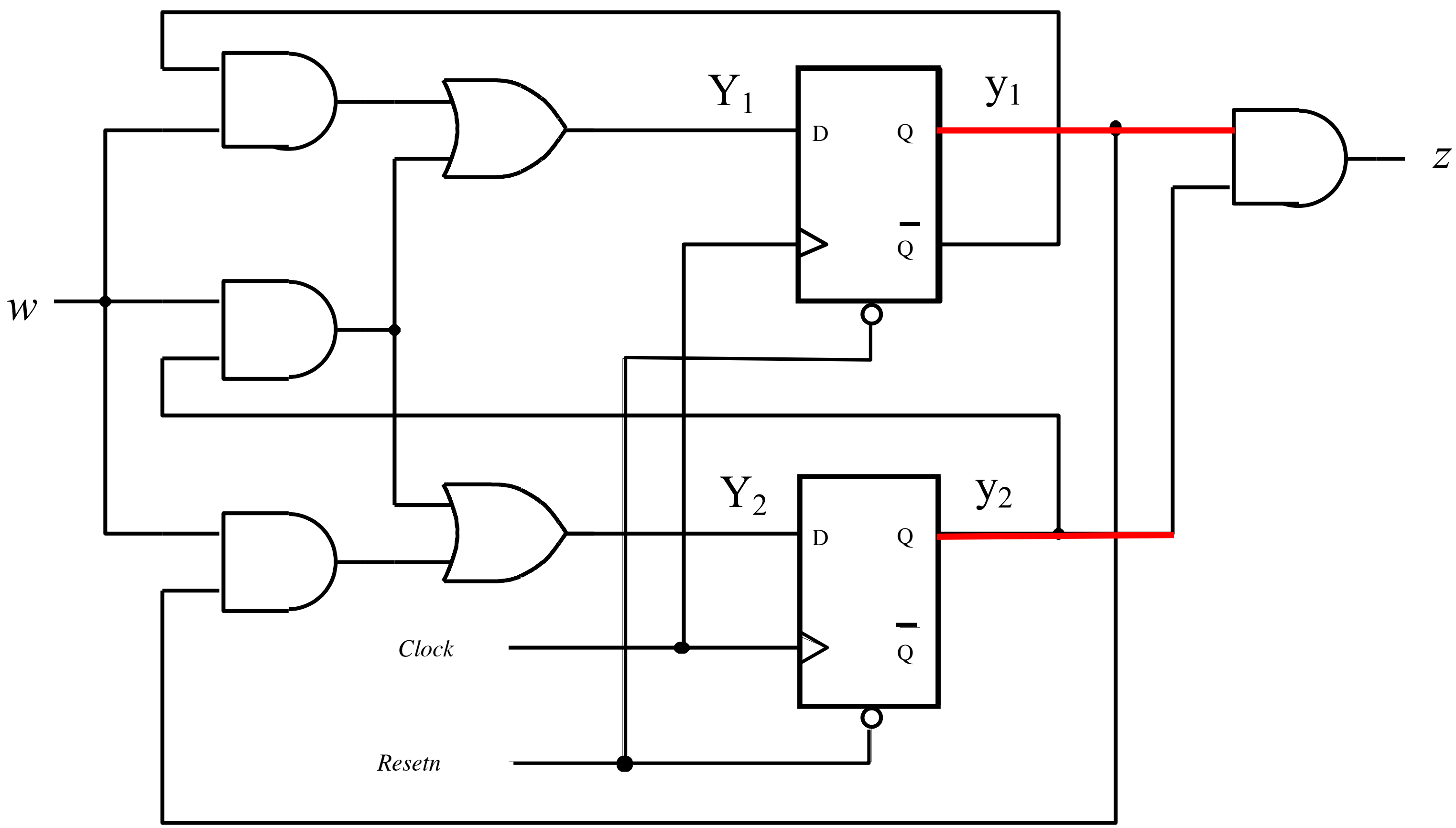


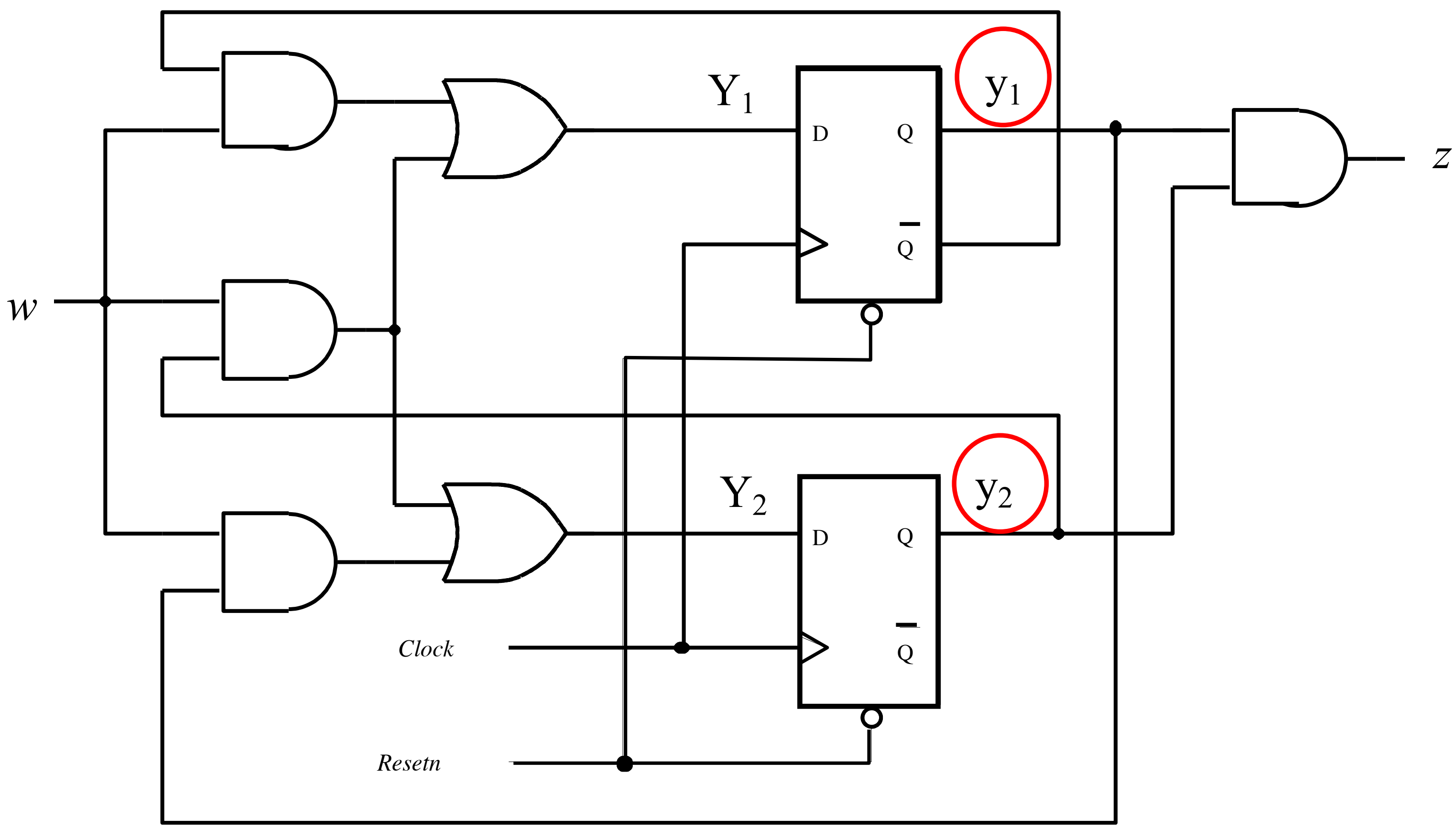
Where are the outputs of the flip-flops?



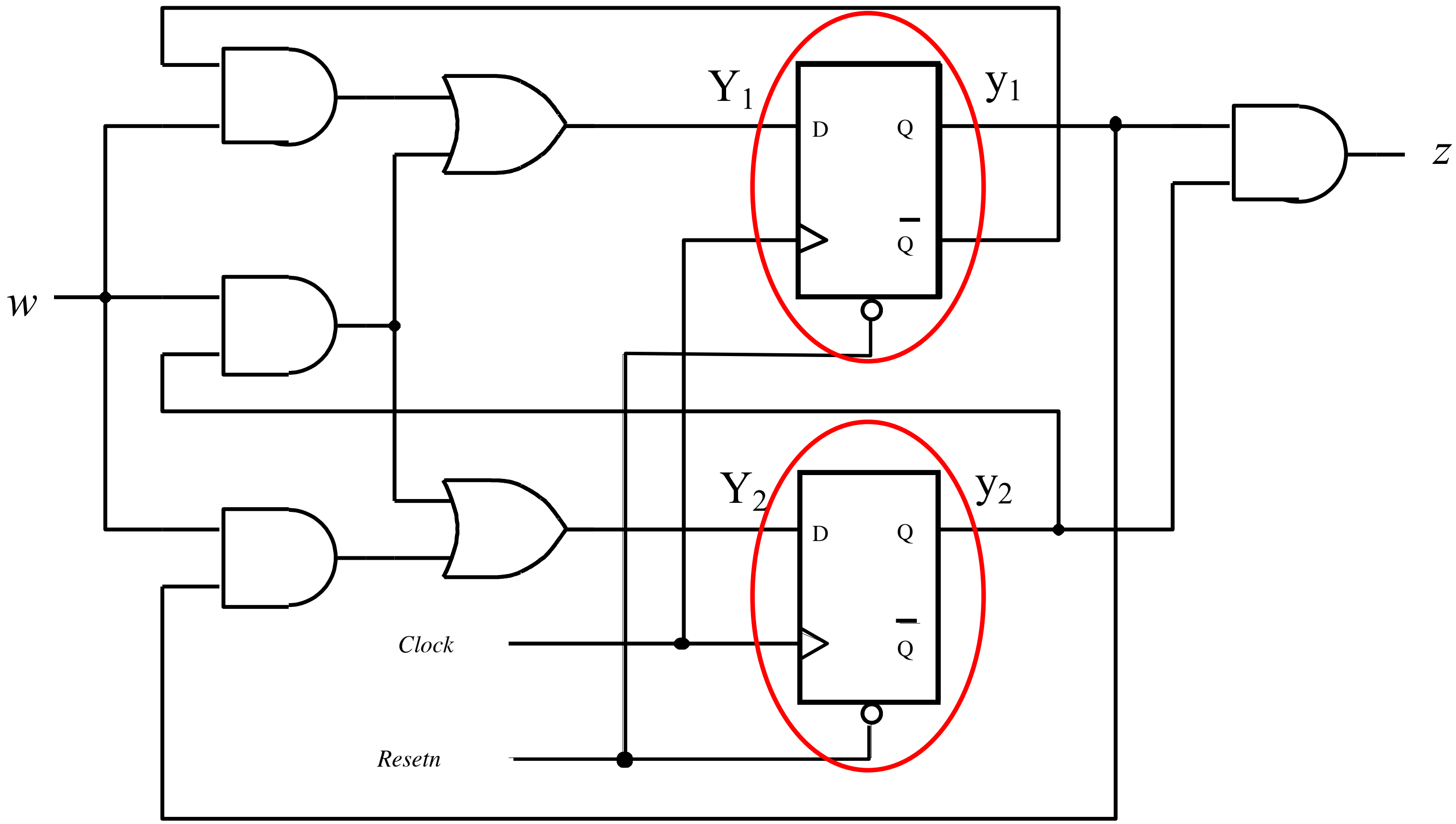
Where are the outputs of the flip-flops?



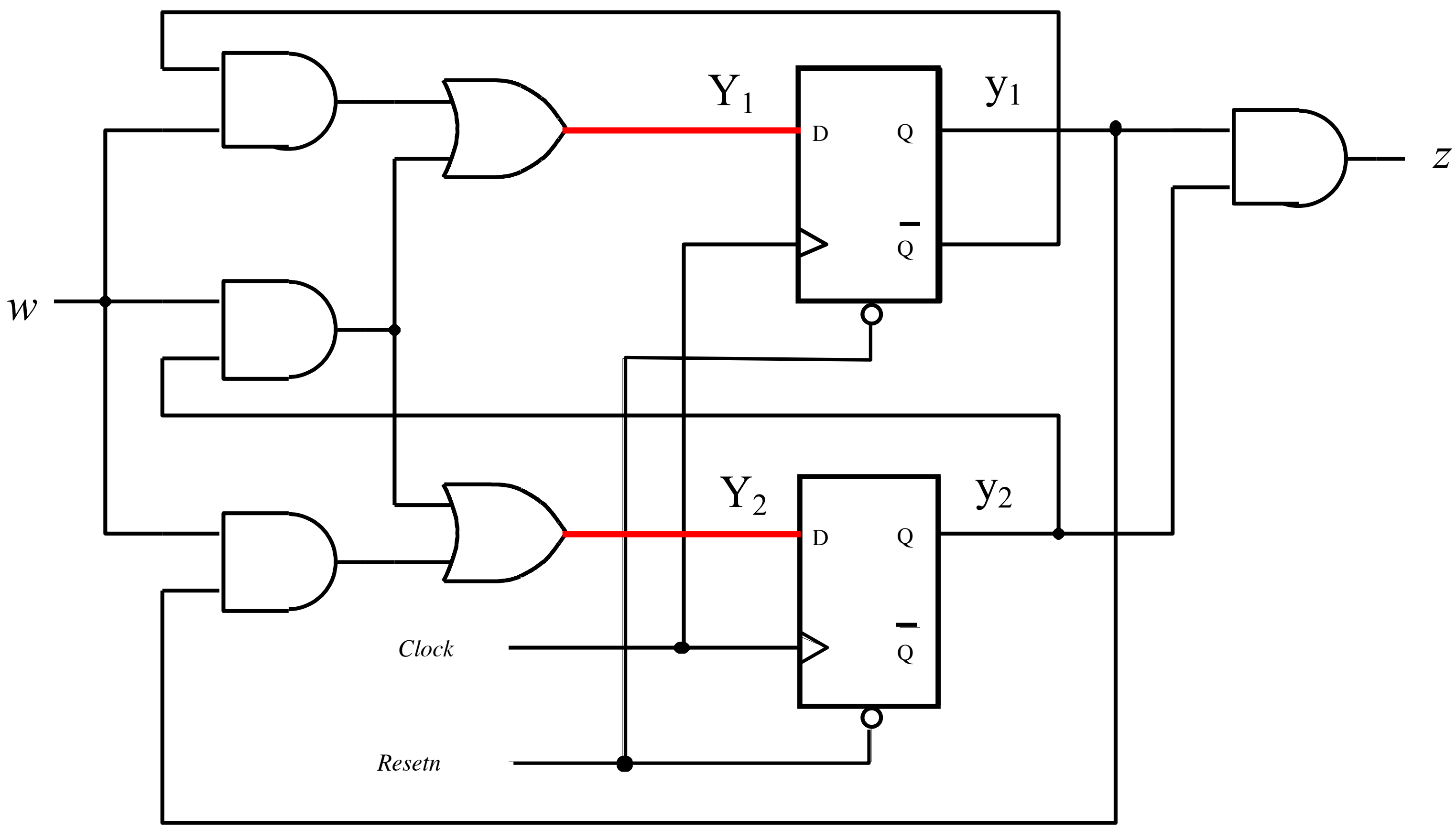
These are the present-state variables



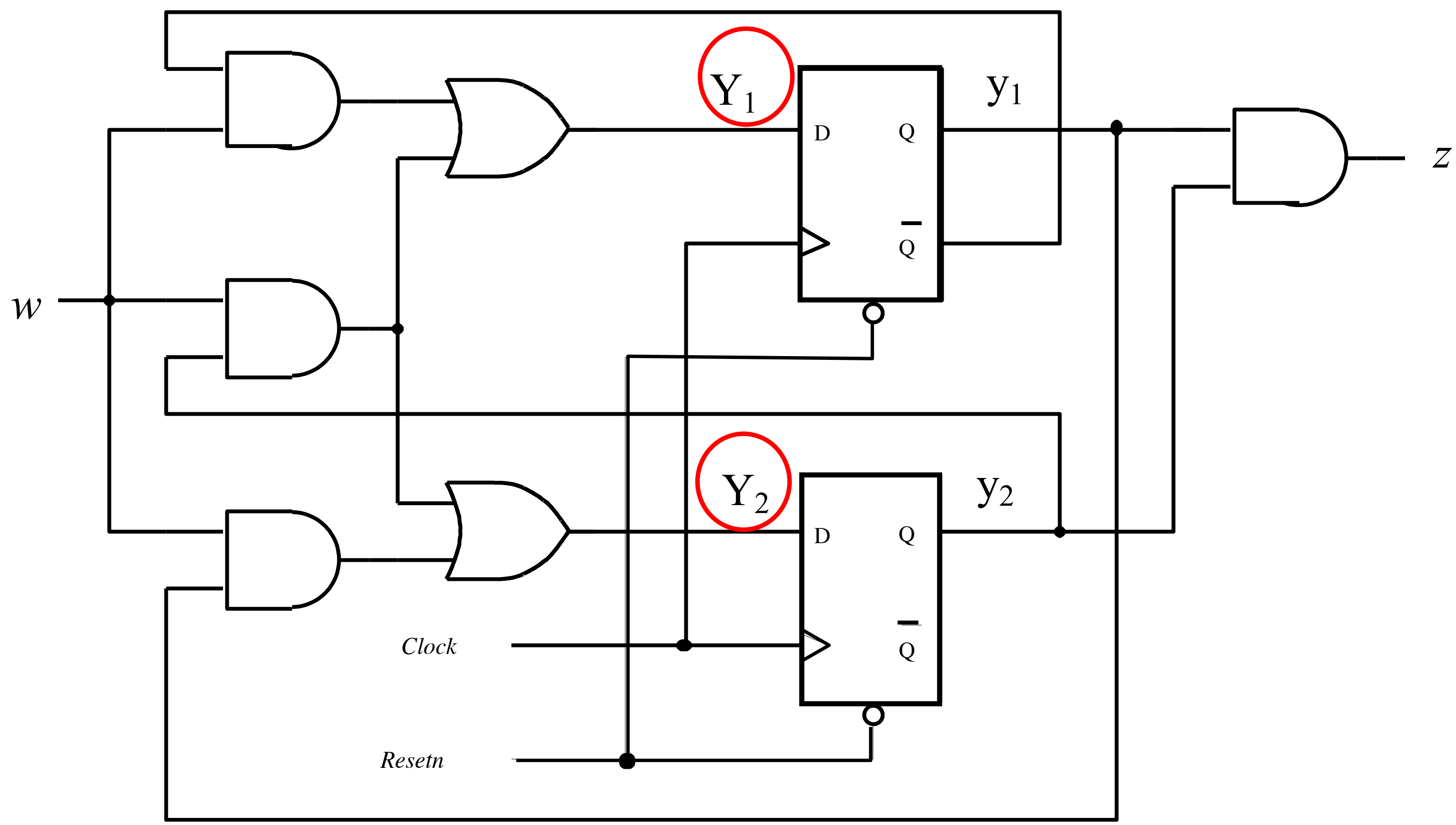
Where are the inputs of the flip-flops?



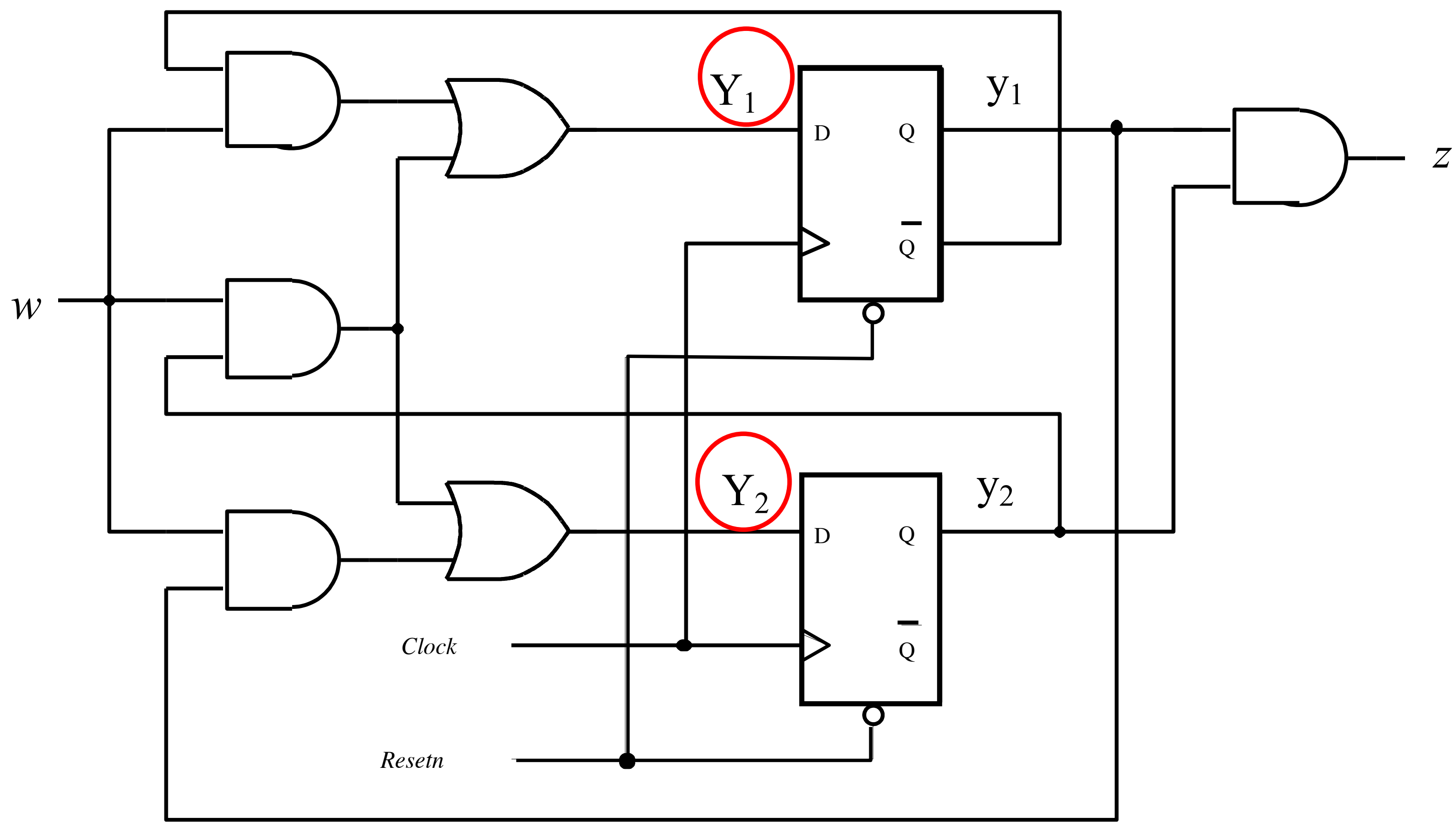
Where are the inputs of the flip-flops?



These are the next-state variables

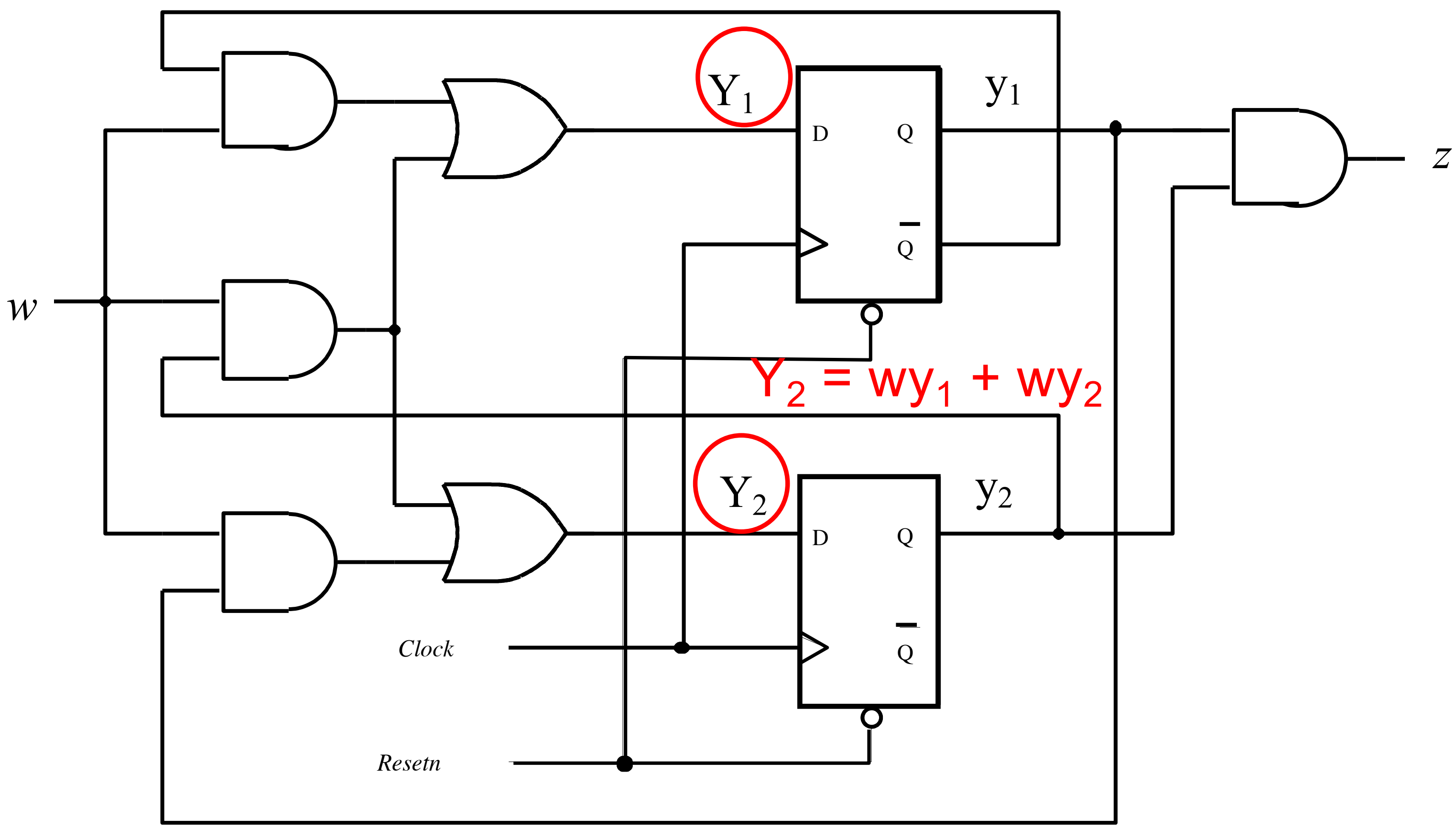


What are their logic expressions?



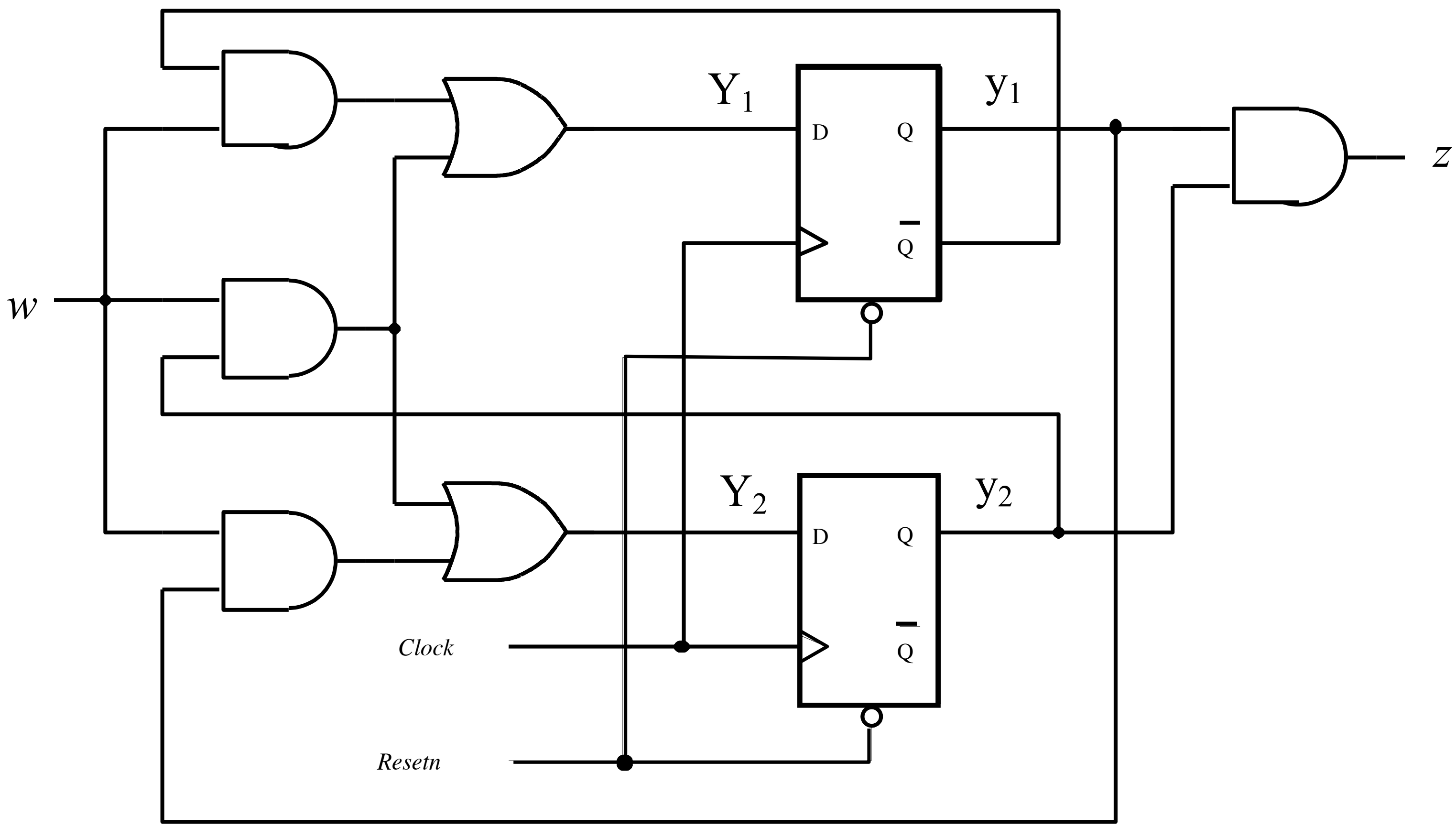
What are their logic expressions?

$Y_1 = w\overline{y_1} + wy_2$

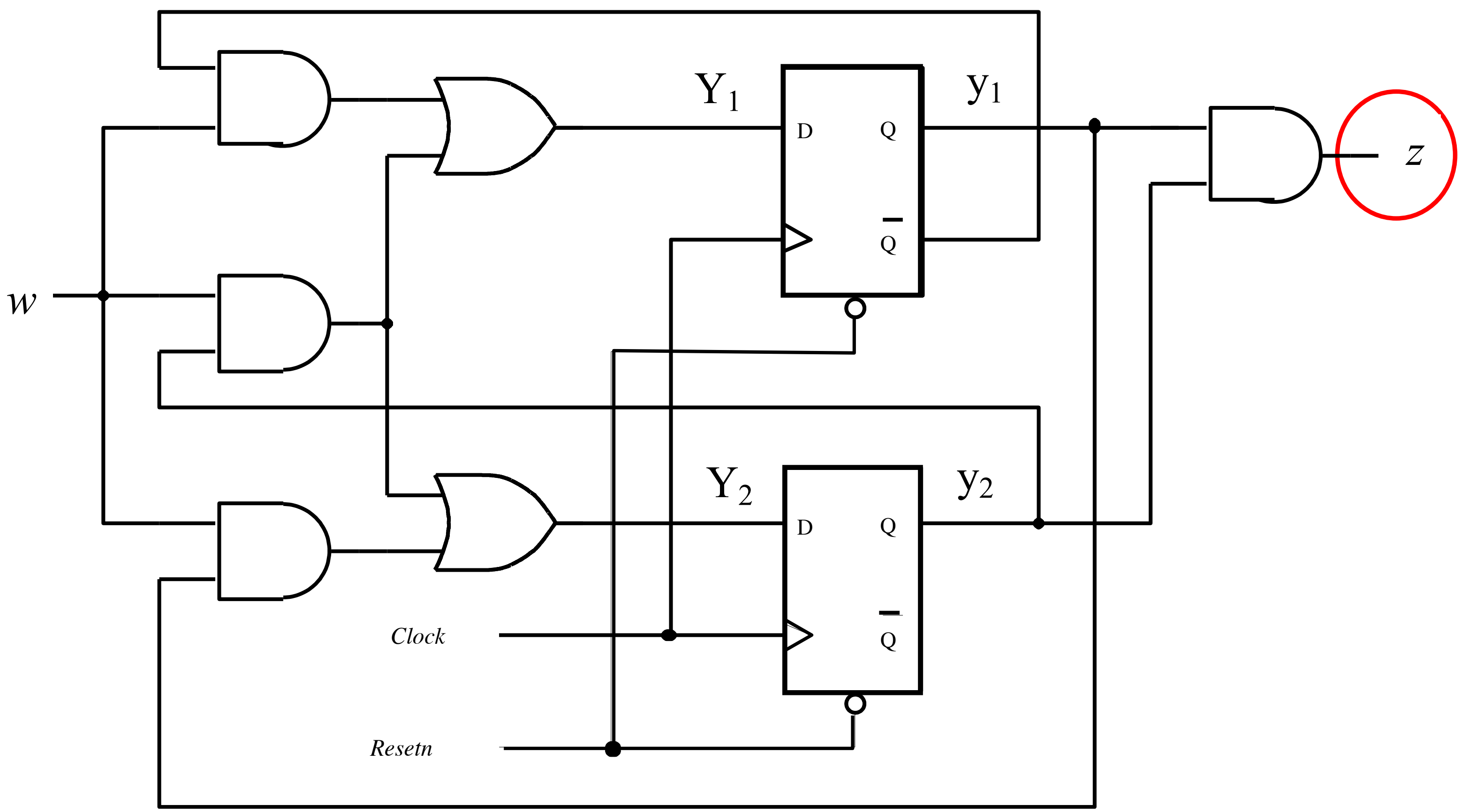


$Y_2 = wy_1 + wy_2$

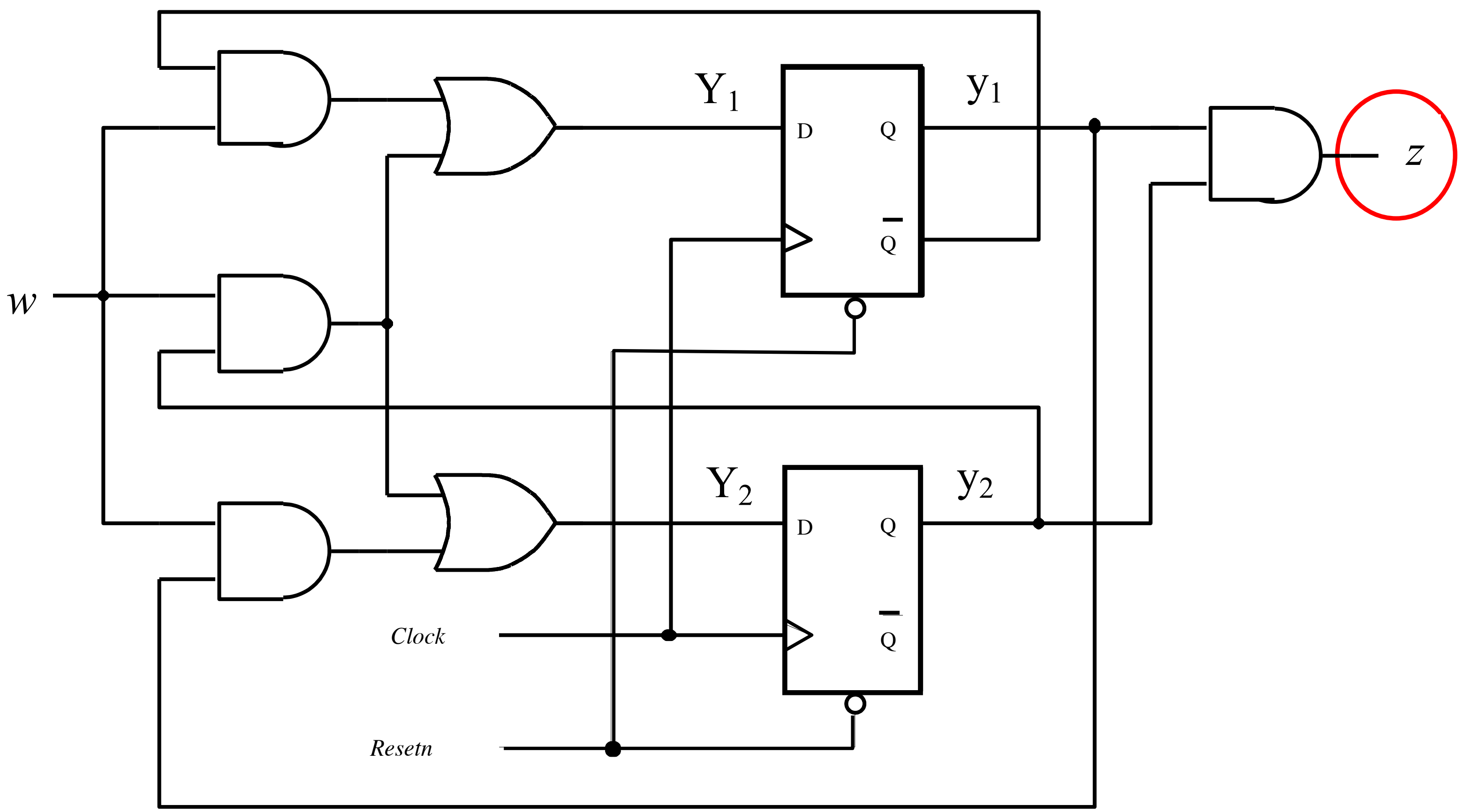
Where is the output, again?



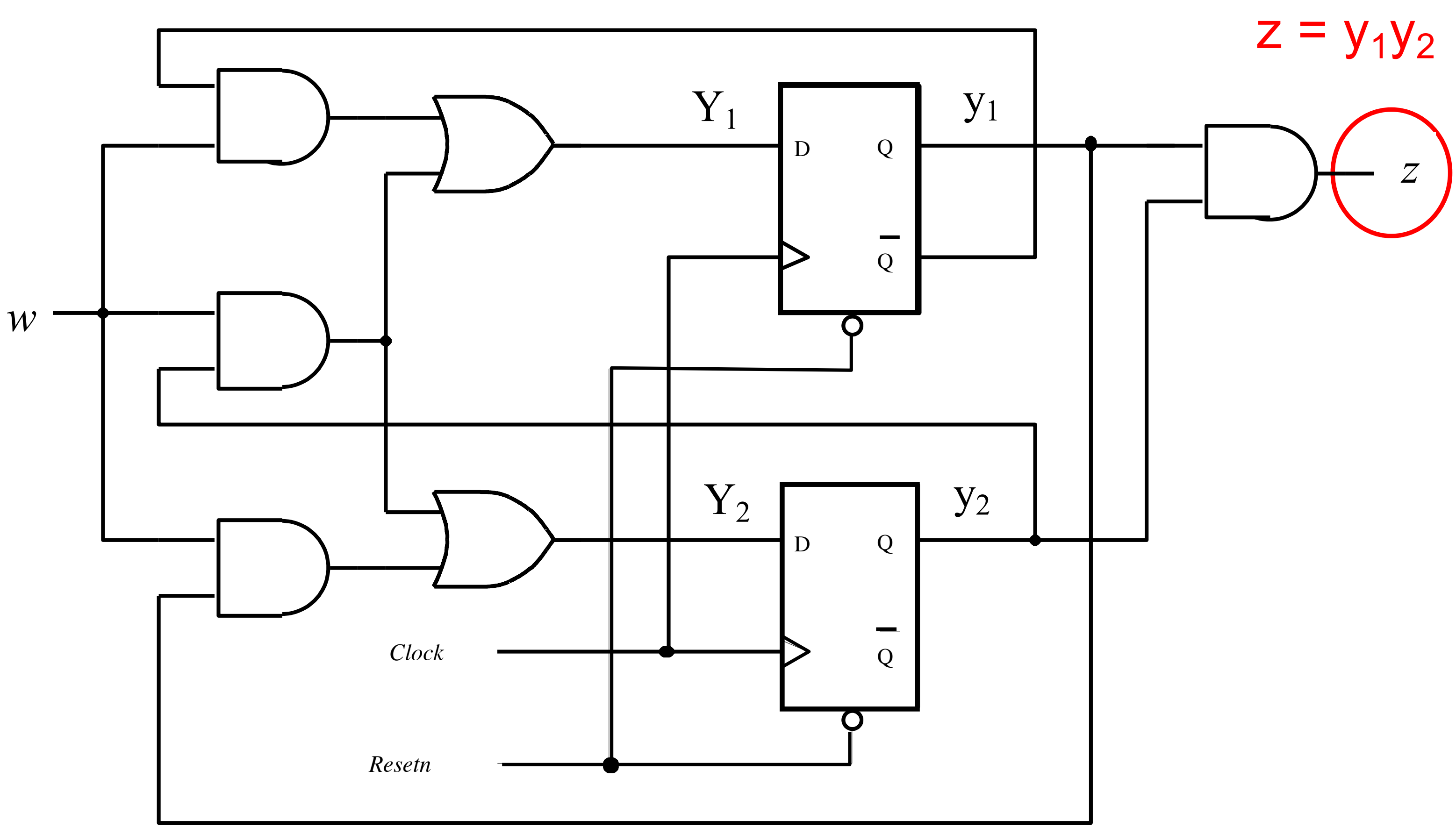
Where is the output, again?



What is its logic expression?



What is its logic expression?



This is what we have to work with now (we don't need the circuit anymore)

$$Y_1 = w\overline{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Let's derive the state-assigned table

$$Y_1 = w\overline{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State		Output z
	$w = 0$	$w = 1$	
	Y_2Y_1	Y_2Y_1	
0 0			
0 1			
1 0			
1 1			

Let's derive the state-assigned table

$$Y_1 = w\overline{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$\boxed{z = y_1 y_2}$$

Present state	Next State		Output
	$w = 0$	$w = 1$	
y_2y_1	Y_2Y_1	Y_2Y_1	z
0 0			
0 1			
1 0			
1 1			

Let's derive the state-assigned table

$$Y_1 = w\overline{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State: $w = 0$ Y_2Y_1	Next State: $w = 1$ Y_2Y_1	Output z
0 0			0
0 1			0
1 0			0
1 1			1

Let's derive the state-assigned table

$$\boxed{Y_1 = w\overline{y}_1 + wy_2}$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State $w = 0$ Y_2Y_1	Next State $w = 1$ Y_2Y_1	Output z
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0 1			0
1 0			0
1 1			1

Let’s derive the state-assigned table

$$Y_1 = w\overline{y}_1 + wy_2$$

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Present state y_2y_1	Next State $w = 0$ Y_2Y_1	Next State $w = 1$ Y_2Y_1	Output z
0 0	0	1	0
0 1	0	0	0
1 0	0	1	0
1 1	0	1	1

Let's derive the state-assigned table

$$Y_1 = w\overline{y}_1 + wy_2$$

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Let’s derive the state-assigned table

$Y_1 = w\overline{y_1} + wy_2$

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$z = y_1 y_2$

Present state y_2y_1	Next State $w = 0$ Y_2Y_1	Next State $w = 1$ Y_2Y_1	Output z
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

We don’t need the logic expressions anymore

$Y_1 = w\overline{y}_1 + wy_2$

$Y_2 = wy_1 + wy_2$

$z = y_1y_2$

Present state y_2y_1	Next State $w = 0$ Y_2Y_1	Next State $w = 1$ Y_2Y_1	Output z
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

We don’t need the logic expressions anymore

Present state y_2y_1	Next State		Output z
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	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
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State-assigned table

Let's derive the state table

Present state	Next state		Output z
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State table

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	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

State-assigned table

Let's derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A			
B			
C			
D			

State table

Present state y_2y_1	Next State w = 0 Y_2Y_1	Next State w = 1 Y_2Y_1	Output z
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

State-assigned table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

State table

Present state y_2y_1	Next State		Output z
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	Y_2Y_1	Y_2Y_1	
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State-assigned table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		
B	A		
C	A		
D	A		

State table

Present state y_2y_1	Next State		Output z
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	Y_2Y_1	Y_2Y_1	
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0 1	0 0	1 0	0
1 0	0 0	1 1	0
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State-assigned table

Let's derive the state table

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	w = 0	w = 1	
A	A		
B	A		
C	A		
D	A		

State table

Present state y_2y_1	Next State		Output z
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	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
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1 1	0 0	1 1	1

State-assigned table

Let's derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state y_2y_1	Next State w = 0 Y_2Y_1	Next State w = 1 Y_2Y_1	Output z
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

State-assigned table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

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0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

State-assigned table

Let’s derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

State-assigned table

The output is the same in both tables

The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
1 1	0 0	1 1	1

State-assigned table

[Figure 6.76 from the textbook]

We don't need the state-assigned table anymore

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
0 0	0 0	0 1	0
0 1	0 0	1 0	0
1 0	0 0	1 1	0
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State-assigned table

[Figure 6.76 from the textbook]

We don't need the state-assigned table anymore

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

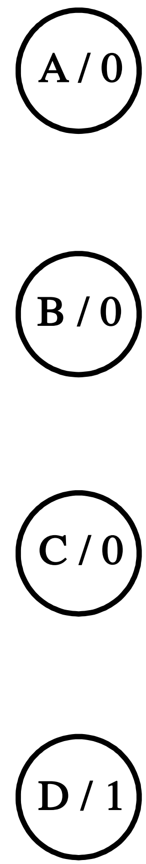
Let’s Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Let’s Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Because this is a Moore machine
the output is tied to the state



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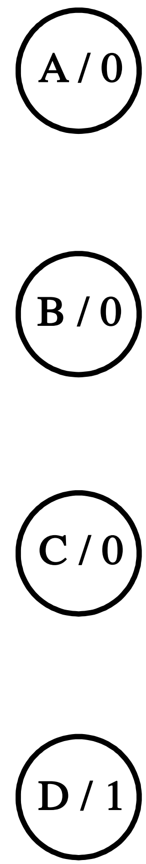
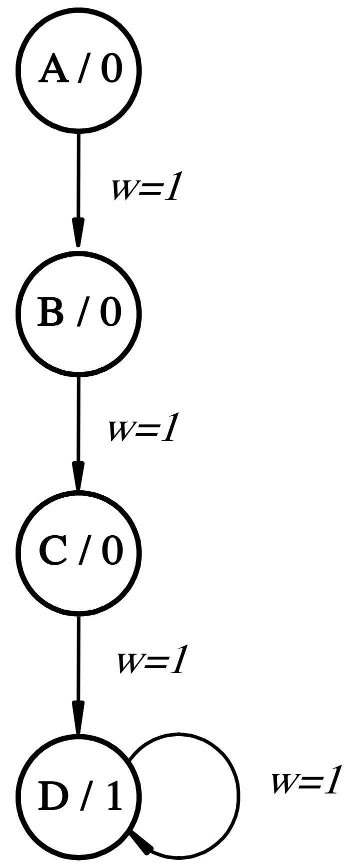


All transitions when the input w is equal to 1

Let’s Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

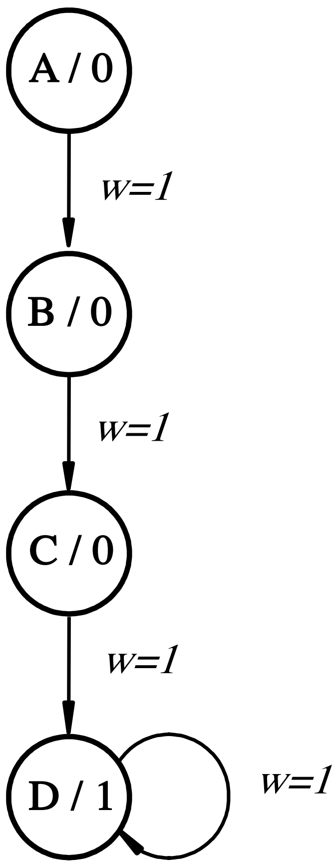
All transitions when the input w is equal to 1



Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

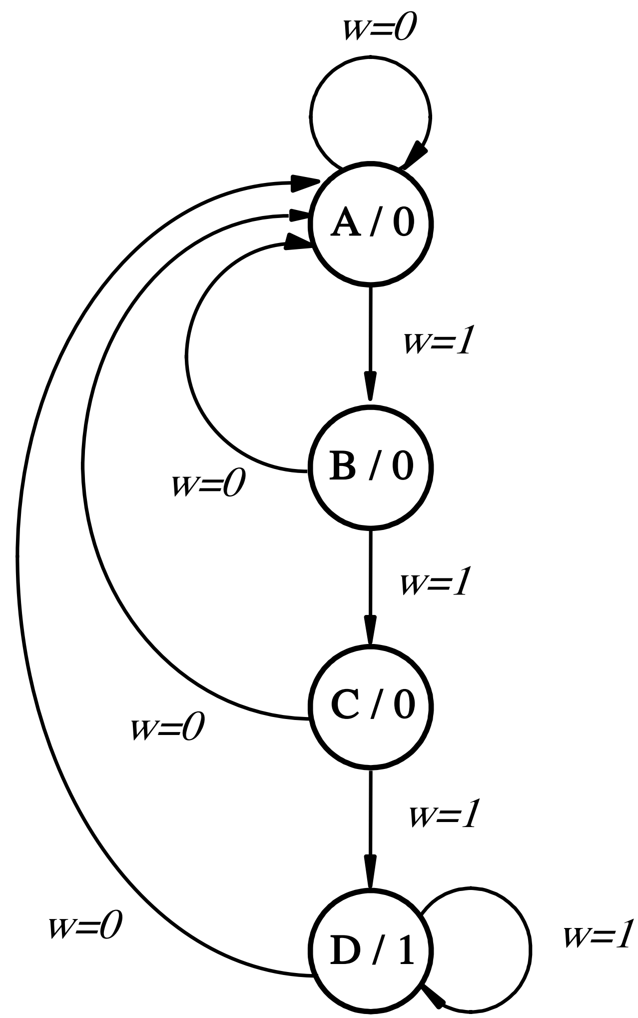
All transitions when the input w is equal to 0



Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

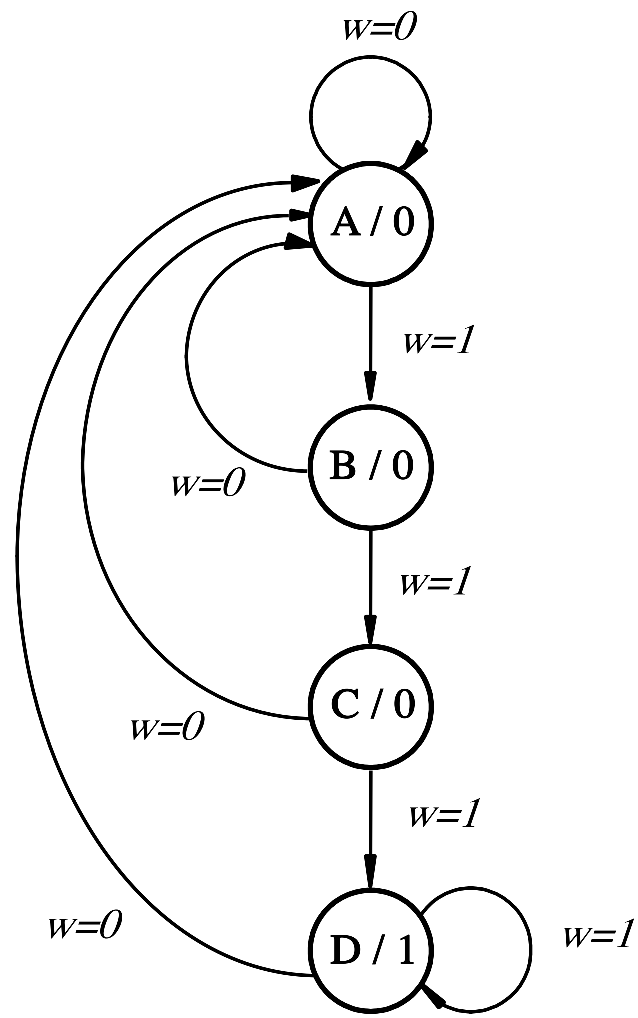
All transitions when the input w is equal to 0



We are done!

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

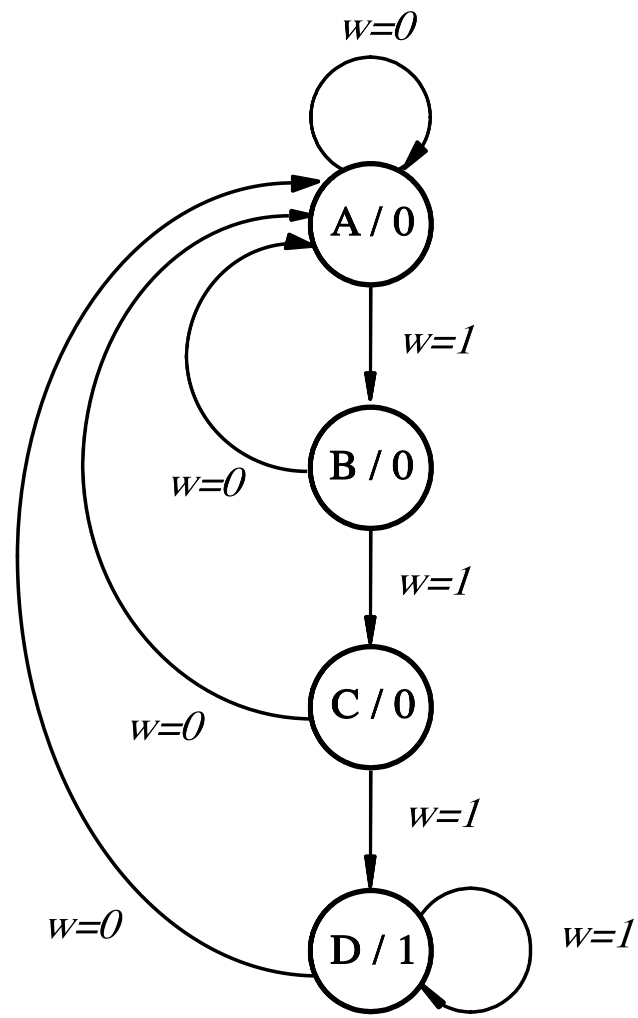


State diagram

Almost done. What does this FSM do?

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



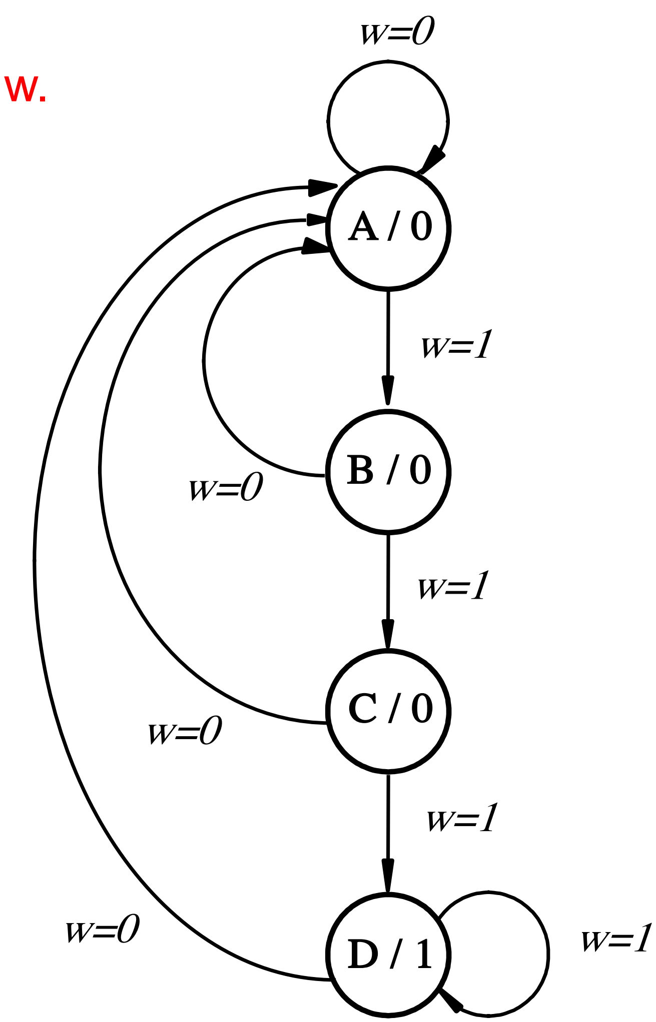
State diagram

Almost done. What does this FSM do?

It sets the output z to 1 when three consecutive 1's occur on the input w. In other words, it is a sequence detector for the input pattern 111.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

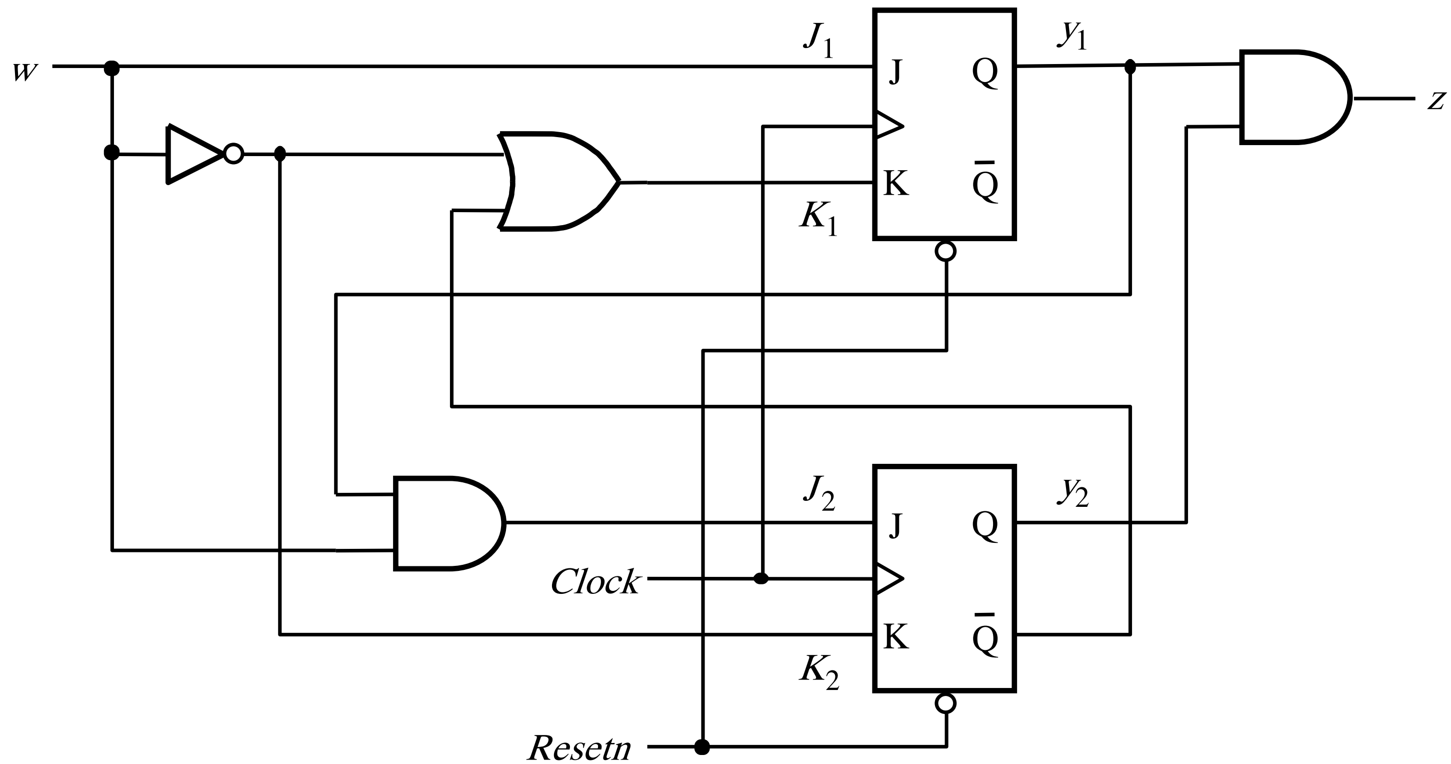
State table



State diagram

Another Example
(with JK flip-flops)

What does this circuit do?

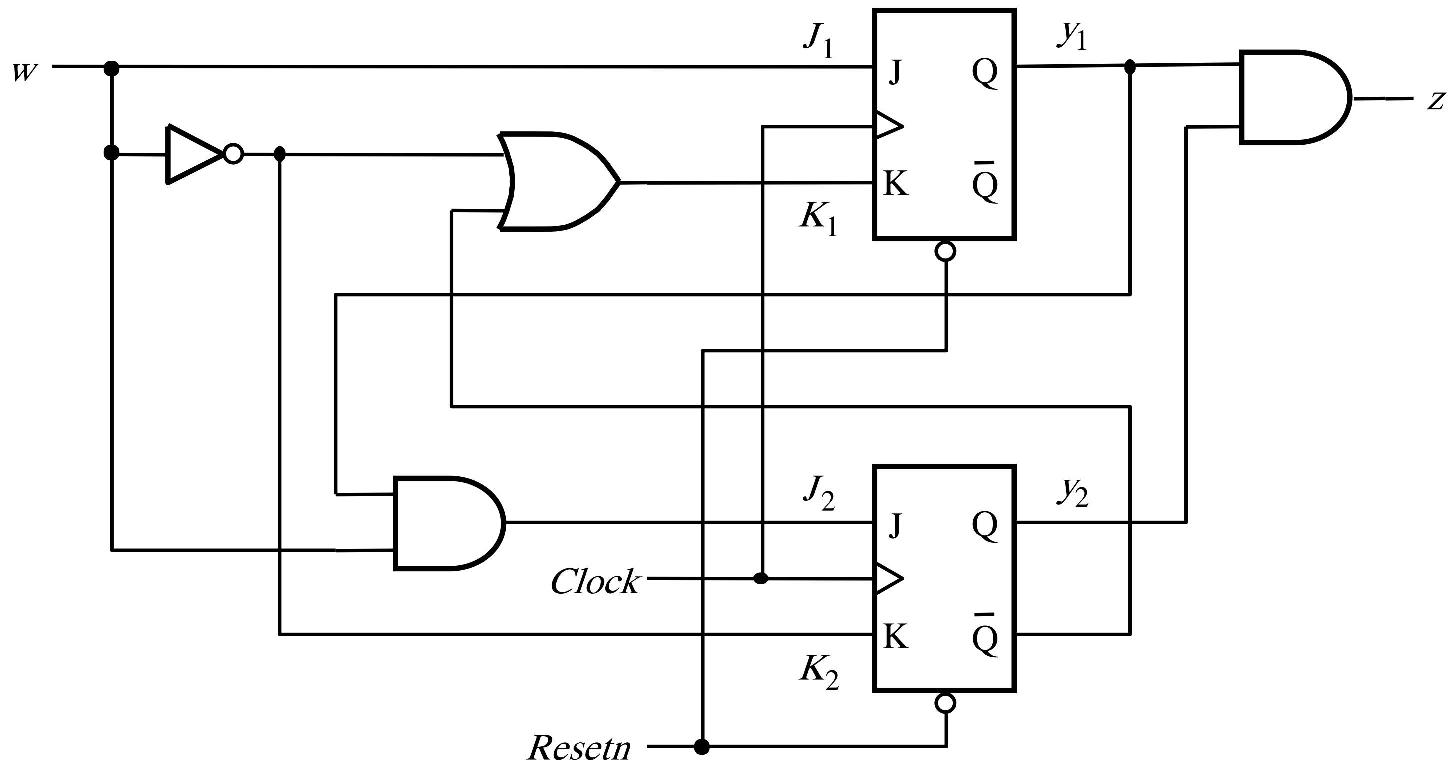


[Figure 6.77 from the textbook]

Approach

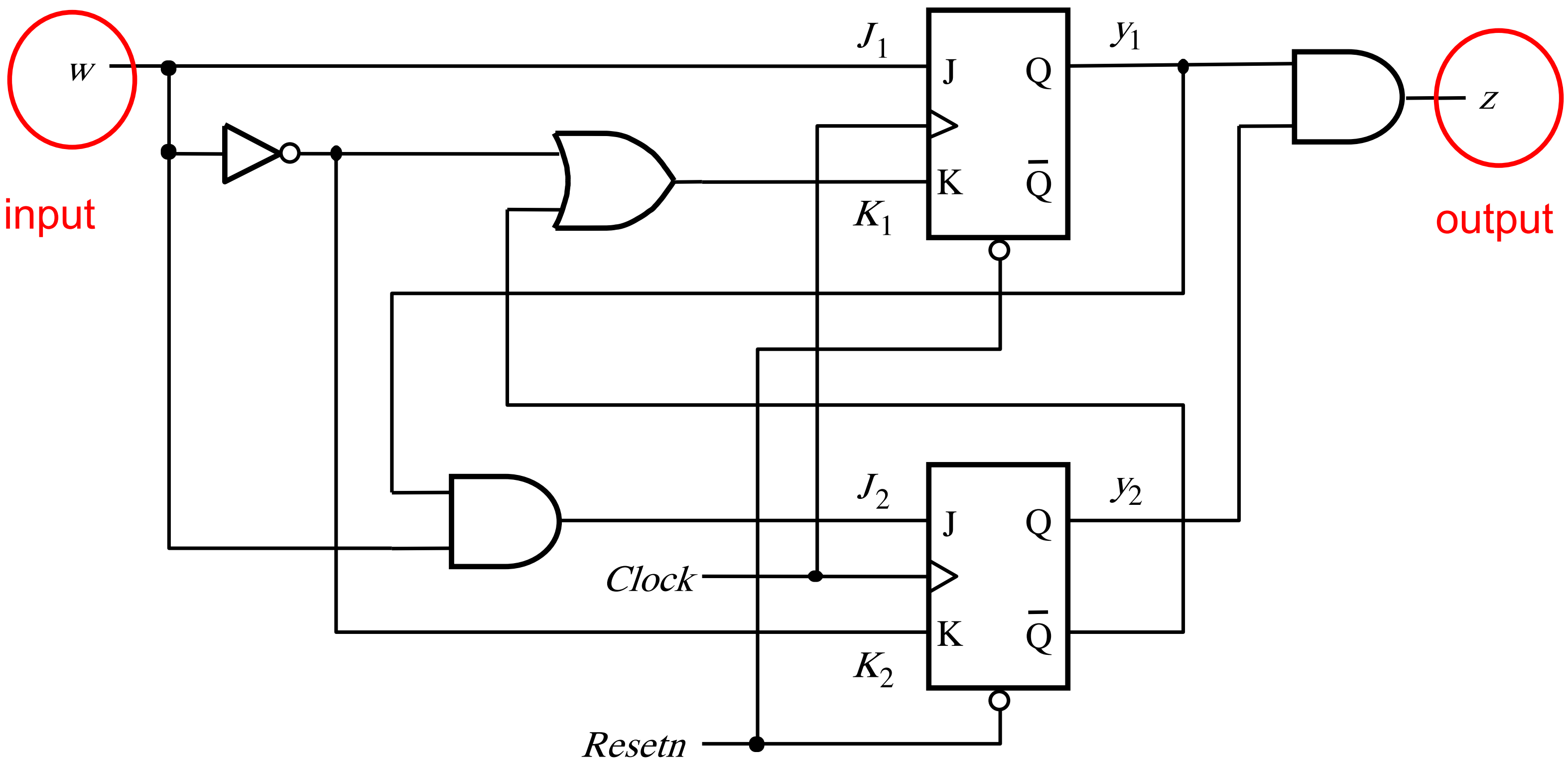
- Find the flip-flops
- Outputs of the flip-flops = present state variables
- Inputs of the flip-flops determine the next state variables
- Determine the logical expressions for the outputs
- Given this info it is easy to do the state-assigned table
- Next do the state table
- Finally, draw the state diagram.

Where are the inputs and outputs?

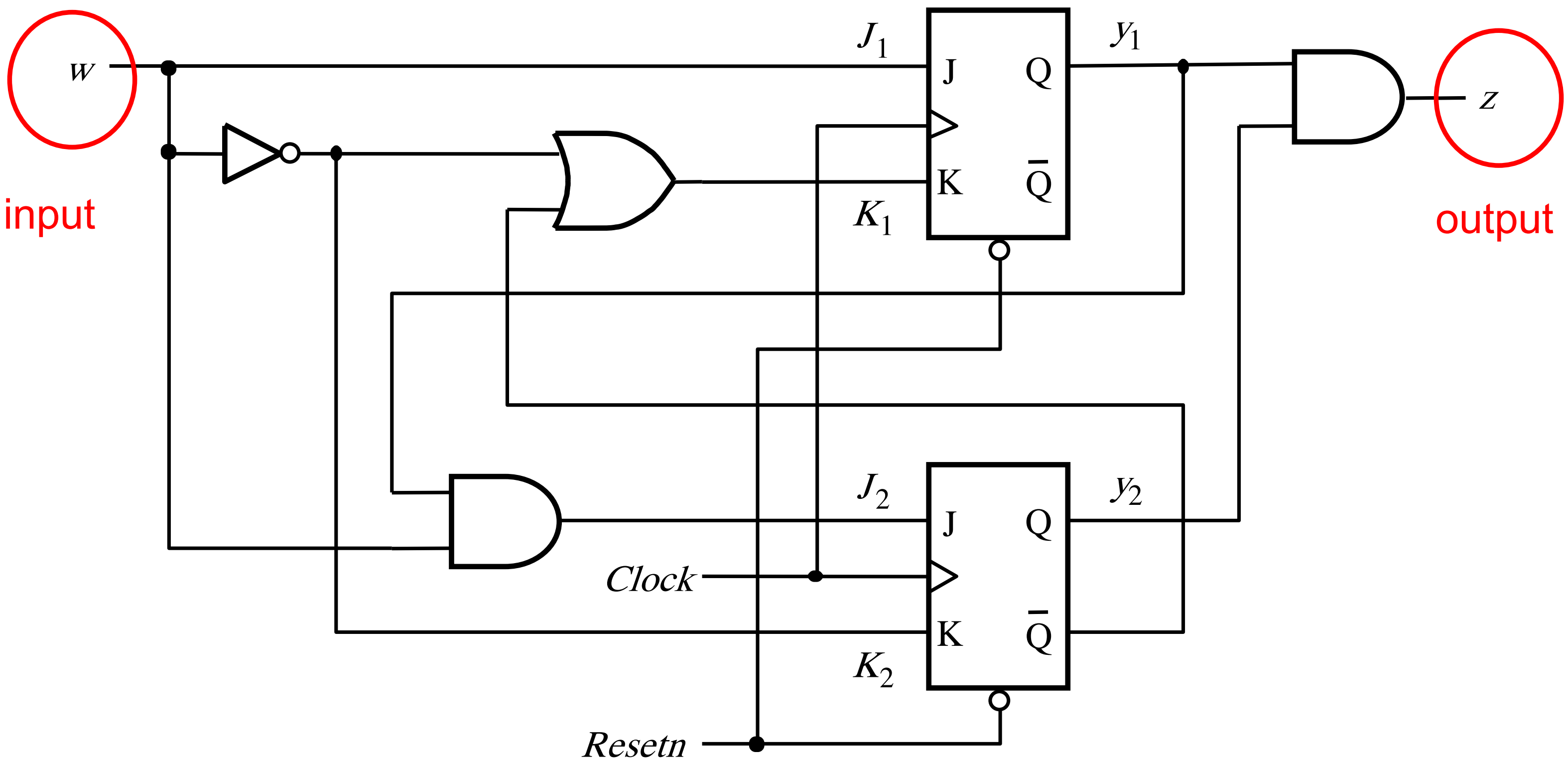


[Figure 6.77 from the textbook]

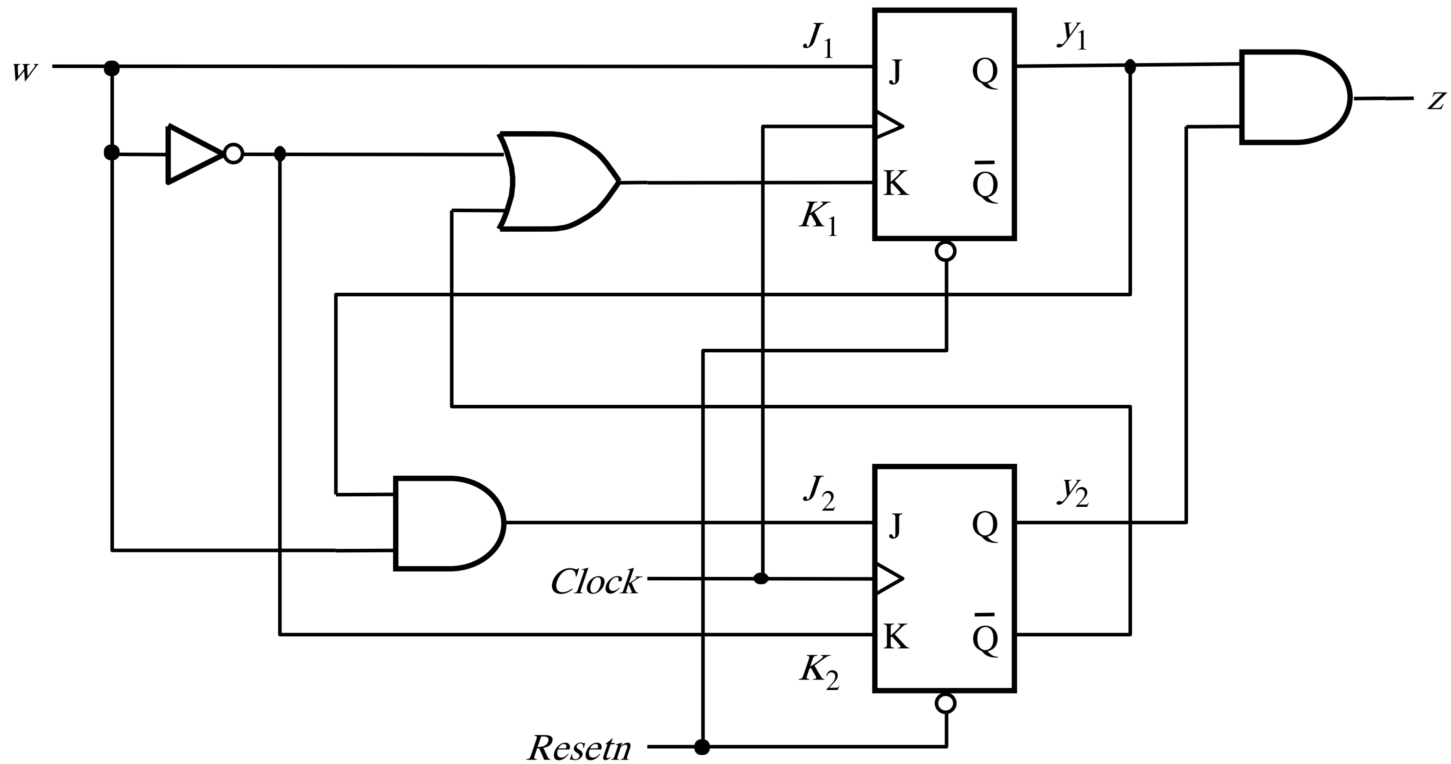
Where are the inputs and outputs?



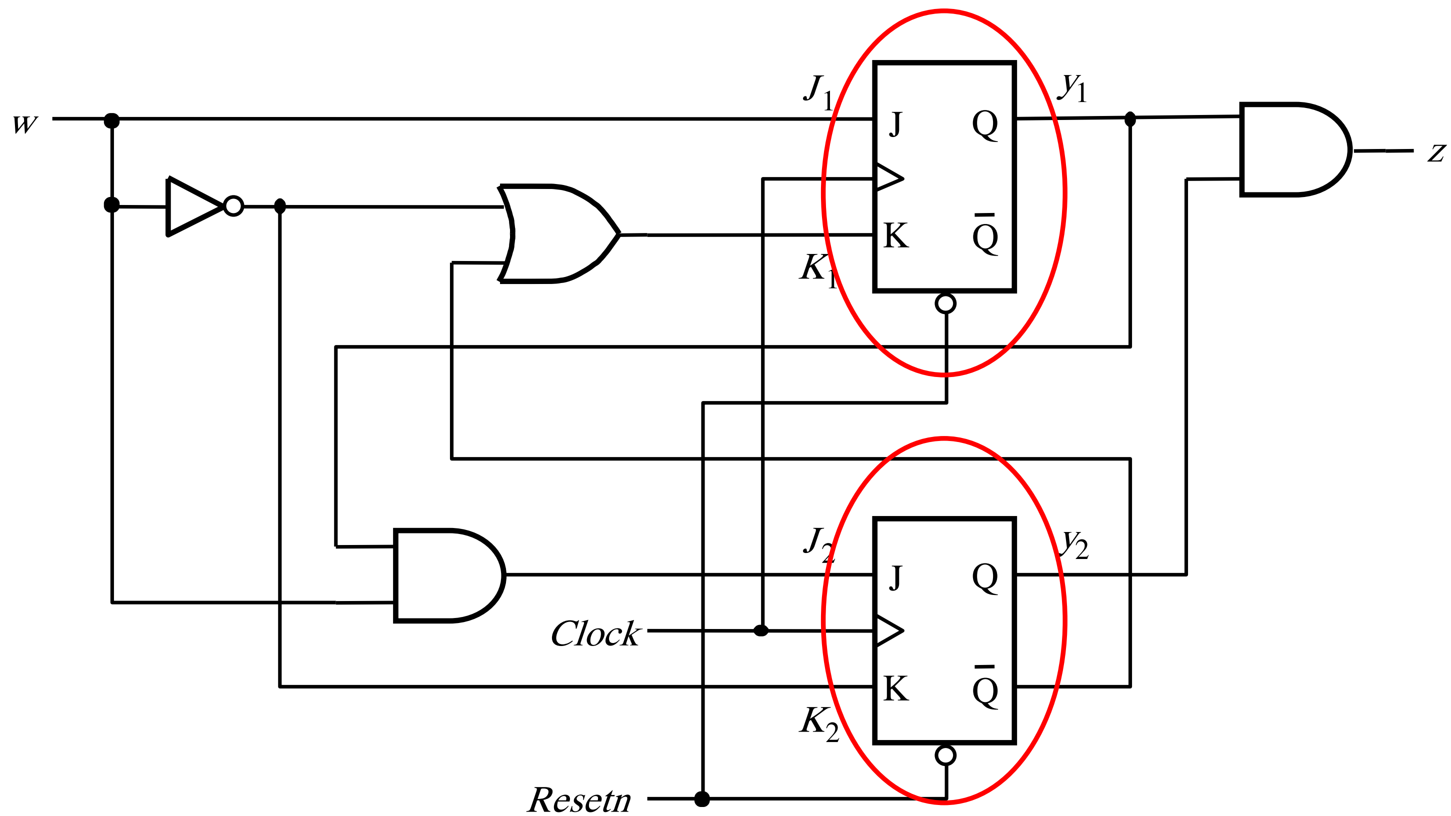
What kind of machine is this?



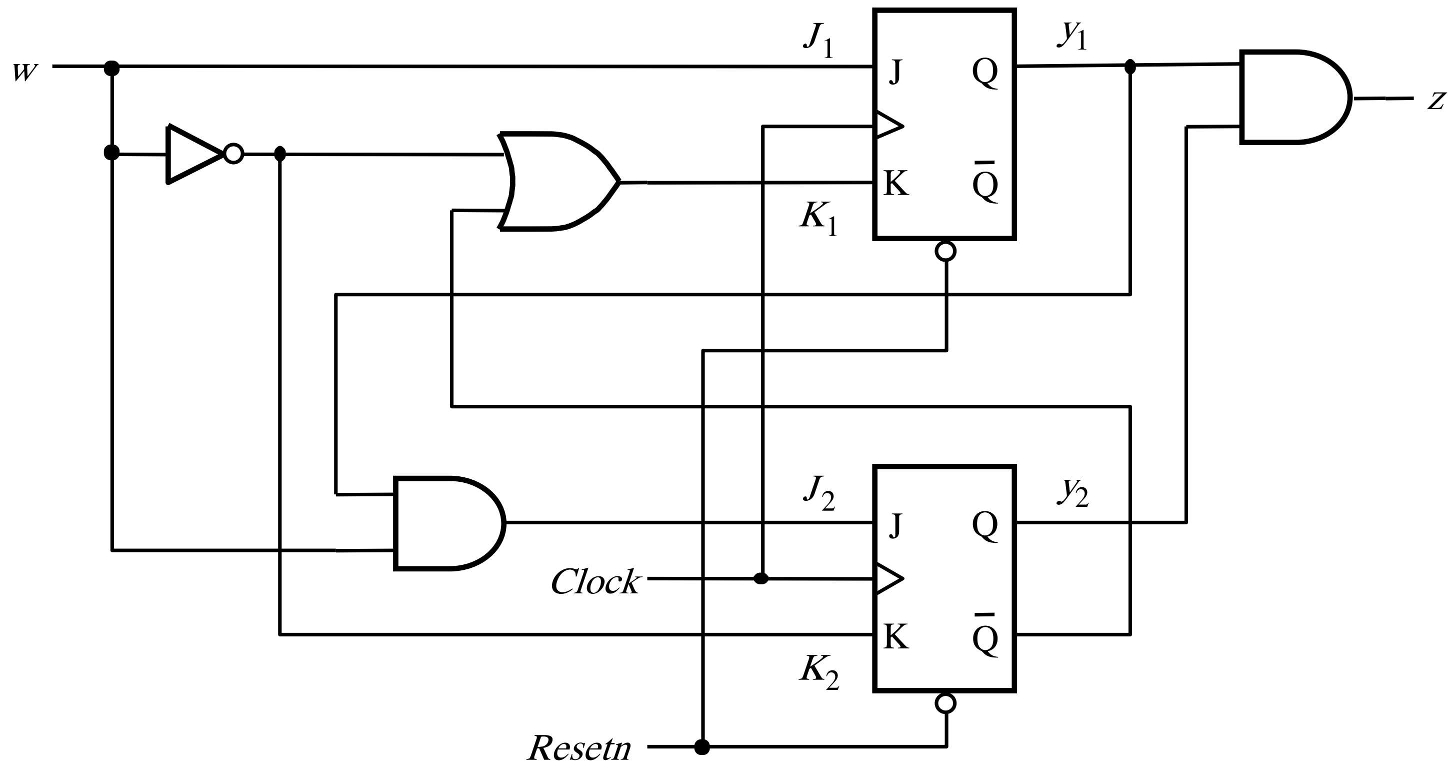
Where are the flip-flops?



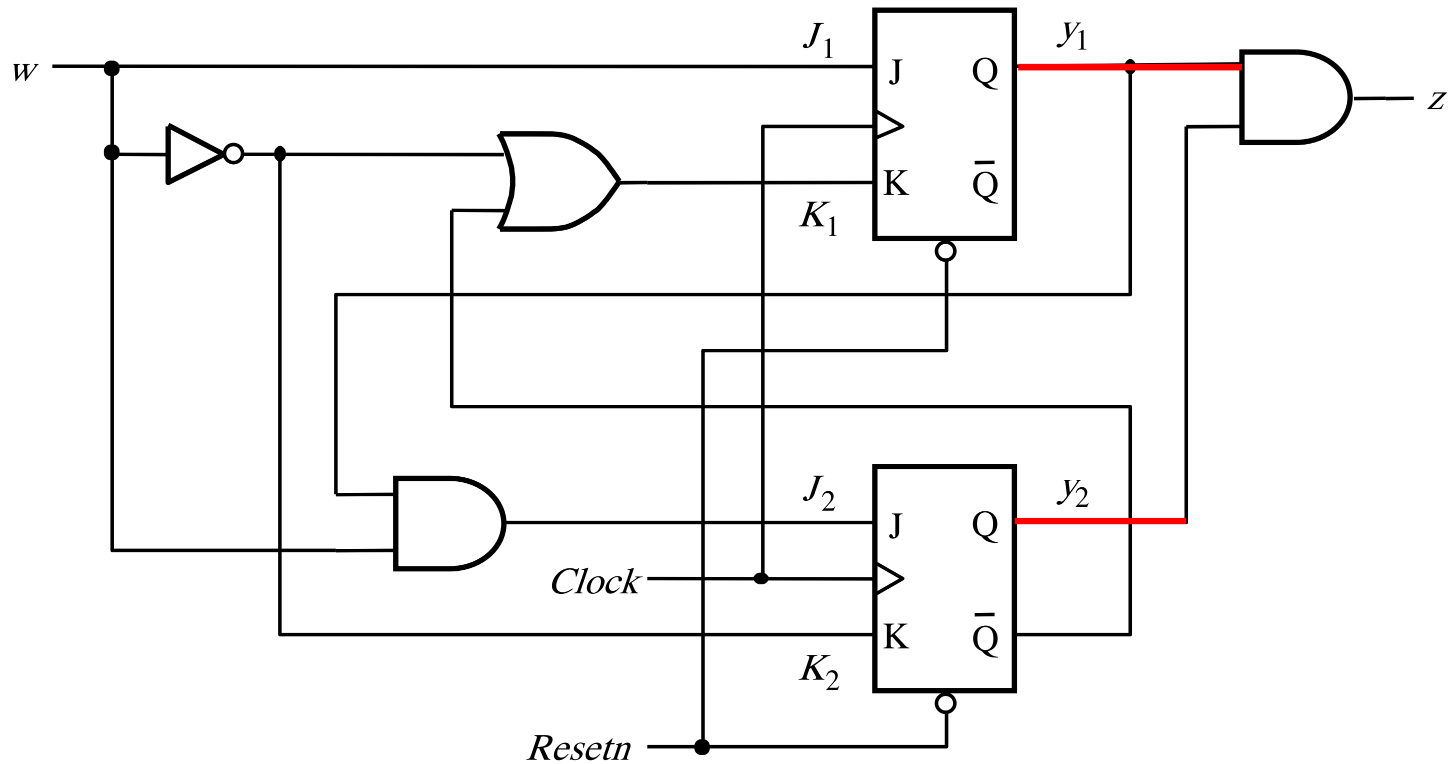
Where are the flip-flops?



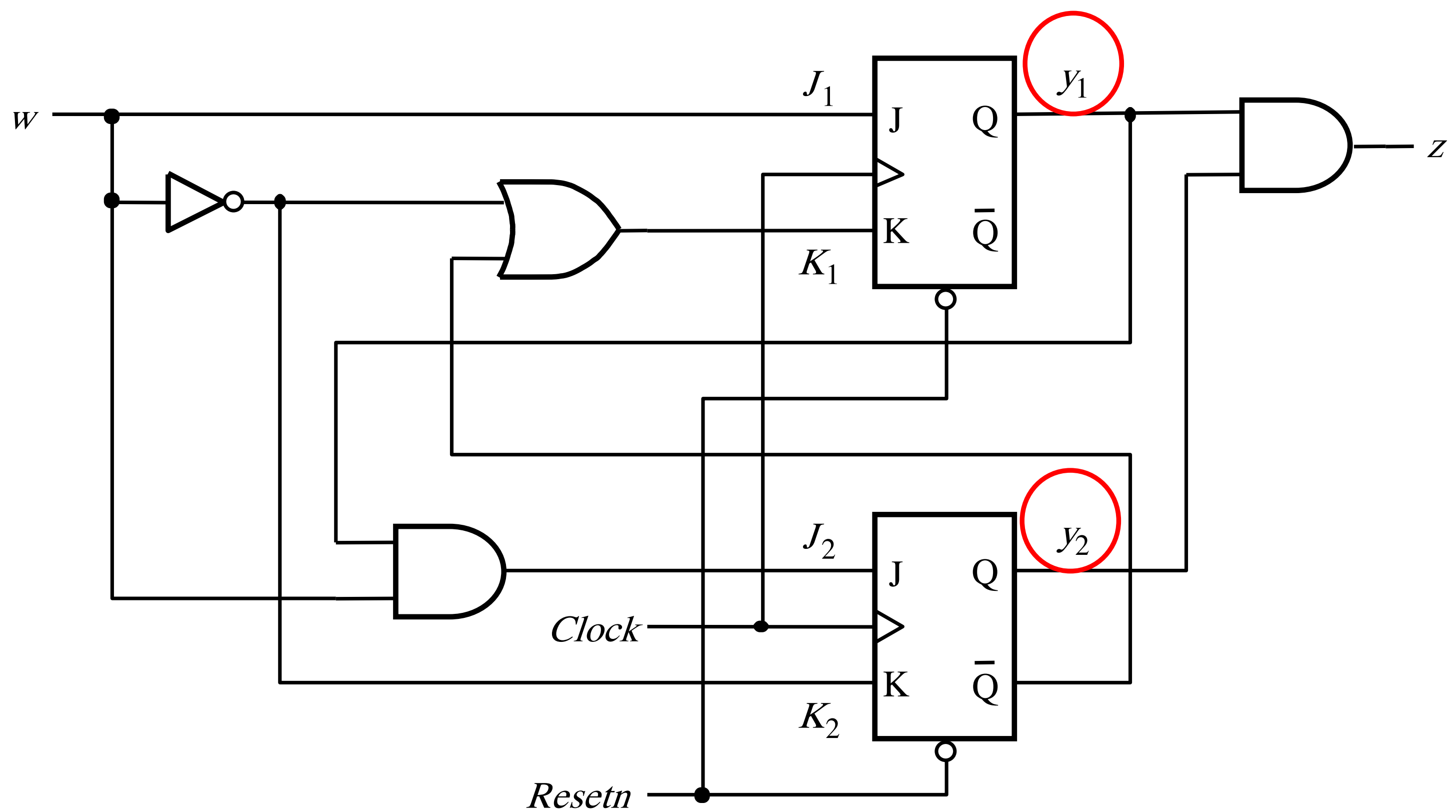
Where are the outputs of the flip-flops?



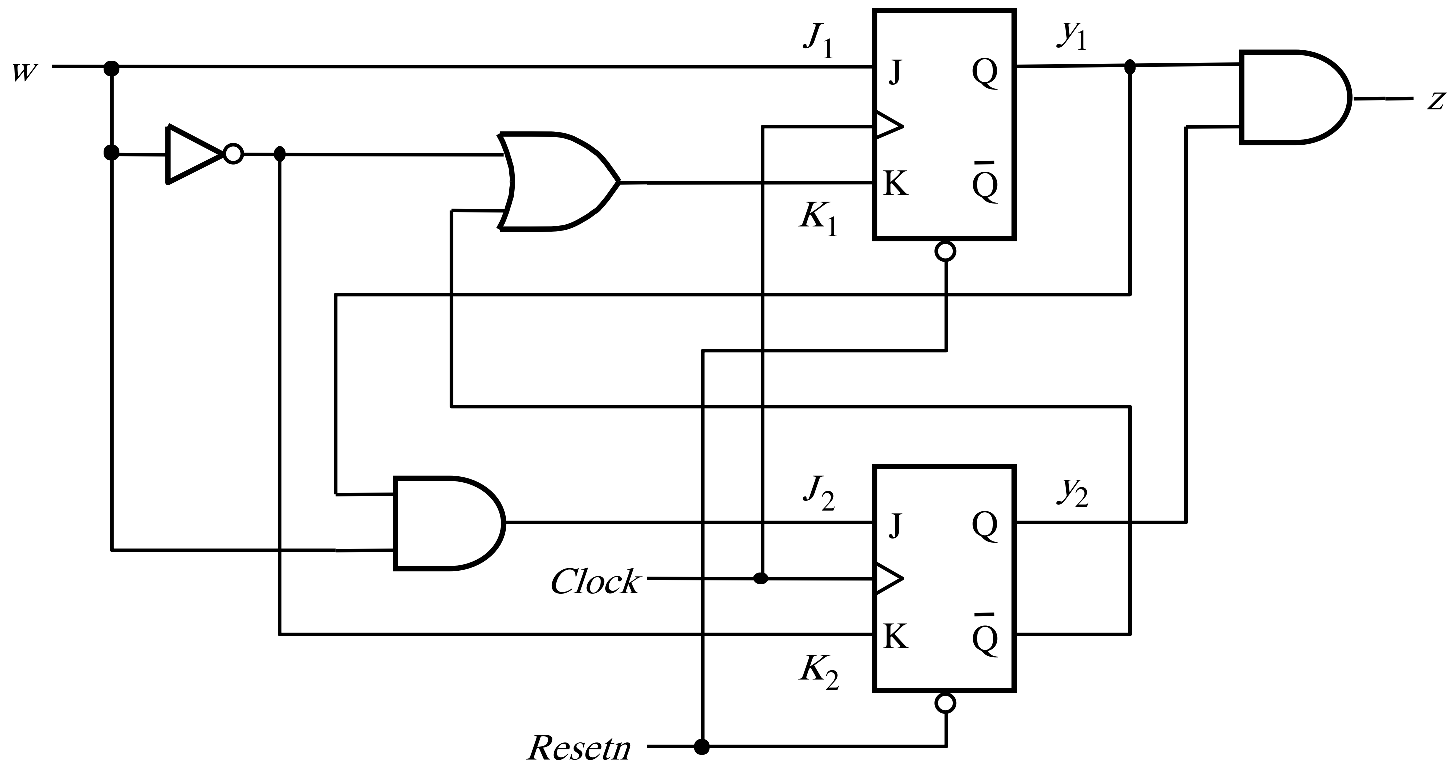
Where are the outputs of the flip-flops?



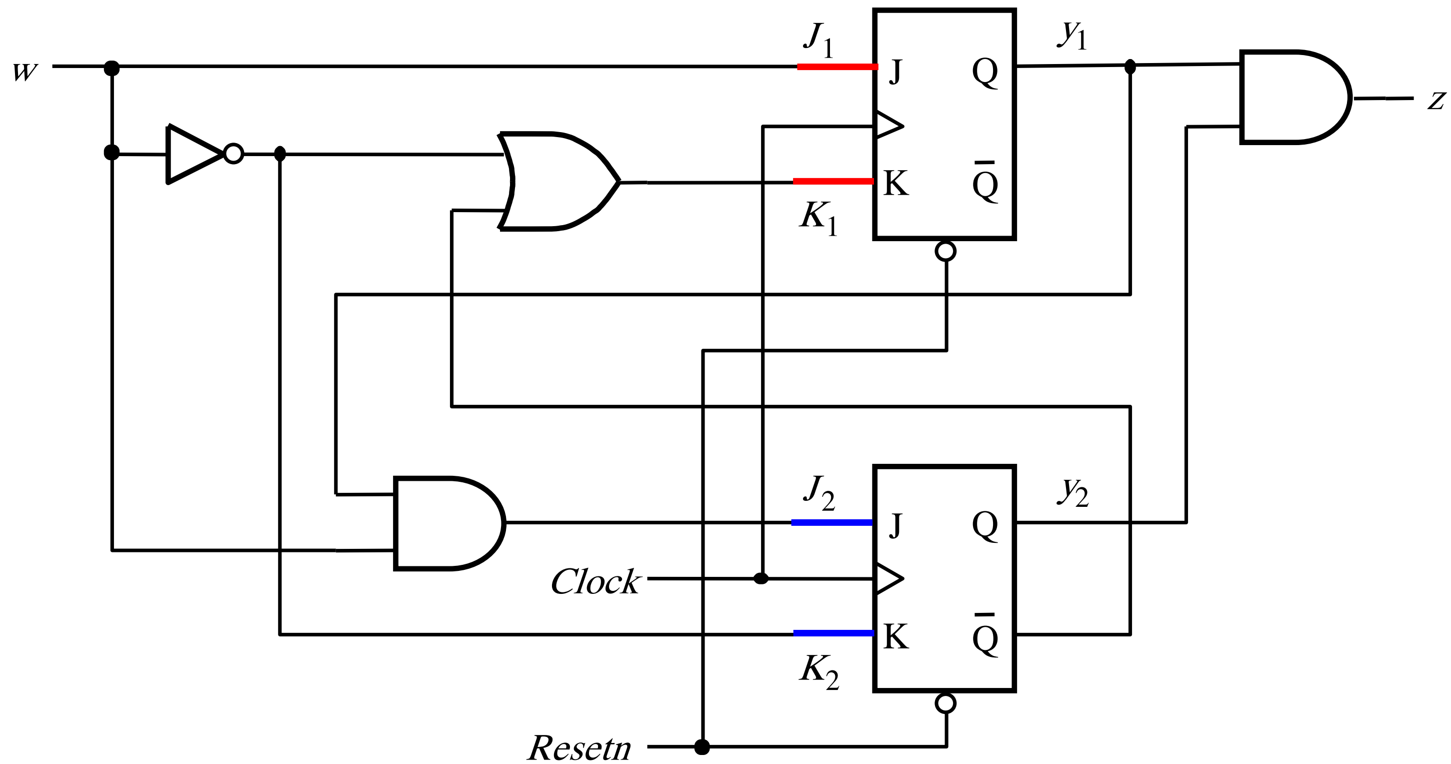
These are the present-state variables



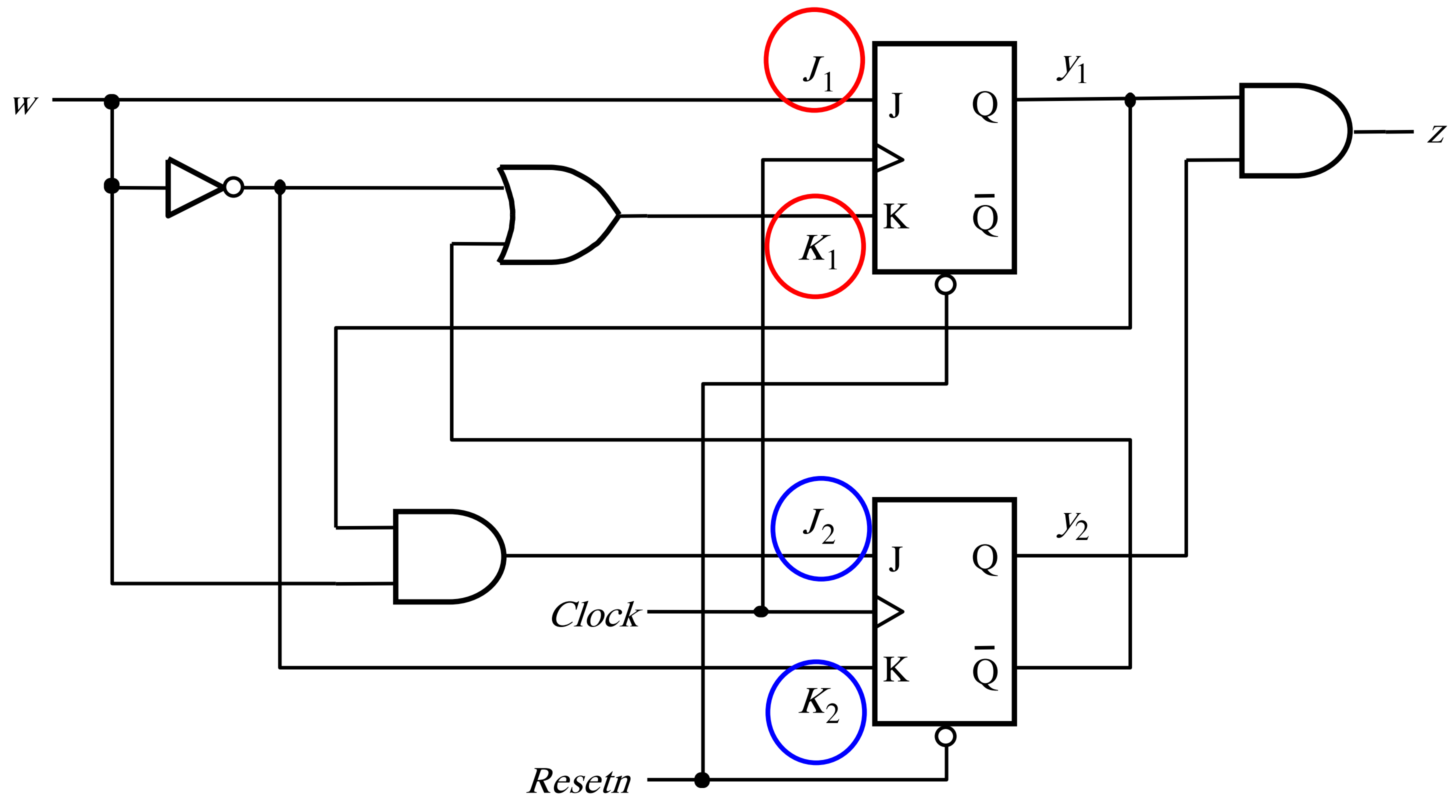
Where are the inputs of the flip-flops?



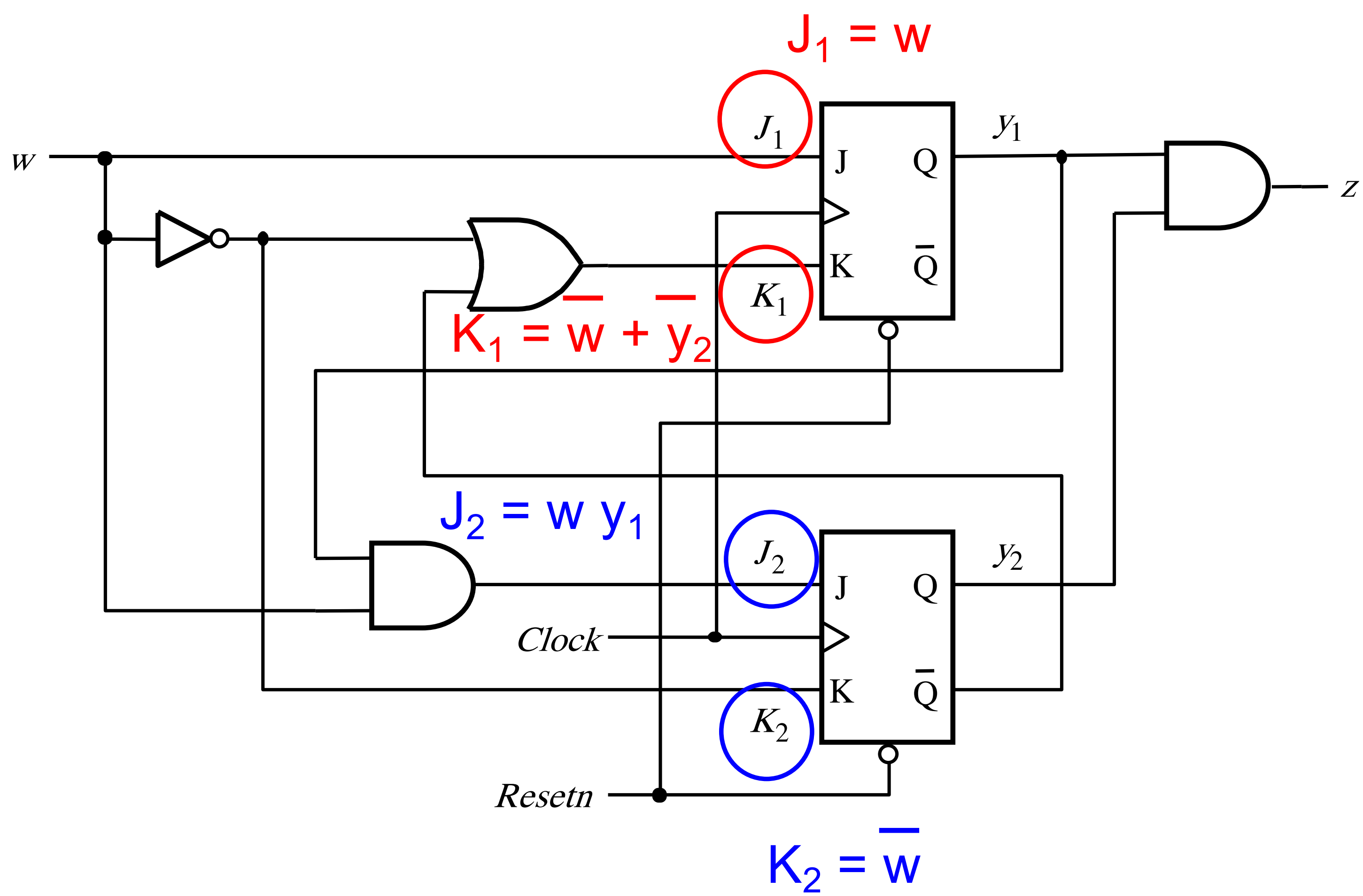
Where are the inputs of the flip-flops?



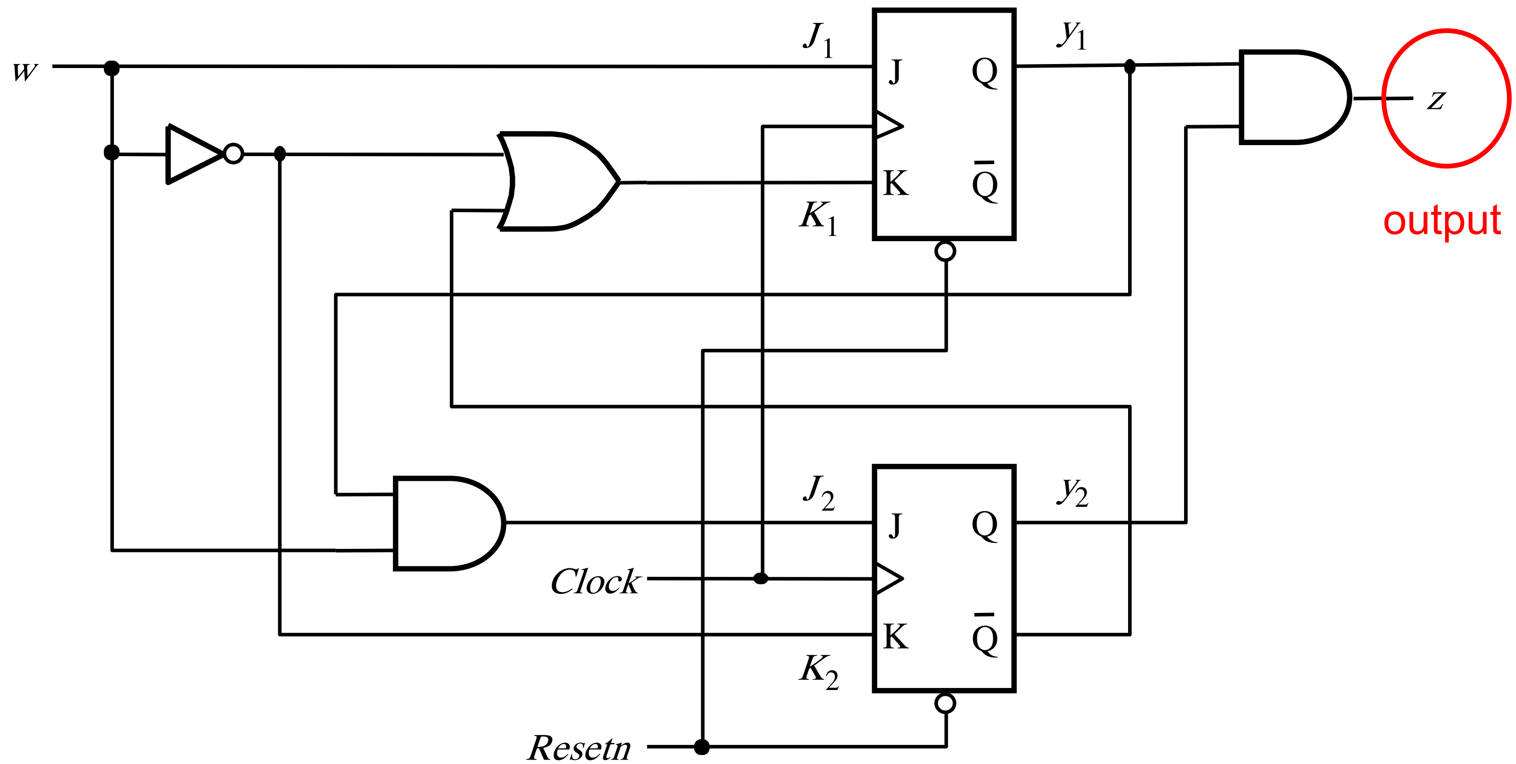
What are their logic expressions?



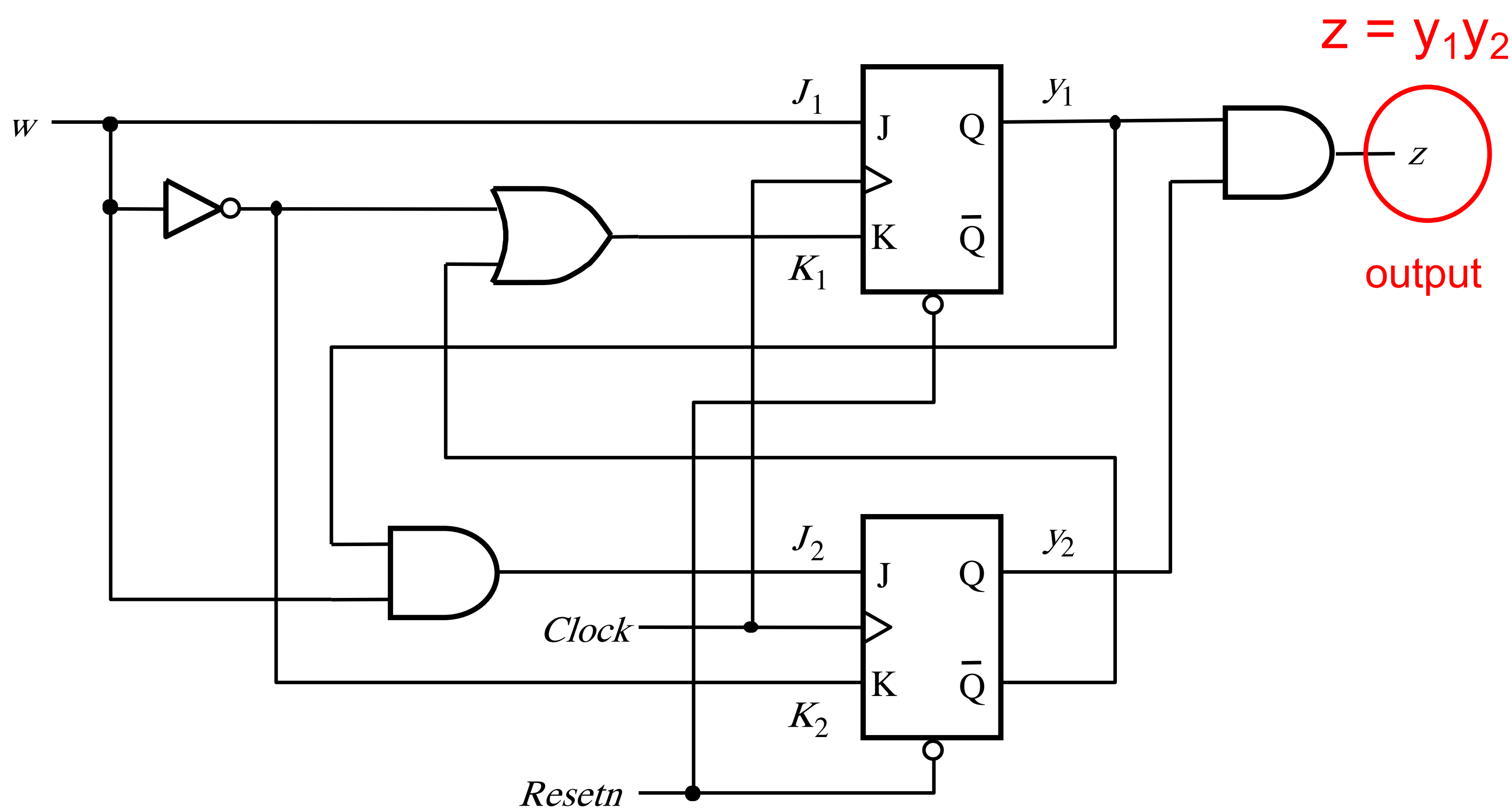
What are their logic expressions?



What is the logic expression of the output?



What is the logic expression of the output?



This is what we have to work with now (we don't need the circuit anymore)

$$J_1 = w$$

$$K_1 = \overline{w} + \overline{y_2}$$

$$J_2 = w\, y_1$$

$$K_2 = \overline{w}$$

$$z = y_1 y_2$$

Let's derive the excitation table

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\ y_1$

$K_2 = \overline{w}$

$z = y_1 y_2$

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00					
01					
10					
11					

Let's derive the excitation table

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\, y_1$

$K_2 = \overline{w}$

$\boxed{z = y_1 y_2}$

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00					
01					
10					
11					

Let's derive the excitation table

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\, y_1$

$K_2 = \overline{w}$

$\boxed{z = y_1 y_2}$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00					0
01					0
10					0
11					1

Let's derive the excitation table

$$\boxed{\begin{aligned} J_1 &= w \\ K_1 &= \overline{w} + \overline{y_2} \end{aligned}}$$

$J_2 = w\, y_1$

$K_2 = \overline{w}$

$z = y_1 y_2$

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00					0
01					0
10					0
11					1

Let's derive the excitation table

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\, y_1$

$K_2 = \overline{w}$

$z = y_1 y_2$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00		0 1		1 1	0
01		0 1		1 1	0
10		0 1		1 0	0
11		0 1		1 0	1

Let's derive the excitation table

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\ y_1$

$K_2 = \overline{w}$

$z = y_1 y_2$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00		0 1		1 1	0
01		0 1		1 1	0
10		0 1		1 0	0
11		0 1		1 0	1

The excitation table

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\, y_1$

$K_2 = \overline{w}$

$z = y_1 y_2$

Present state	Flip-flop inputs				Output
	$w = 0$		$w = 1$		
$y_2 y_1$	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	z
00	01	0 1	0 0	1 1	0
01	01	0 1	1 0	1 1	0
10	01	0 1	0 0	1 0	0
11	01	0 1	1 0	1 0	1

[Figure 6.78 from the textbook]

We don’t need the logic expressions anymore

$J_1 = w$

$K_1 = \overline{w} + \overline{y_2}$

$J_2 = w\, y_1$

$K_2 = \overline{w}$

$z = y_1 y_2$

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	0 1	0 0	1 1	0
01	01	0 1	1 0	1 1	0
10	01	0 1	0 0	1 0	0
11	01	0 1	1 0	1 0	1

[Figure 6.78 from the textbook]

We don’t need the logic expressions anymore

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	0 1	0 0	1 1	0
01	01	0 1	1 0	1 1	0
10	01	0 1	0 0	1 0	0
11	01	0 1	1 0	1 0	1

[Figure 6.78 from the textbook]

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A			
B			
C			
D			

State table

Present state	Flip-flop inputs				Output
	$w = 0$		$w = 1$		
y_2y_1	J_2K_2	J_1K_1	J_2K_2	J_1K_1	z
00	01	0 1	0 0	1 1	0
01	01	0 1	1 0	1 1	0
10	01	0 1	0 0	1 0	0
11	01	0 1	1 0	1 0	1

Excitation table

This step is easy
(map 2-bit numbers to 4 letters)

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

State table

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

This step is easy too
(the outputs are the same in both tables)

Let’s derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	$\boxed{?}$		0
B			0
C			0
D			1

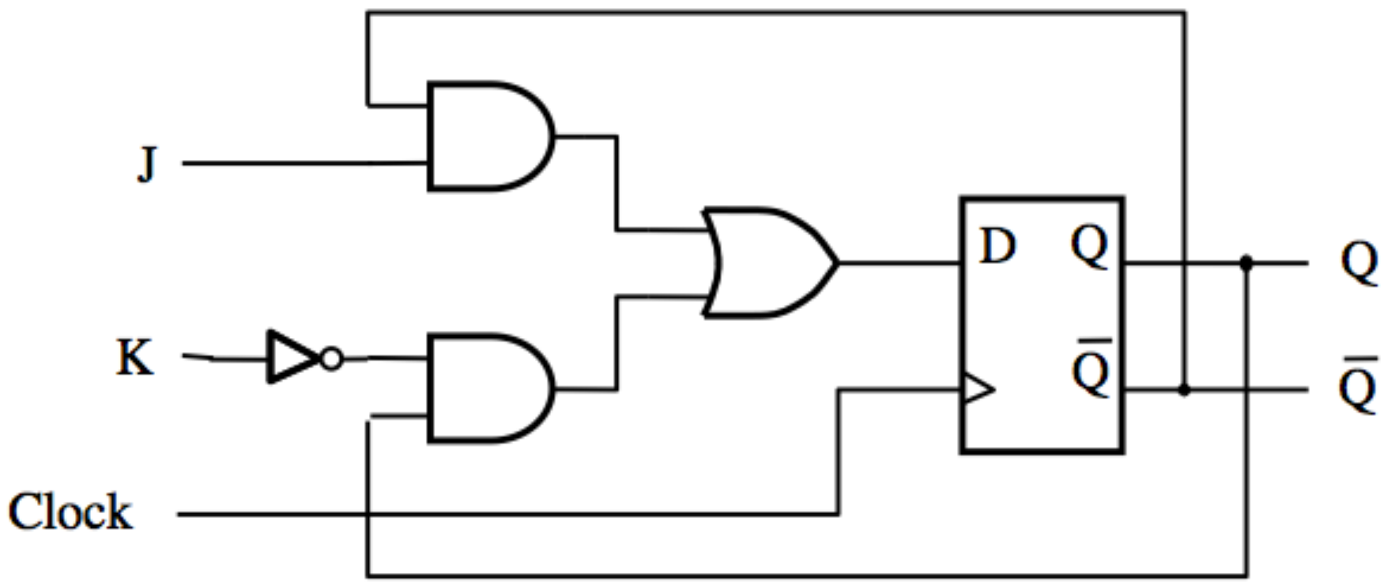
State table

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

How should we do this?

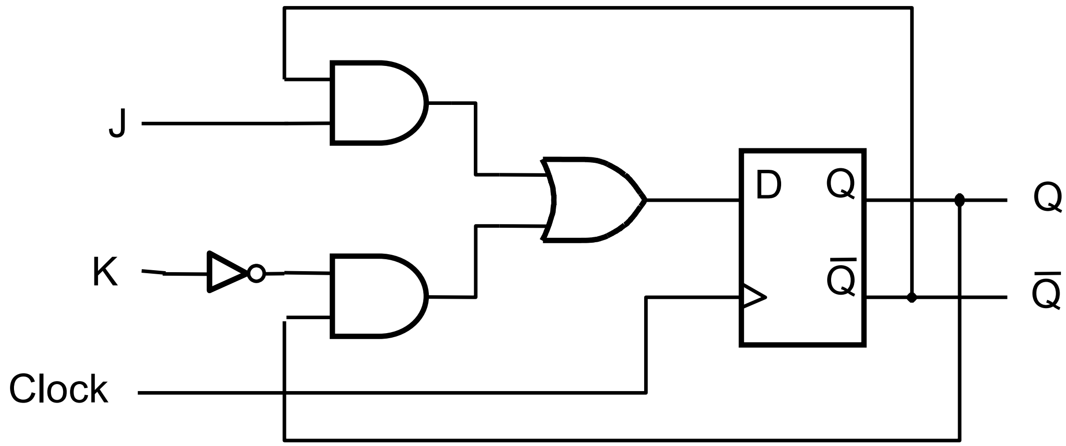
JK Flip-Flop Refresher



$$D = J\overline{Q} + \overline{K}Q$$

[Figure 5.16a from the textbook]

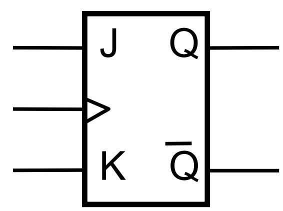
JK Flip-Flop Refresher



(a) Circuit

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

(b) Truth table



(c) Graphical symbol

[Figure 5.16 from the textbook]

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	?		0
B			0
C			0
D			1

State table

Present state	Flip-flop inputs				Output
	$w = 0$		$w = 1$		
y_2y_1	J_2K_2	J_1K_1	J_2K_2	J_1K_1	z
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

How should we do this?

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	$\boxed{01}$	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
$\boxed{0\ 1}$	0
1 0	1
1 1	$\overline{Q}(t)$

Let’s derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A			0
B			0
C			0
D			1

Present state	Flip-flop inputs				Output
	$w = 0$		$w = 1$		z
y_2y_1	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

Let’s derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state	Flip-flop inputs				Output
	$w = 0$		$w = 1$		
$y_2 y_1$	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	z
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Note that A = 00

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A	$\boxed{?}$	0
B			0
C			0
D			1

Present state	Flip-flop inputs				Output
	$w = 0$		$w = 1$		
$y_2 y_1$	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	z
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

Let's derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs $w = 0$: J_2K_2	$w = 0$: J_1K_1	$w = 1$: J_2K_2	$w = 1$: J_1K_1	Output z
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	$\boxed{10}$	$\boxed{11}$	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
$\boxed{1\ 0}$	$\boxed{1}$
1 1	$\bar{Q}(t)$

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
$\boxed{1\ 1}$	$\boxed{\bar{Q}(t)}$

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs w = 0: J_2K_2	w = 0: J_1K_1	w = 1: J_2K_2	w = 1: J_1K_1	Output z
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

$= \bar{1} = 0$

Let’s derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A	A		0
B		C	0
C			0
D			1

Present state y_2y_1	Flip-flop inputs $w = 0$ J_2K_2	$w = 0$ J_1K_1	$w = 1$ J_2K_2	$w = 1$ J_1K_1	Output z
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Note that C = 10

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q}(t)$

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q}(t)$ =0

The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

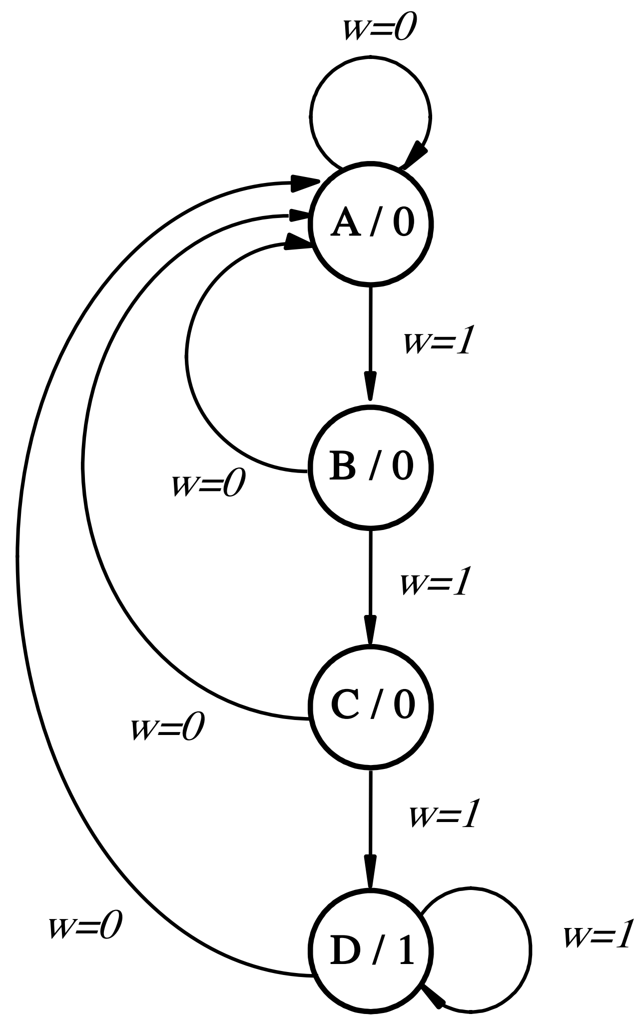
Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	0 1	0 0	1 1	0
01	01	0 1	1 0	1 1	0
10	01	0 1	0 0	1 0	0
11	01	0 1	1 0	1 0	1

Excitation table

The state diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



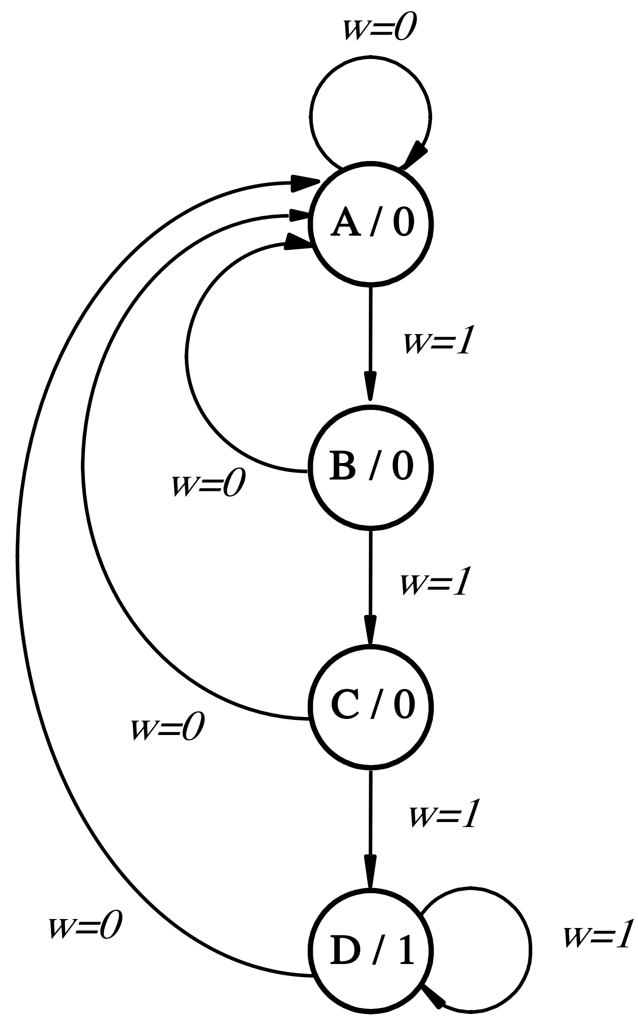
State diagram

The state diagram

Thus, this FSM is identical to the one in the previous example, even though the circuit uses JK flip-flops.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

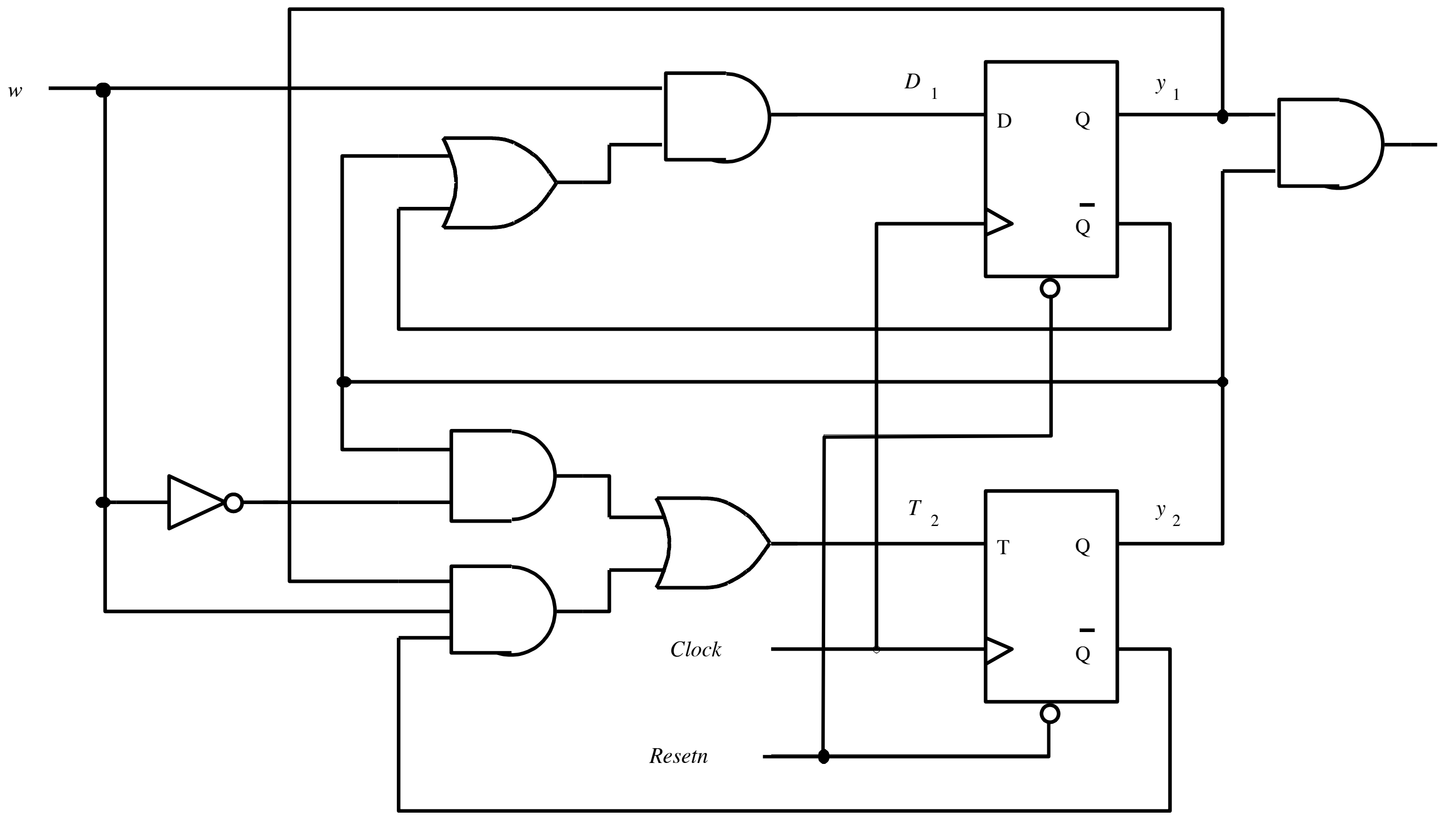
State table



State diagram

Yet Another Example
(with mixed flip-flops)

What does this circuit do?

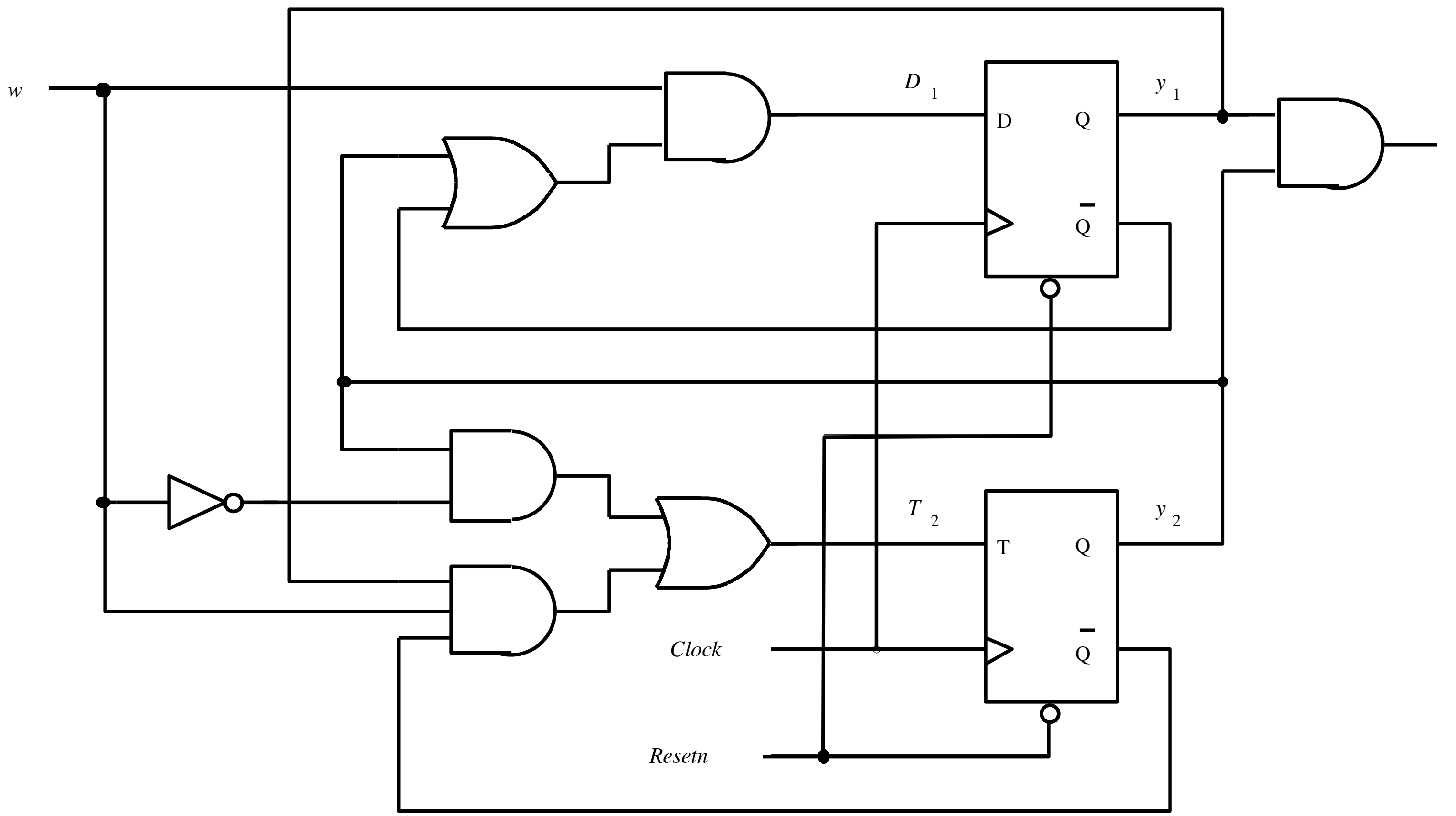


[Figure 6.79 from the textbook]

Approach

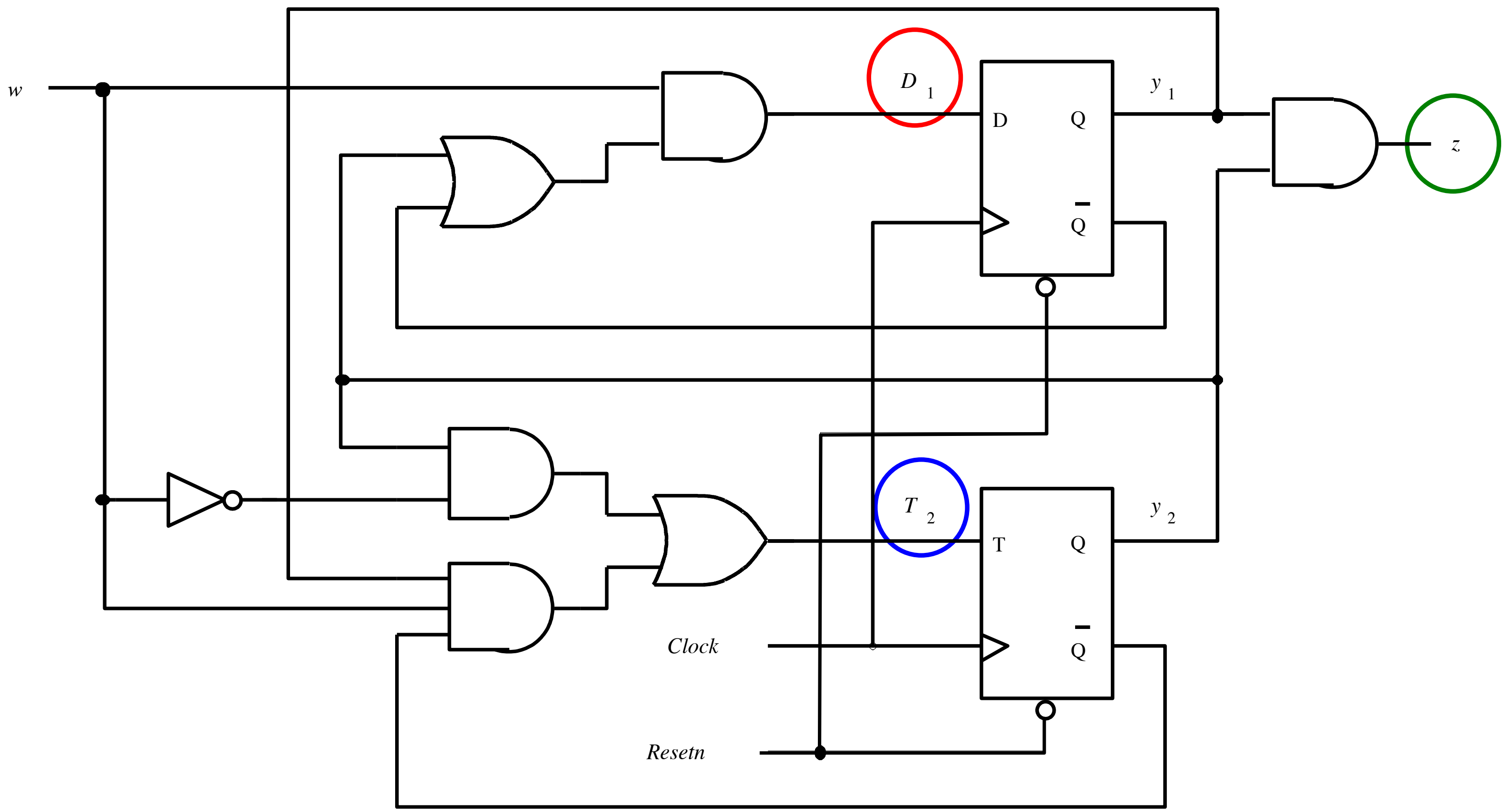
- Find the flip-flops
- Outputs of the flip-flops = present state variables
- Inputs of the flip-flops determine the next state variables
- Determine the logical expressions for the outputs
- Given this info it is easy to do the state-assigned table
- Next do the state table
- Finally, draw the state diagram.

What are the logic expressions?



[Figure 6.79 from the textbook]

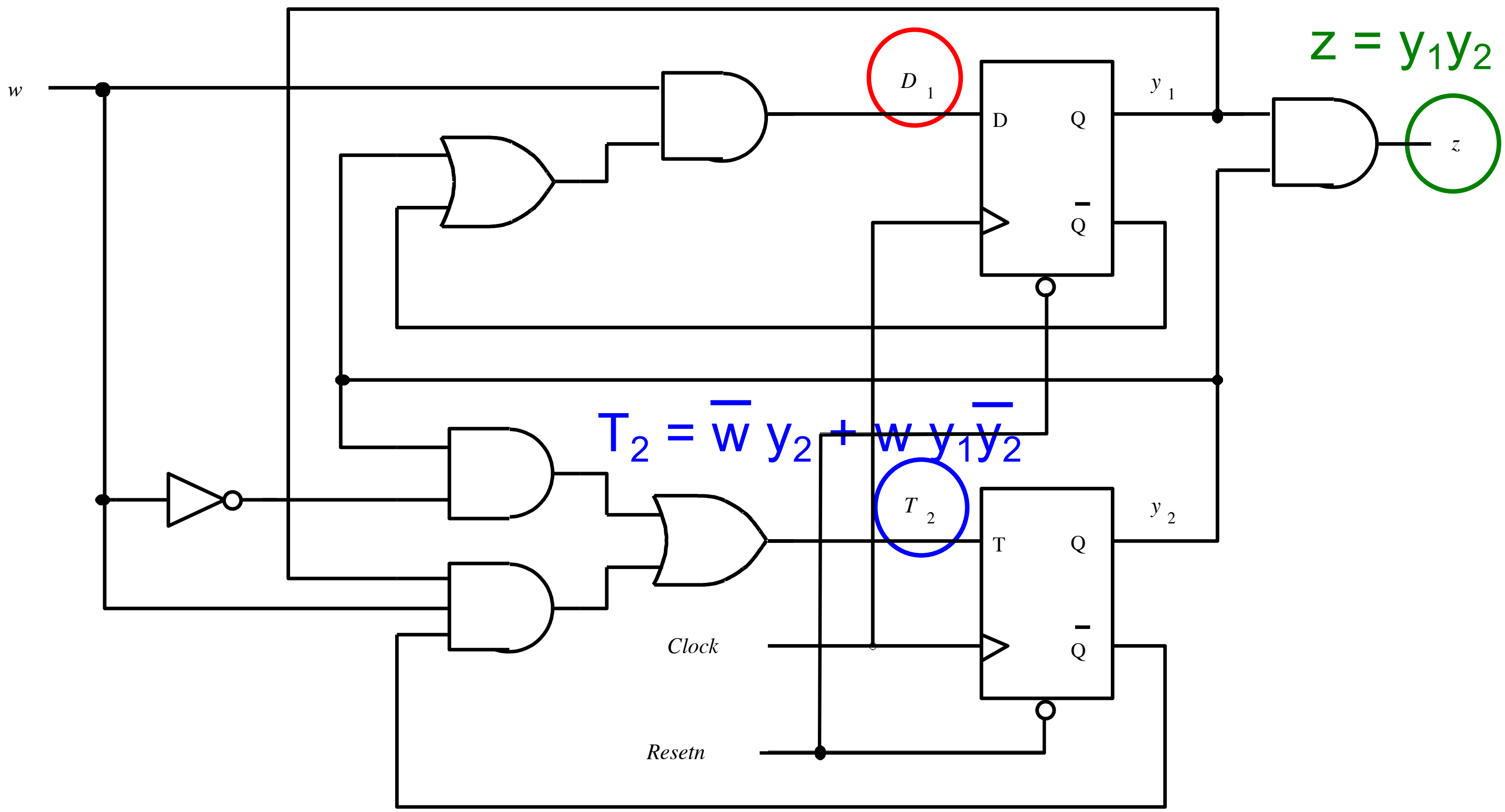
What are the logic expressions?



What are the logic expressions?

$D_1 = w\,(\overline{y_1} + y_2)$

$z = y_1 y_2$



$T_2 = \overline{w}\,y_2 + w\,y_1\overline{y_2}$

The Excitation Table

$D_1 = w\,(\overline{y_1} + y_2)$

$T_2 = \overline{w}\,y_2 + w\,y_1\overline{y_2}$

$z = y_1 y_2$

Present state $y_2 y_1$	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

Excitation table

Let's derive the state table

Present state	Next state		Output z
	$w = 0$	$w = 1$	

Present state $y_2 y_1$	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

Present state y_2y_1	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	T_2D_1	T_2D_1	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

This step is easy
(map 2-bit numbers to 4 letters)

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A			0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
$y_2 y_1$	$T_2 D_1$	$T_2 D_1$	z
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

This step is easy too
(the outputs are the same in both tables)

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	$\boxed{?}$		0
B			0
C			0
D			1

Present state $y_2 y_1$	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	
0 0	$\boxed{0\,0}$	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

What should we do here?

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	$\boxed{?}$		0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
y_2y_1	T_2D_1	T_2D_1	z
0 0	$\boxed{0\,0}$	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

What should we do here?

T	$Q(t+1)$
0	$Q(t)$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A			0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
$y_2 y_1$	$T_2 D_1$	$T_2 D_1$	z
0 0	$\boxed{0}$ 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$Q(t)$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	T_2D_1	T_2D_1	
0 0	$\boxed{0}$ 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{Q(t)}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let’s derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	T_2D_1	T_2D_1	
$\boxed{0}0$	$\boxed{0}0$	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{Q(t)}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs $w = 0$: T_2D_1	Flip-flop inputs $w = 1$: T_2D_1	Output z
0 0	$\boxed{0}$ 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{0}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A			0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
$y_2 y_1$	$T_2 D_1$	$T_2 D_1$	z
0 0	$\boxed{0}\boxed{0}$	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{0}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
$\boxed{0}$	0
1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	T_2D_1	T_2D_1	
0 0	$\boxed{0}\boxed{0}$	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{0}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
$\boxed{0}$	$\boxed{0}$
1	1

Let’s derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs $w = 0$: T_2D_1	Flip-flop inputs $w = 1$: T_2D_1	Output z
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

Note that A = 00

T	$Q(t+1)$
0	0
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C		$\boxed{?}$	0
D			1

Present state y_2y_1	Flip-flop inputs		Output
	$w = 0$	$w = 1$	z
	T_2D_1	T_2D_1	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	$\boxed{01}$	0
1 1	1 0	01	1

What should we do here?

T	$Q(t+1)$
0	$Q(t)$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2 y_1$	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	$\boxed{0}$1	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$Q(t)$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
y_2y_1	T_2D_1	T_2D_1	z
00	00	01	0
01	00	10	0
10	10	$\boxed{0}$1	0
11	10	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{Q(t)}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output
	$w = 0$	$w = 1$	z
	T_2D_1	T_2D_1	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
0	$Q(t)$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
$y_2 y_1$	$T_2 D_1$	$T_2 D_1$	z
0 0	0 0	01	0
0 1	0 0	10	0
$\boxed{1}$ 0	1 0	$\boxed{0}$1	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{1}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
y_2y_1	T_2D_1	T_2D_1	z
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

T	$Q(t+1)$
0	1
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C			0
D			1

Present state $y_2 y_1$	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	z
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	$\boxed{0}\boxed{1}$	0
1 1	1 0	01	1

T	$Q(t+1)$
$\boxed{0}$	$\boxed{1}$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
$\boxed{1}$	$\boxed{1}$

Let's derive the state table

Present state	Next state		Output
	w = 0	w = 1	z
A	A		0
B			0
C		D	0
D			1

Present state	Flip-flop inputs		Output
	$w = 0$	$w = 1$	
y_2y_1	T_2D_1	T_2D_1	z
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

Note that D = 11

T	$Q(t+1)$
0	1
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

Let's derive the state table

Present state	Next state w = 0	Next state w = 1	Output z
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Present state y_2y_1	Flip-flop inputs $w = 0$: T_2D_1	Flip-flop inputs $w = 1$: T_2D_1	Output z
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$
0	$Q(t)$
1	$\overline{Q}(t)$

D	$Q(t+1)$
0	0
1	1

The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state y_2y_1	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	T_2D_1	T_2D_1	
0 0	0 0	01	0
0 1	0 0	10	0
1 0	1 0	01	0
1 1	1 0	01	1

Excitation table

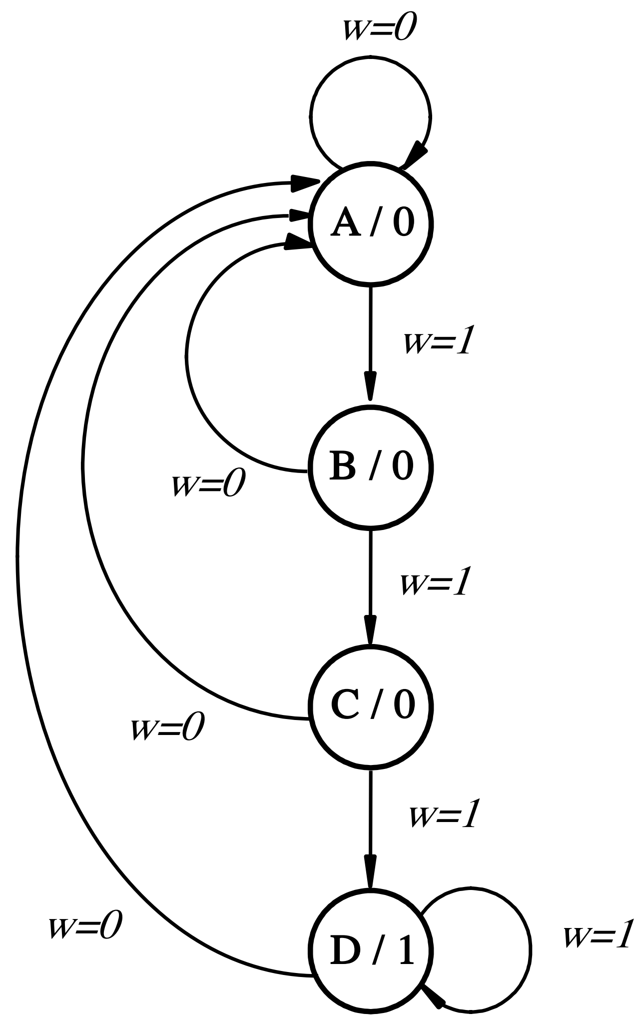
[Figure 6.75b from the textbook]

[Figure 6.80 from the textbook]

The state diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



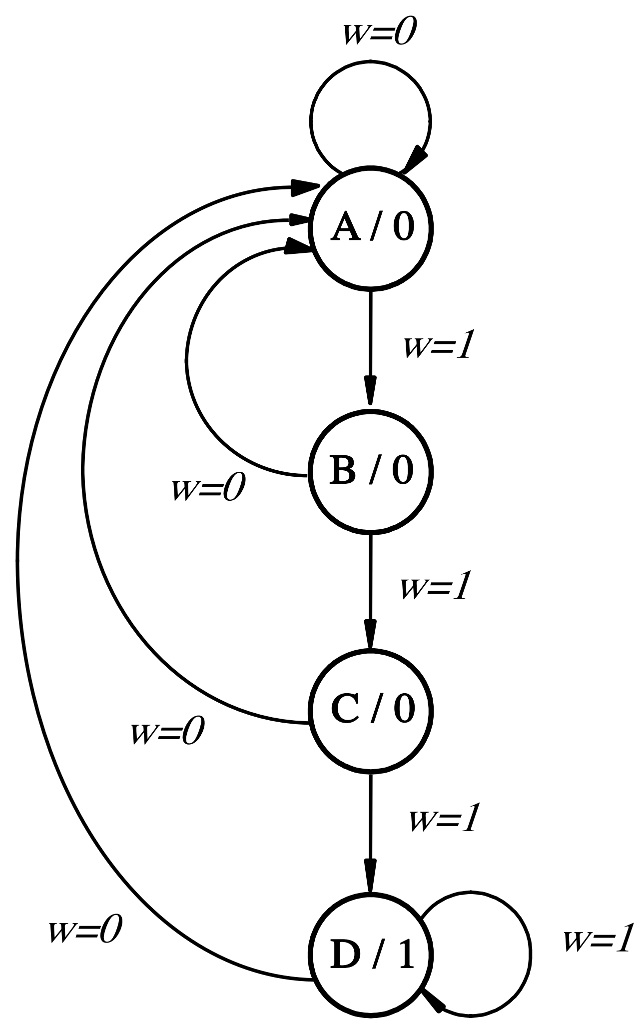
State diagram

The state diagram

Thus, this FSM is identical to the ones in the previous examples, even though the circuit uses one D and one T flip-flop.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



State diagram

Questions?

THE END