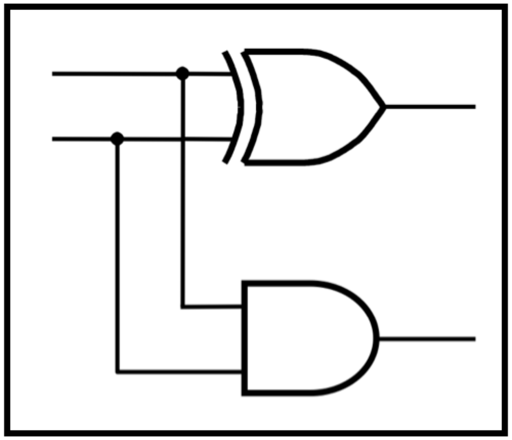


# CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

# Karnaugh Maps

*CprE 2810: Digital Logic*
*Iowa State University, Ames, IA*

# Administrative Stuff

- HW3 is due today

# Administrative Stuff

- HW4 is out
- It is due on Monday Sep 22 @ 10 pm
- It is posted on the class web page
- I also sent you an e-mail with the link.

# Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 26.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **Sample exams are posted on the class web page.**
- **More details to follow.**

# Quick Review

# Do You Still Remember This Boolean Algebra Theorem?

14a. $x \bullet y + x \bullet \overline{y} = x$ Combining

14b. $(x + y) \bullet (x + \overline{y}) = x$

# Let's prove 14.a

| $x$ | $y$ | $\mathbf{x} \bullet \mathbf{y} + \mathbf{x} \bullet \overline{\mathbf{y}} = \mathbf{x}$ |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | |
| 1 | 0 | |
| 1 | 1 | |

# Let's prove 14.a

| $x$ | $y$ | $x \cdot y + x \cdot \overline{y} = x$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Let’s prove 14.a

| $x$ | $y$ | $\mathbf{x \cdot y}$ | $+$ | $\mathbf{x \cdot \bar{y}}$ | $=$ | $\mathbf{x}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | | 0 | | |
| 0 | 1 | 0 | | 0 | | |
| 1 | 0 | 0 | | 1 | | |
| 1 | 1 | 1 | | 0 | | |

# Let’s prove 14.a

| $x$ | $y$ | $\mathbf{x} \bullet \mathbf{y}$ | $+$ | $\mathbf{x} \bullet \overline{\mathbf{y}}$ | $= \mathbf{x}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | |
| 0 | 1 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 1 | 1 | |
| 1 | 1 | 1 | 1 | 0 | |

# Let’s prove 14.a

| $x$ | $y$ | $\mathbf{x}$ | $\bullet$ | $\mathbf{y}$ | $+$ | $\mathbf{x}$ | $\bullet$ | $\overline{\mathbf{y}}$ | $=$ | $\mathbf{x}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | | 0 | | 0 | | 0 | | | 0 |
| 0 | 1 | | 0 | | 0 | | 0 | | | 0 |
| 1 | 0 | | 0 | | 1 | | 1 | | | 1 |
| 1 | 1 | | 1 | | 1 | | 0 | | | 1 |

# Let’s prove 14.a

| $x$ | $y$ | $\mathbf{x \cdot y}$ | $\mathbf{+}$ | $\mathbf{x \cdot \bar{y}}$ | $\mathbf{= x}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 |

They are equal.

# Motivation

| A | B | C | D | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

**An approach for simplifying logic expressions.**

**How do we guarantee that we have reached the minimum SOP/POS representation?**

# This method was described in 1953

M. Karnaugh, “The map method for synthesis of combinational logic circuits”

Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics (pages 593 – 599, Volume: 72, Issue: 5, Nov. 1953)

https://ieeexplore.ieee.org/document/6371932

# Two-Variable K-Map

# Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(a) Truth table

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

(b) Karnaugh map

# Karnaugh Map (K-map)

- **A visual representation of the function**
- **Same information as the truth table**
- **Easier to group minterms**

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | $m_0$ |
| 0 | 1 | $m_1$ |
| 1 | 0 | $m_2$ |
| 1 | 1 | $m_3$ |

(a) Truth table

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | $m_0$ | $m_2$ |
| 1 | $m_1$ | $m_3$ |

(b) Karnaugh map

[ Figure 2.49 from the textbook ]

# Minterms

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | $m_0$ |
| 0 | 1 | $m_1$ |
| 1 | 0 | $m_2$ |
| 1 | 1 | $m_3$ |

| $x_1$ | $x_2$ | $m_0$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |

# Minterm Addition Example

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $x_1$ | $x_2$ | $m_0$ | $m_1$ | $m_2$ | $m_3$ | $m_1 + m_3$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |

# Minterm Addition Example

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $x_1$ | $x_2$ | $m_0$ | $m_1$ | $m_2$ | $m_3$ | $m_1 + m_3$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 |

$$\overline{x}_1 x_2 + x_1 x_2 = x_2$$

# Minterm Addition Example

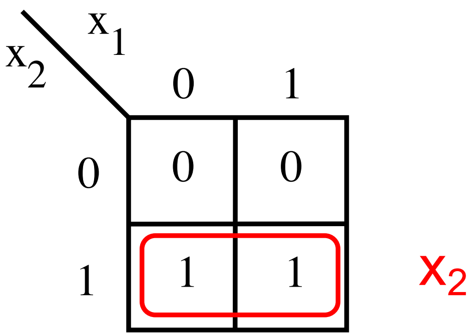
| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

$$\overline{x}_1x_2 + x_1x_2 = x_2$$

# Minterm Addition Example

| $x_1$ | $x_2$ | f |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |




$$\bar{x}_1 x_2 + x_1 x_2 = x_2$$

# Another Grouping Example

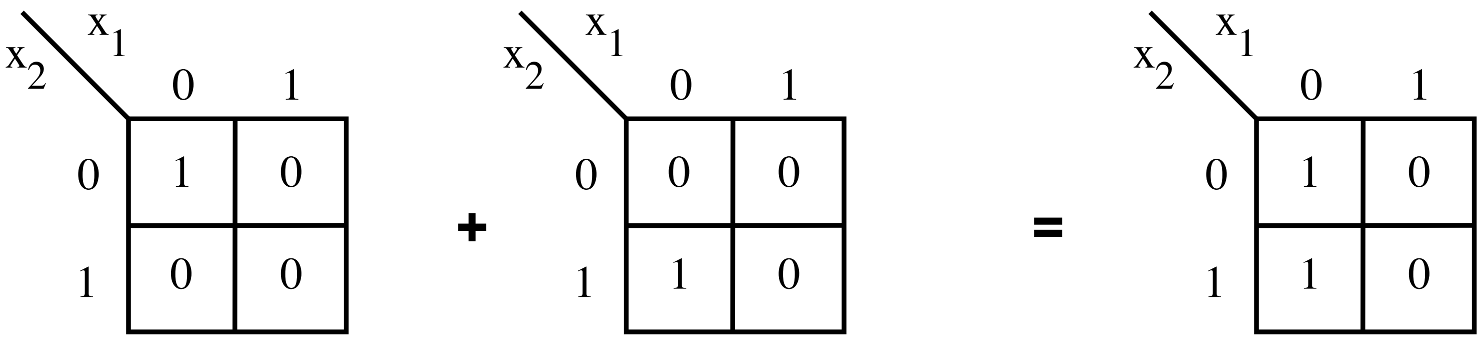
| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

**$m_0$**

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 0 |

**$m_1$**

# Another Grouping Example




$$m_0 \quad + \quad m_1 \quad = \quad m_0 + m_1$$

# Another Grouping Example

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

+

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 0 |

=

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 0 |

$$m_0 \quad + \quad m_1 \quad = \quad m_0 + m_1$$

# Another Grouping Example

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

$+$

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 0 |

$=$

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 0 |

$$m_0 \quad + \quad m_1 \quad = \quad m_0 + m_1$$

$$\bar{x}_1\bar{x}_2 \quad + \quad \bar{x}_1 x_2 \quad = \quad \bar{x}_1$$

Property 14a (Combining)

# Grouping Rules

- **Group "1"s with rectangles**
- **Both sides must be a power of 2:**
  - **1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4**
- **Can use/cover the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
  - **Try to use as few groups as possible to cover all "1"s.**
  - **For each group, try to make it as large as you can (i.e., if you can use a 2x2, don't use a 2x1 even if that is enough).**

# Two-Variable K-map

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | $m_0$ |
| 0 | 1 | $m_1$ |
| 1 | 0 | $m_2$ |
| 1 | 1 | $m_3$ |

(a) Truth table

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | $m_0$ | $m_2$ |
| 1 | $m_1$ | $m_3$ |

(b) Karnaugh map

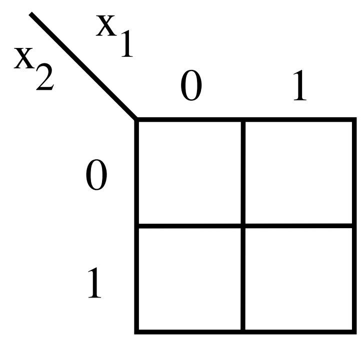
[ Figure 2.49 from the textbook ]

# Step-By-Step Example

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# 1. Draw The Map

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



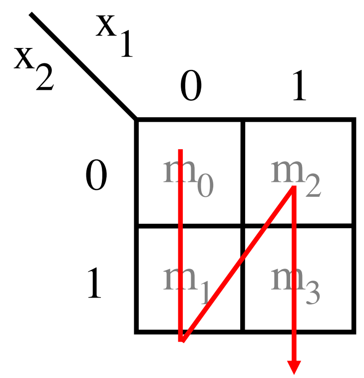
# 2. Fill The Map

| | $x_1$ | $x_2$ | $f$ |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 0 |
| $m_3$ | 1 | 1 | 1 |

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | $m_0$ | $m_2$ |
| 1 | $m_1$ | $m_3$ |

# 2. Fill The Map

| | $x_1$ | $x_2$ | f |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 0 |
| $m_3$ | 1 | 1 | 1 |



# 2. Fill The Map

| | $x_1$ | $x_2$ | $f$ |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 0 |
| $m_3$ | 1 | 1 | 1 |

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

# 3. Group

| | $x_1$ | $x_2$ | $f$ |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 0 |
| $m_3$ | 1 | 1 | 1 |

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

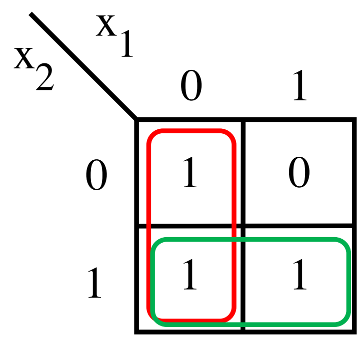
# 3. Group

| | $x_1$ | $x_2$ | f |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 0 |
| $m_3$ | 1 | 1 | 1 |

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

# 3. Group

| | $x_1$ | $x_2$ | $f$ |
|---|---|---|---|
| $m_0$ | 0 | 0 | 1 |
| $m_1$ | 0 | 1 | 1 |
| $m_2$ | 1 | 0 | 0 |
| $m_3$ | 1 | 1 | 1 |



# 4. Write The Expression

| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

# 4. Write The Expression

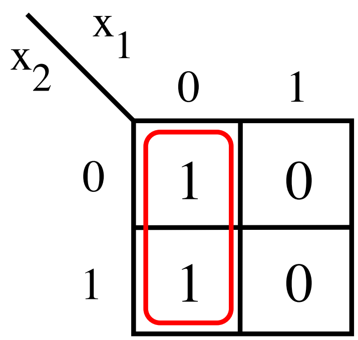
| $x_1$ | $x_2$ | $f$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $x_2$ \ $x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |

$\overline{x}_1 + x_2$

# Writing The Expression

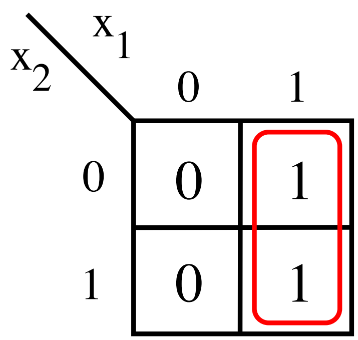
- **Find which variable is constant**




$\overline{x}_1$

# Writing The Expression

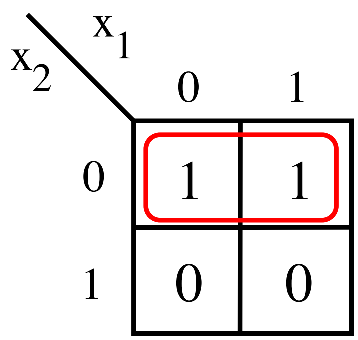
- **Find which variable is constant**




$x_1$

# Writing The Expression

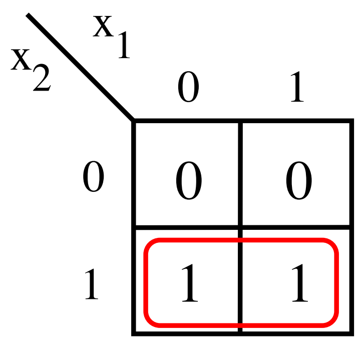
- **Find which variable is constant**




$\overline{x}_2$

# Writing The Expression

- **Find which variable is constant**




$x_2$

# These are all valid groupings

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

$\bar{x}_1 \bar{x}_2$

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 0 |

$\bar{x}_1 x_2$

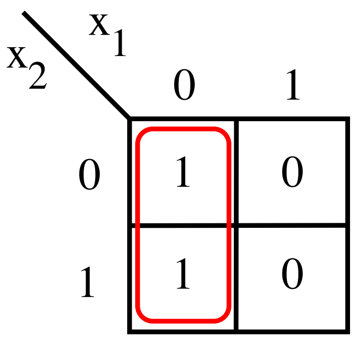
| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 0 |

$x_1 \bar{x}_2$

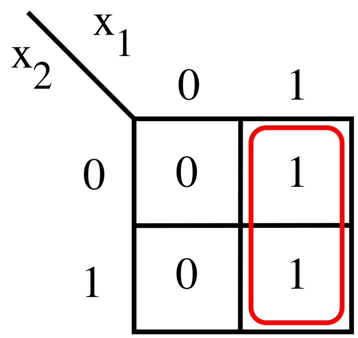
| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

$x_1 x_2$

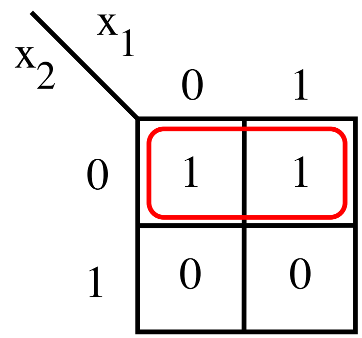
# These are all valid groupings



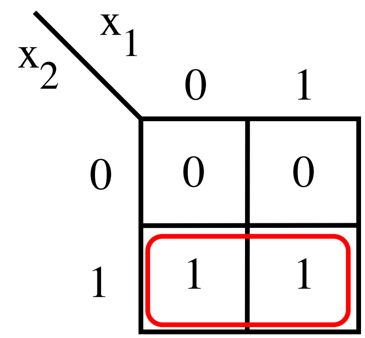

$\overline{x}_1$




$x_1$

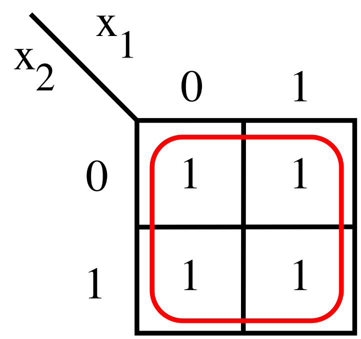



$\overline{x}_2$



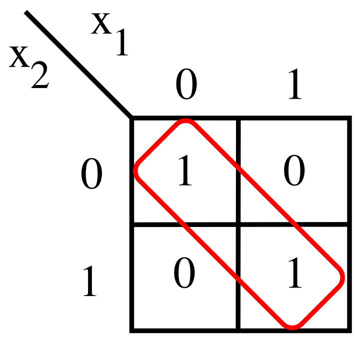

$x_2$

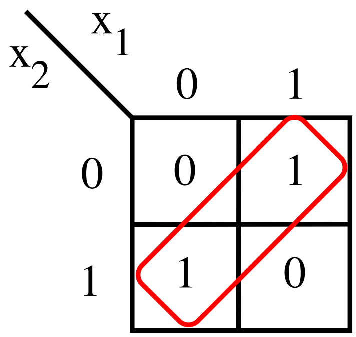
# This one is valid too




In this case the result is the constant function 1.

# Invalid Groupings



# Can’t group diagonally. Why?

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

**$m_0$**

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

**$m_3$**

# Can’t group diagonally. Why?

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

+

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

=

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |

$$m_0 \quad + \quad m_3 \quad = \quad m_0 + m_3$$

# Can’t group diagonally. Why?

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 0 |

+

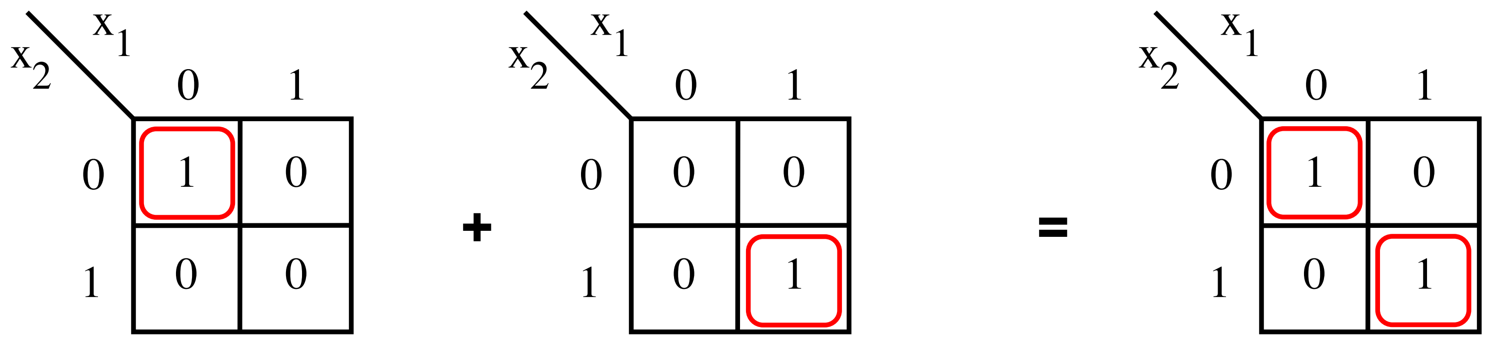
| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

=

| $x_2 \backslash x_1$ | 0 | 1 |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |

$\mathbf{m_0}$ **+** $\mathbf{m_3}$ **=** $\mathbf{m_0 + m_3}$

# Can’t group diagonally. Why?




$$m_0 \quad + \quad m_3 \quad = \quad m_0 + m_3$$

$$\overline{x}_1\overline{x}_2 \quad + \quad x_1x_2 \quad = \quad \overline{x}_1\overline{x}_2 \; + \; x_1x_2$$

We can’t use Property 14a here. This can’t be simplified.

# Three-Variable K-Map

# Location of three-variable minterms

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table

| $x_3$ \ $x_1 x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

[ Figure 2.51 from the textbook ]

# Location of three-variable minterms

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table

| $x_3$ \ $x_1 x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Location of three-variable minterms

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table

| $x_3$ \ $x_1 x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

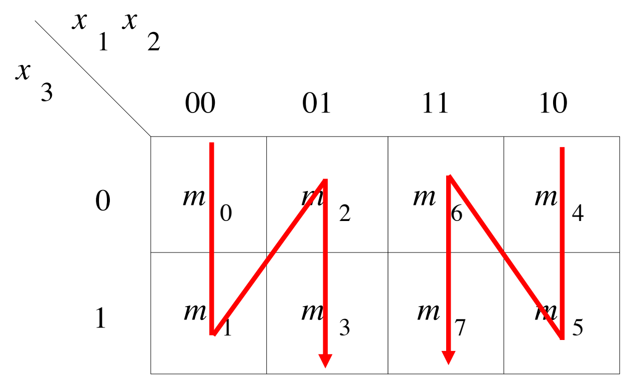
**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Location of three-variable minterms

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table




(b) Karnaugh map

**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
00
01
11
10
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

```
000
001
011
010
110
111
101
100
```

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

# Gray Code & K-map

| | $s\,x_1\,x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\,x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

These two neighbors differ only in the LAST bit

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

These two neighbors
differ only in the LAST bit

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

These two neighbors
differ only in the FIRST bit

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

# Adjacency Rules

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

adjacent
columns

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

These two neighbors
differ only in the FIRST bit

# Gray Code & K-map

| | $s\ x_1\ x_2$ |
|---|---|
| $m_0$ | 0 0 0 |
| $m_1$ | 0 0 1 |
| $m_2$ | 0 1 0 |
| $m_3$ | 0 1 1 |
| $m_4$ | 1 0 0 |
| $m_5$ | 1 0 1 |
| $m_6$ | 1 1 0 |
| $m_7$ | 1 1 1 |

| $x_2$ \ $s\ x_1$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 000 | 010 | 110 | 100 |
| 1 | 001 | 011 | 111 | 101 |

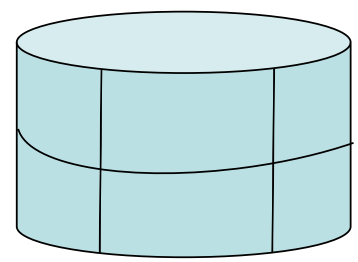
These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

# Adjacency Rules

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

adjacent
columns

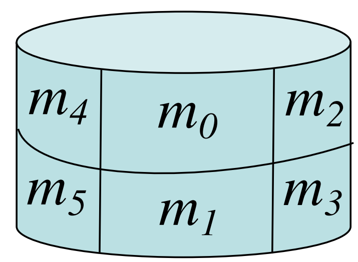


As if the K-map were
drawn on a cylinder

# Adjacency Rules

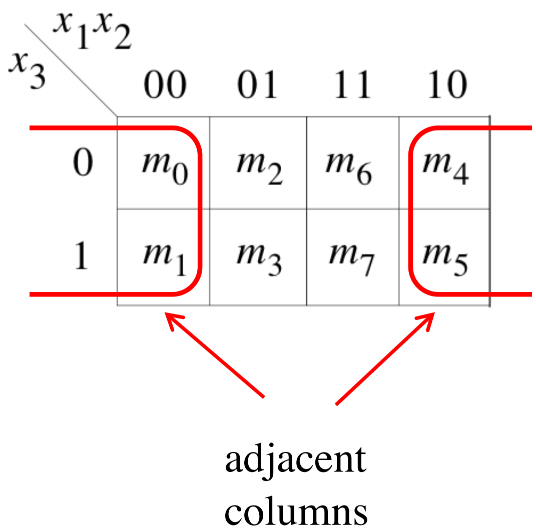
| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

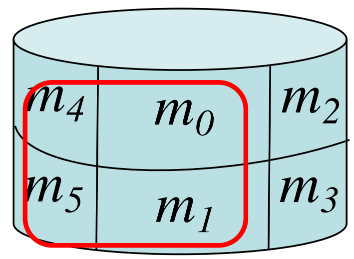
adjacent
columns




As if the K-map were
drawn on a cylinder

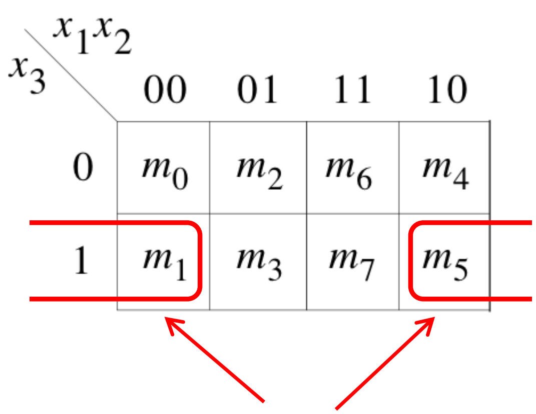
# Adjacency Rules



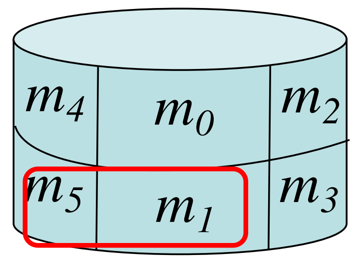




As if the K-map were
drawn on a cylinder

# Adjacency Rules




adjacent
columns




As if the K-map were
drawn on a cylinder

# Examples of Valid Groupings

# Groupings and Expressions

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$\bar{x}_1 \bar{x}_2 \bar{x}_3$

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$\bar{x}_1 x_2 \bar{x}_3$

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$x_1 x_2 \bar{x}_3$

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |

$x_1 \bar{x}_2 \bar{x}_3$

# Groupings and Expressions

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |

$\bar{x}_1 \bar{x}_2 x_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |

$\bar{x}_1 x_2 x_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |

$x_1 x_2 x_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |

$x_1 \bar{x}_2 x_3$

# Groupings and Expressions

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |

$\overline{x}_1 \overline{x}_2$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |

$\overline{x}_1 x_2$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |

$x_1 x_2$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |

$x_1 \overline{x}_2$

# Groupings and Expressions

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$\bar{x}_1 \bar{x}_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$x_2 \bar{x}_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |

$x_1 \bar{x}_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |

$\bar{x}_2 \bar{x}_3$

# Groupings and Expressions

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$\bar{x}_1 x_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |

$x_2 x_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |

$x_1 x_3$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |

$\bar{x}_2 x_3$

# Groupings and Expressions

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$\bar{x}_1$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |

$x_2$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |

$x_1$

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |

$\bar{x}_2$

# Groupings and Expressions

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |

$\overline{x_3}$

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$x_3$

# Groupings and Expressions

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |

0

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

1

# Examples of Invalid Groupings

# Some Invalid Groupings

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
| --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
| --- | --- | --- | --- | --- |
| 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |

Can’t group diagonally.

# Some Invalid Groupings

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |

Can’t group three in a row.
Each side must be a power of 2.

# Some Invalid Groupings

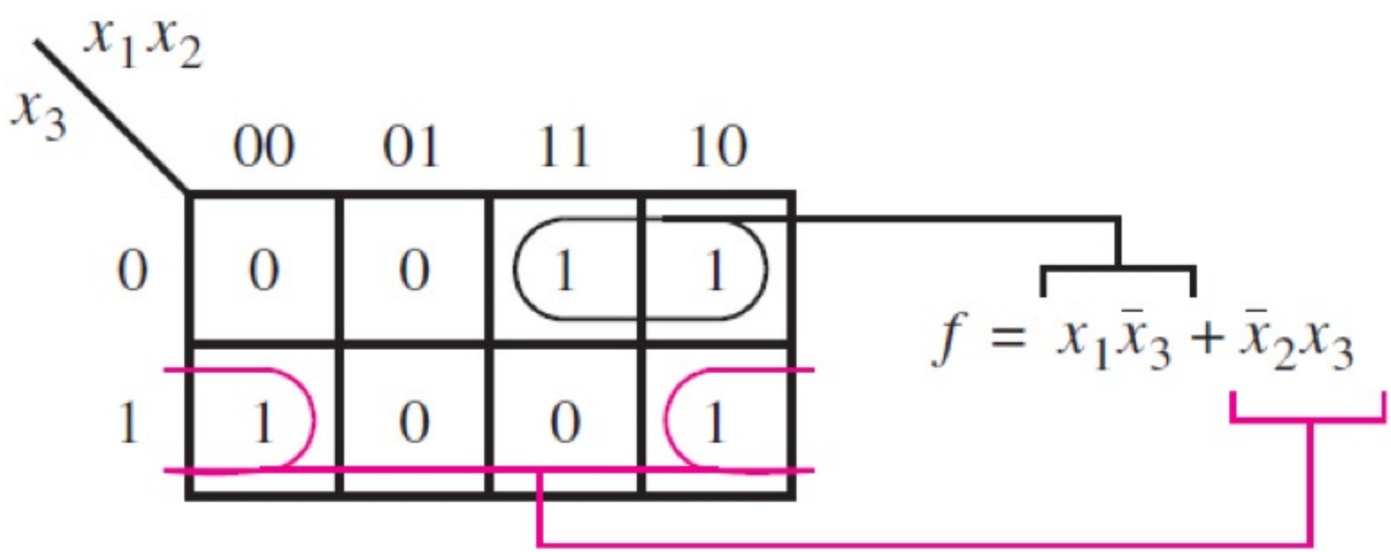
| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | **0** | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | **0** | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |

Can’t group zeros and ones together.

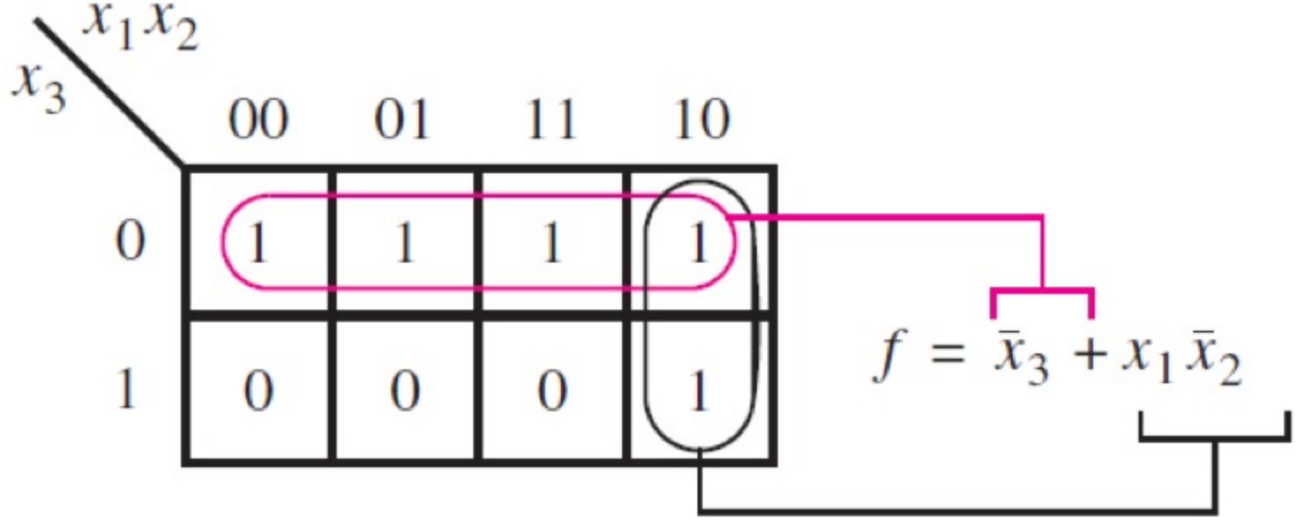
# Minimization Examples with 3-variable K-Maps

# Examples of three-variable Karnaugh maps




[ Figure 2.52a from the textbook ]

# Examples of three-variable Karnaugh maps




[ Figure 2.52b from the textbook ]

# Four-Variable K-Map

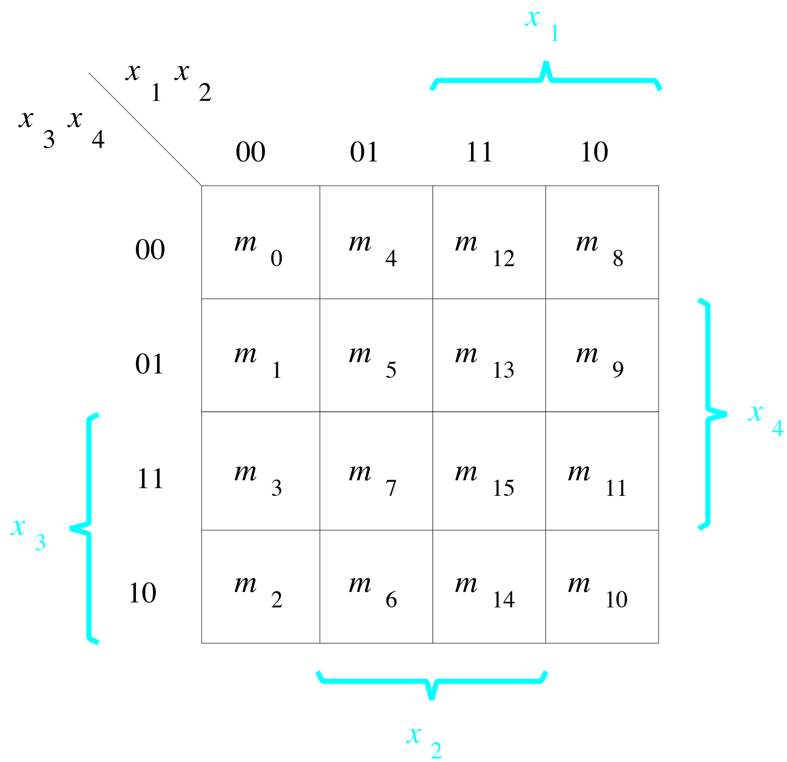
# A four-variable Karnaugh map

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

[ Figure 2.53 from the textbook ]

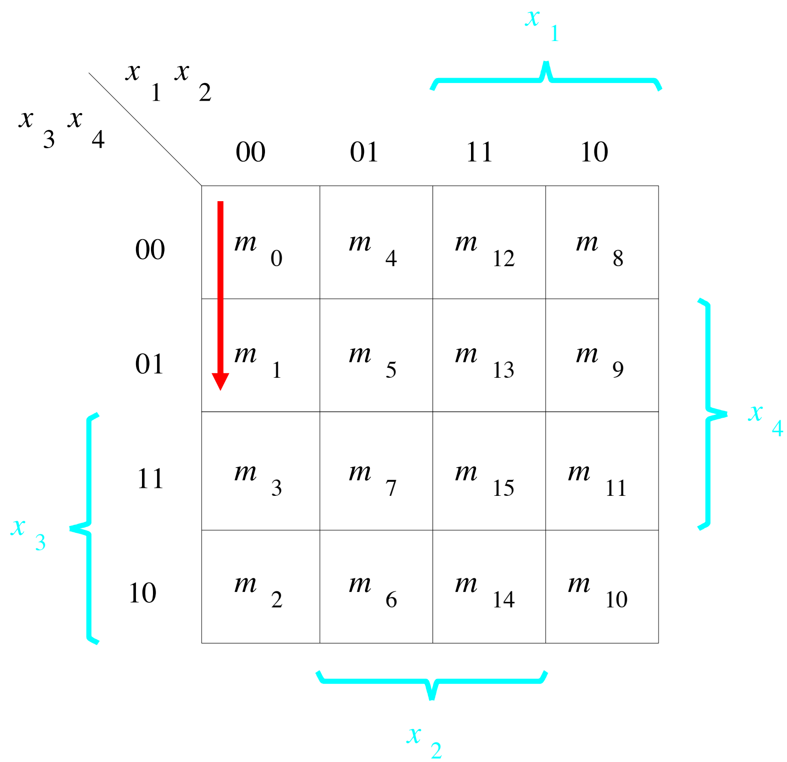
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



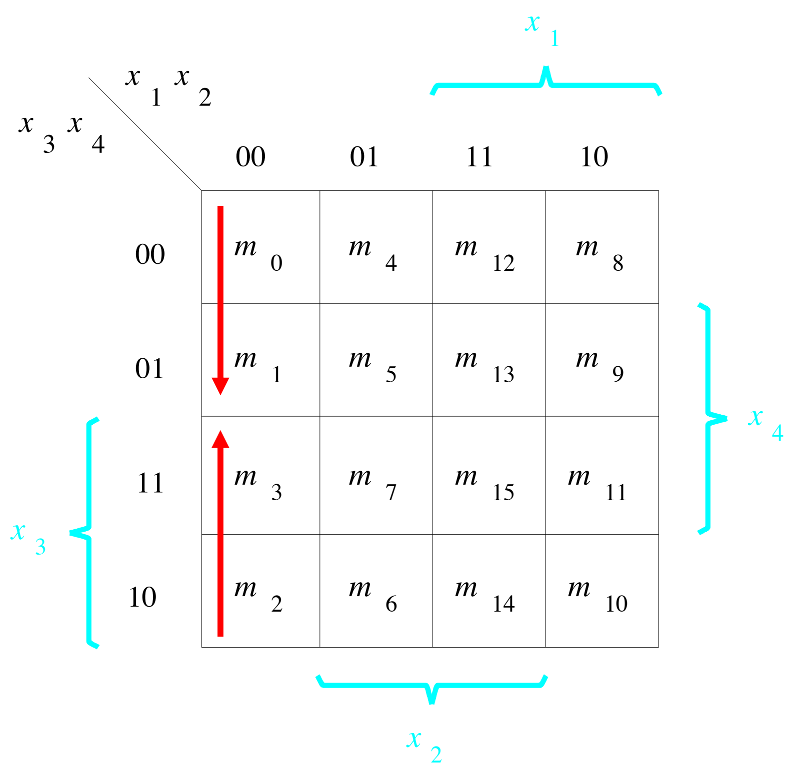
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



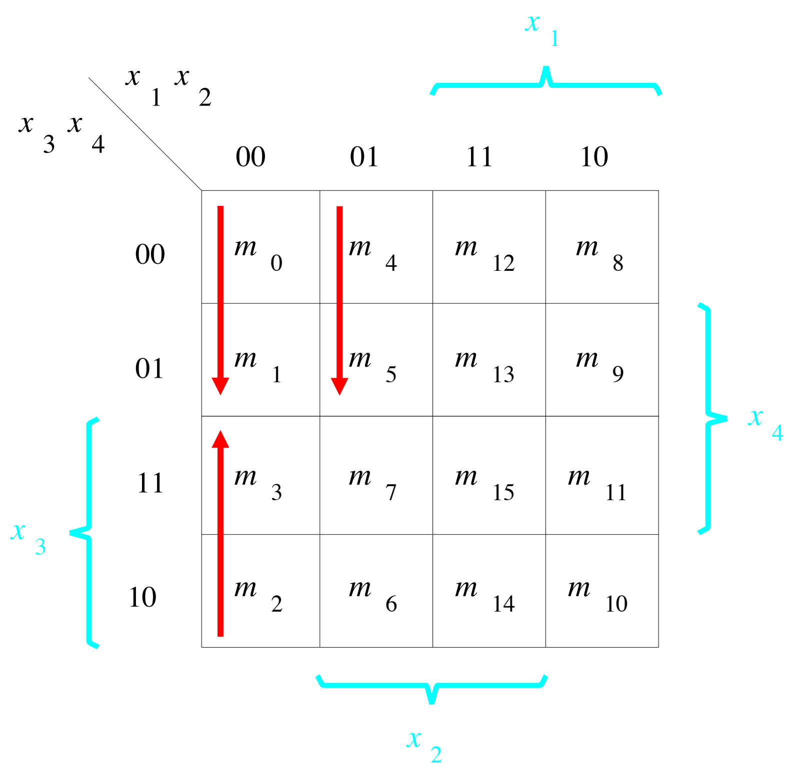
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



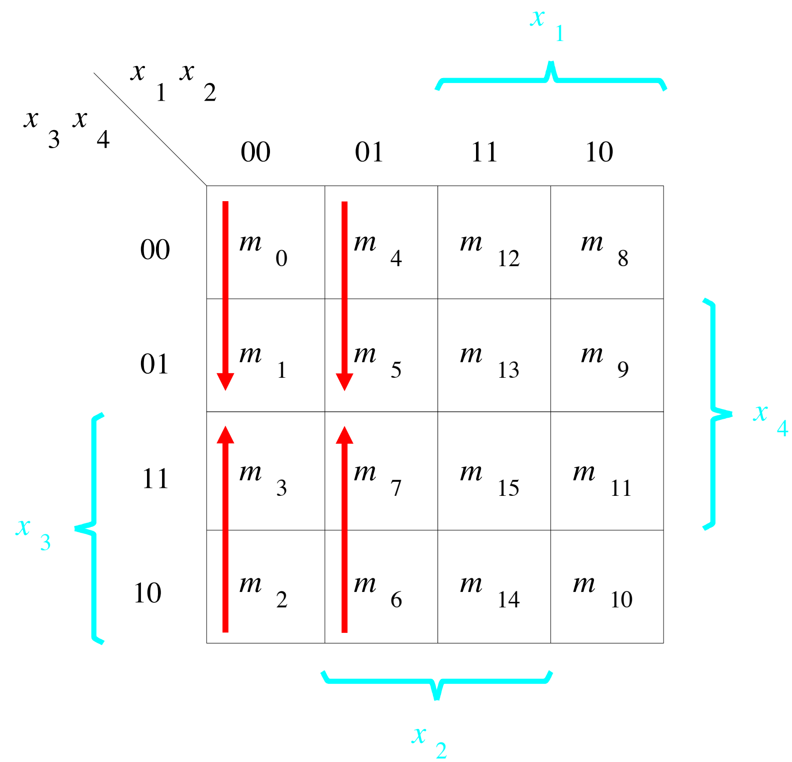
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



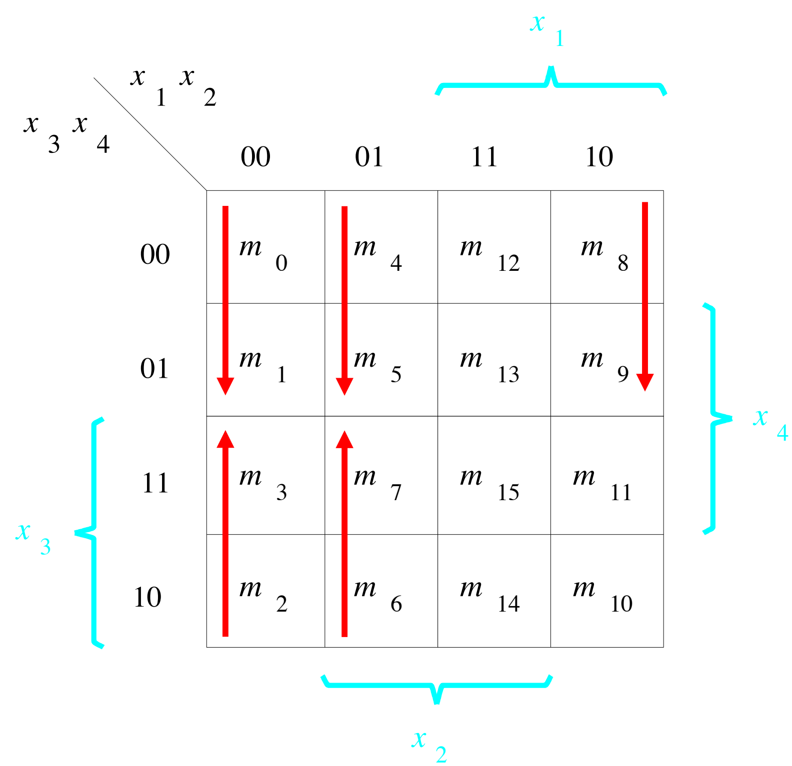
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



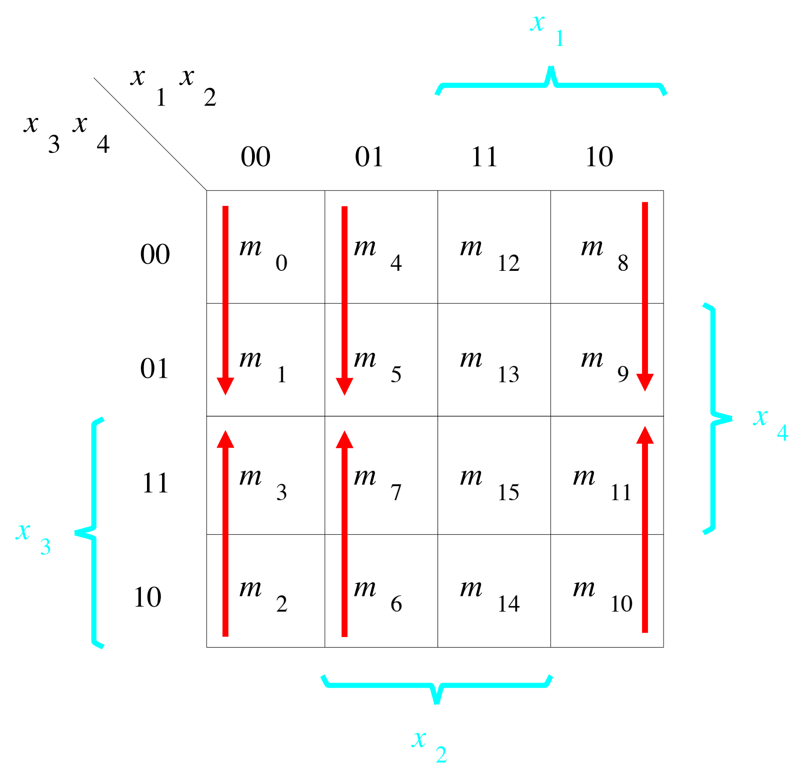
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



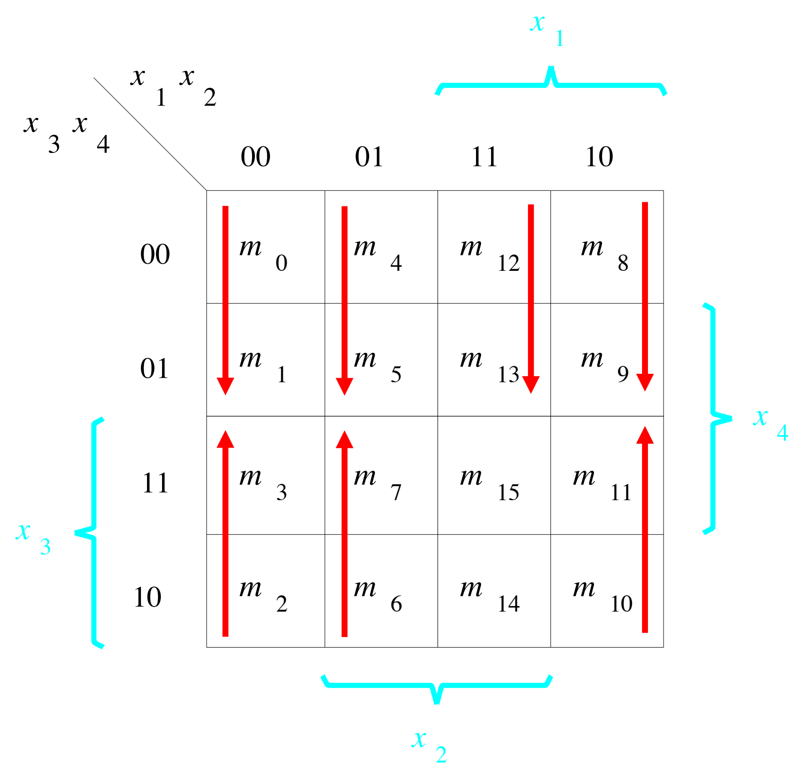
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



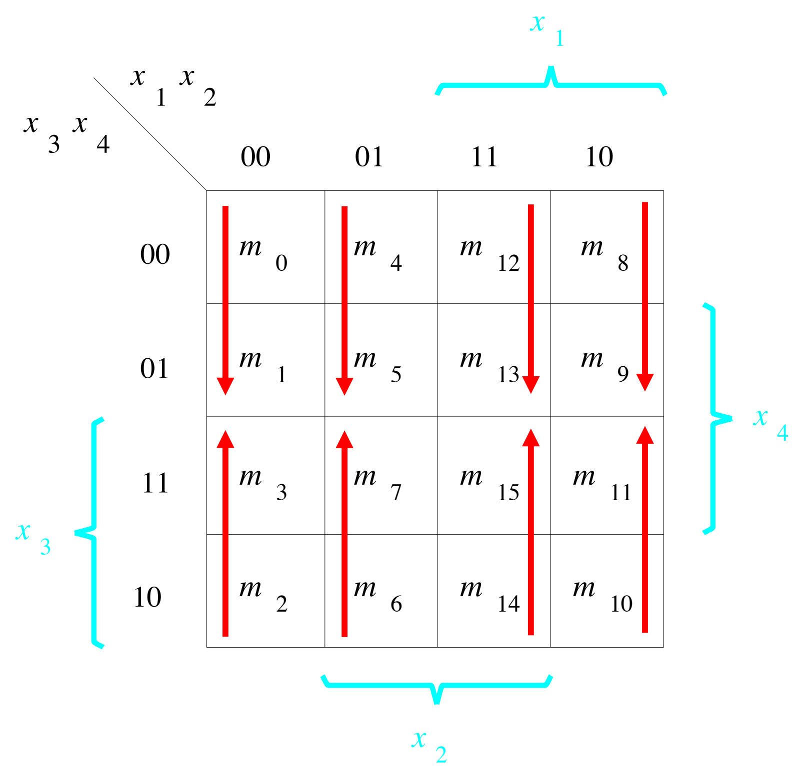
# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



# A four-variable Karnaugh map

| x1 | x2 | x3 | x4 | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | m0 |
| 0 | 0 | 0 | 1 | m1 |
| 0 | 0 | 1 | 0 | m2 |
| 0 | 0 | 1 | 1 | m3 |
| 0 | 1 | 0 | 0 | m4 |
| 0 | 1 | 0 | 1 | m5 |
| 0 | 1 | 1 | 0 | m6 |
| 0 | 1 | 1 | 1 | m7 |
| 1 | 0 | 0 | 0 | m8 |
| 1 | 0 | 0 | 1 | m9 |
| 1 | 0 | 1 | 0 | m10 |
| 1 | 0 | 1 | 1 | m11 |
| 1 | 1 | 0 | 0 | m12 |
| 1 | 1 | 0 | 1 | m13 |
| 1 | 1 | 1 | 0 | m14 |
| 1 | 1 | 1 | 1 | m15 |



# Adjacency Rules

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

adjacent
columns

| $x_3x_4 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

adjacent
rows

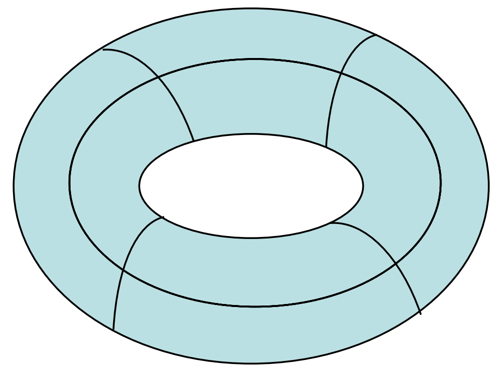
adjacent
columns

# Adjacency Rules

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

adjacent rows

adjacent columns



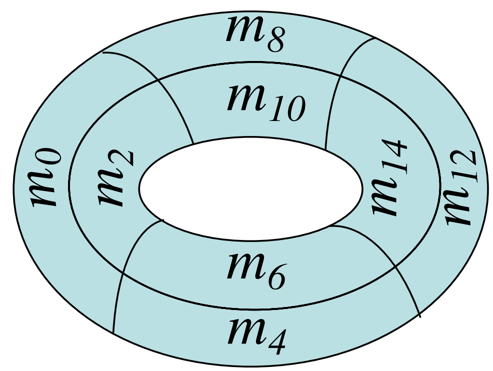
As if the K-map were drawn on a torus

# Adjacency Rules

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

adjacent rows

adjacent columns



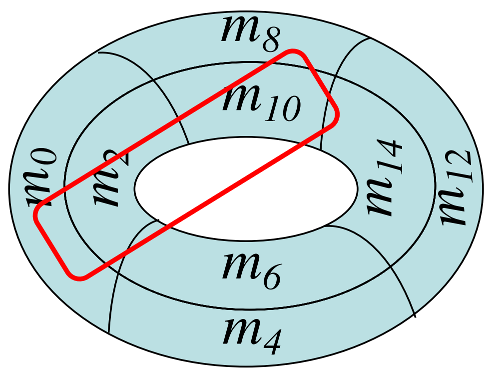

As if the K-map were drawn on a torus

# Adjacency Rules

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

adjacent rows

adjacent columns



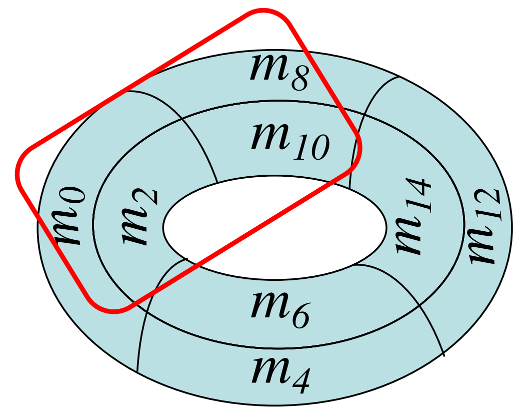

As if the K-map were drawn on a torus

# Adjacency Rules

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

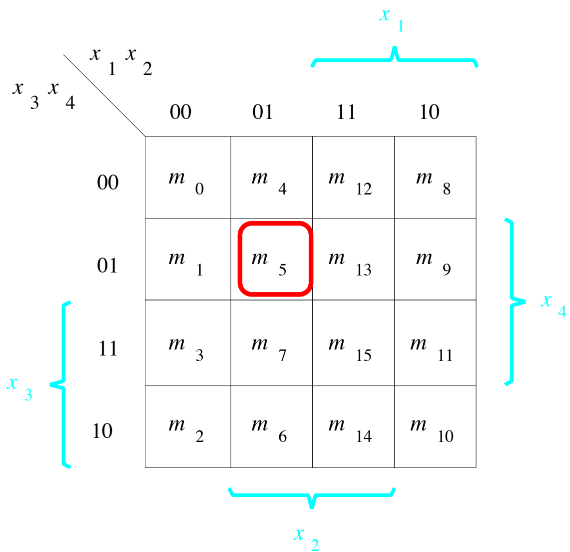
adjacent
rows

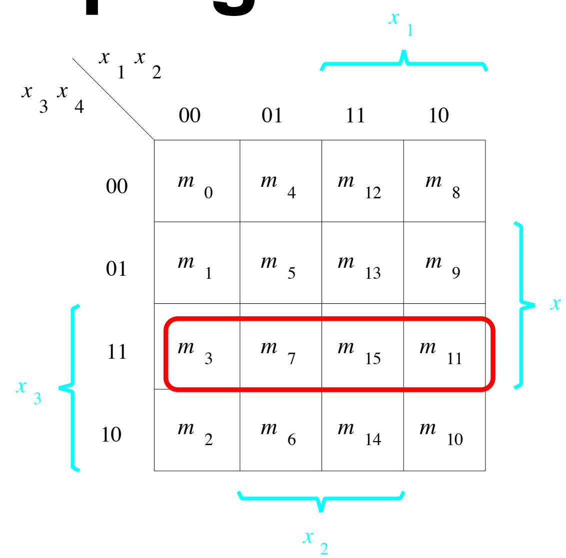
adjacent
columns

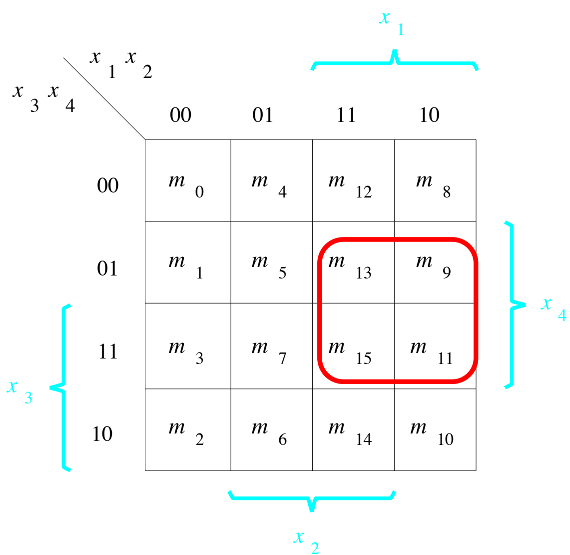


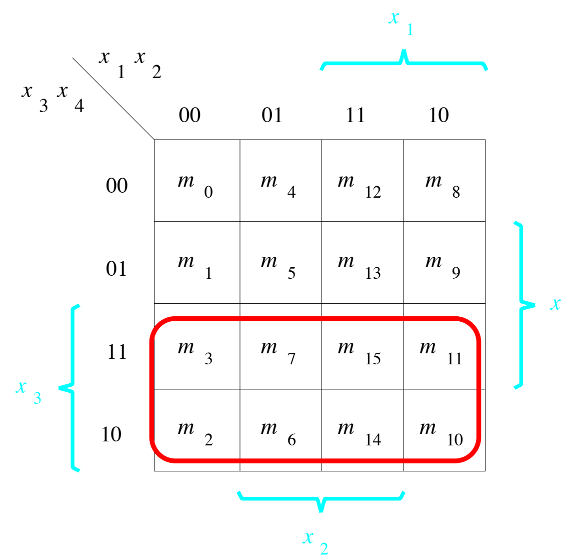

As if the K-map were
drawn on a torus

# Some Valid Groupings



# Some Valid Groupings

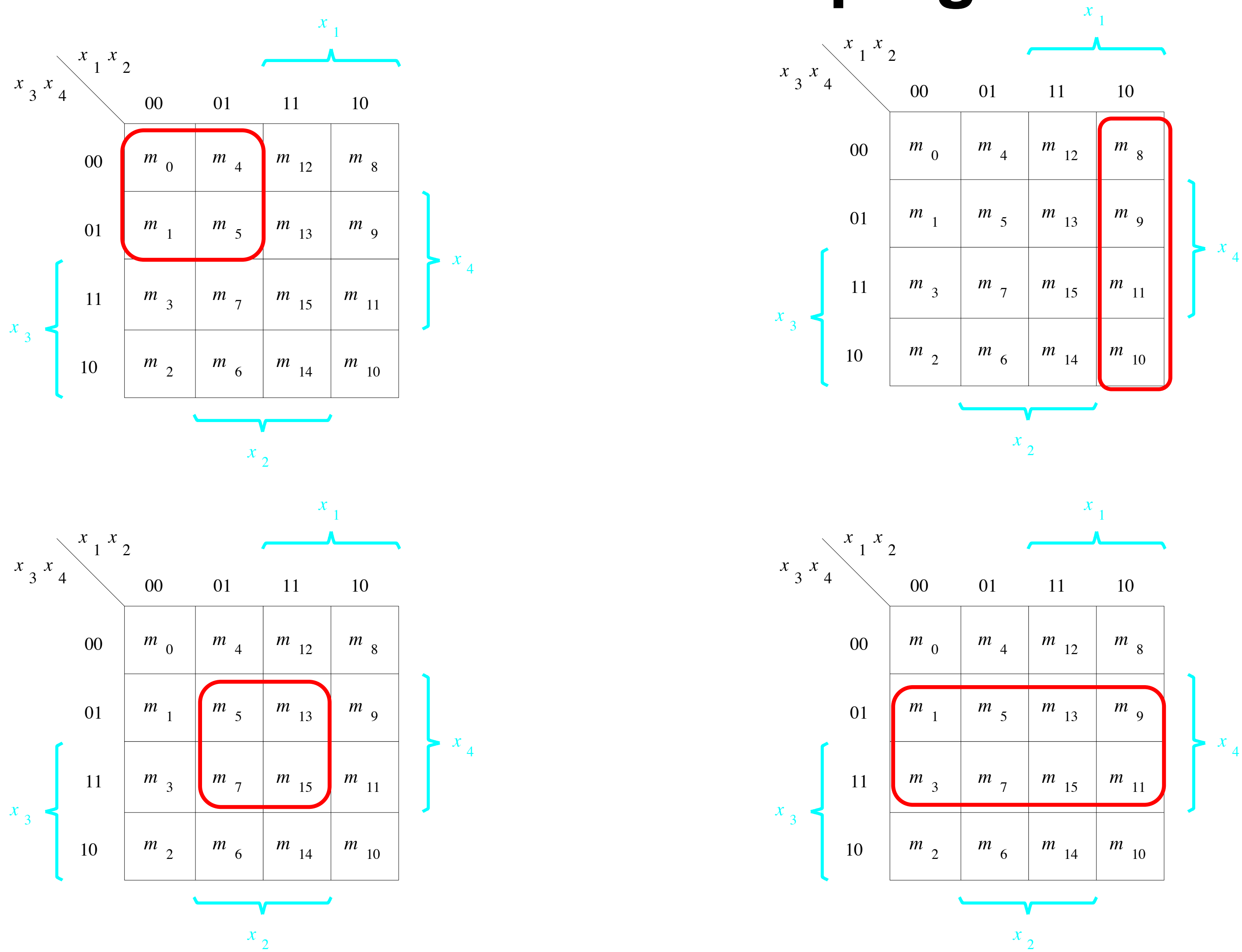
| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $\boxed{m_5}$ | $\boxed{m_{13}}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $\boxed{m_3}$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $\boxed{m_2}$ | $m_6$ | $m_{14}$ | $m_{10}$ |

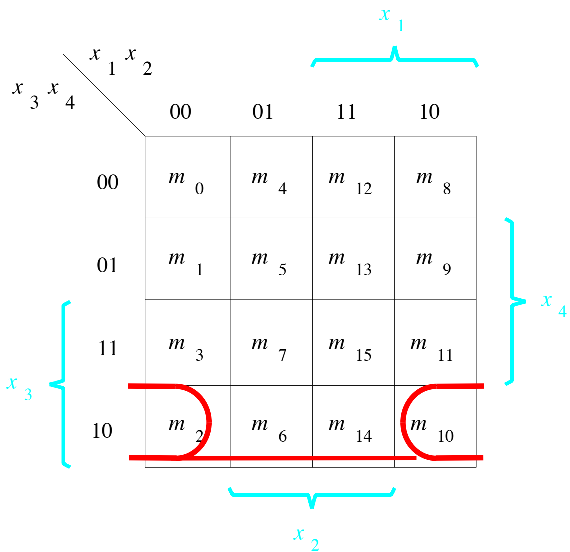
| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $\boxed{m_{15}}$ | $\boxed{m_{11}}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

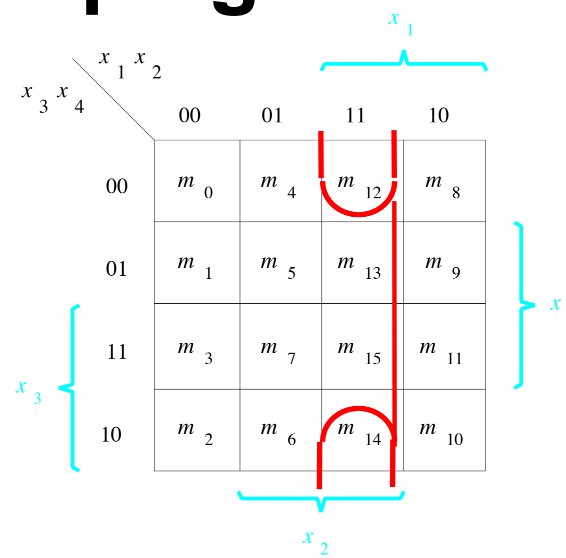
| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $\boxed{m_4}$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $\boxed{m_5}$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

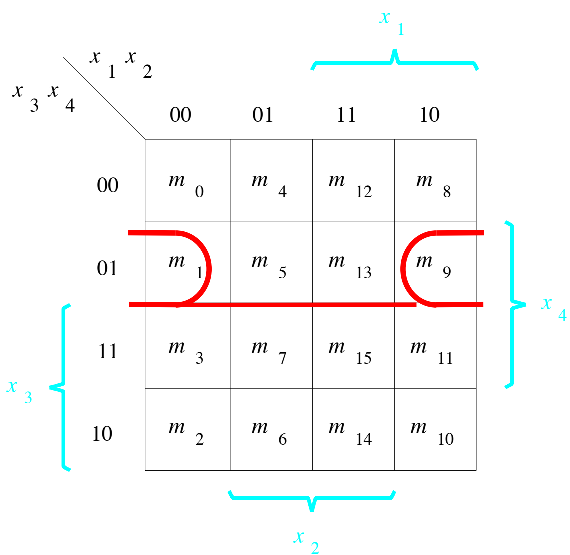
# Some Valid Groupings

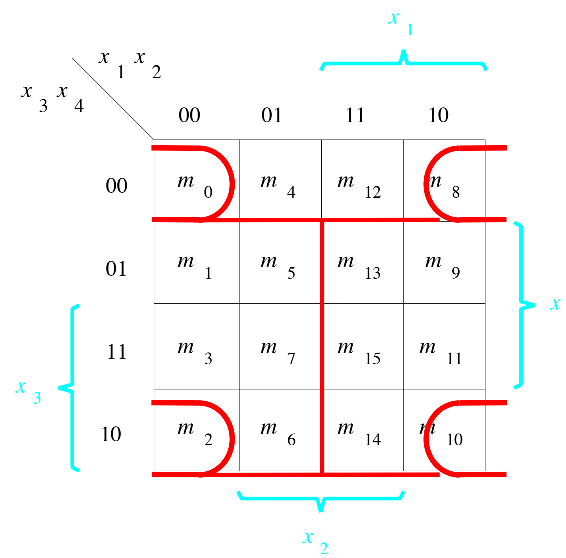


# Some Valid Groupings

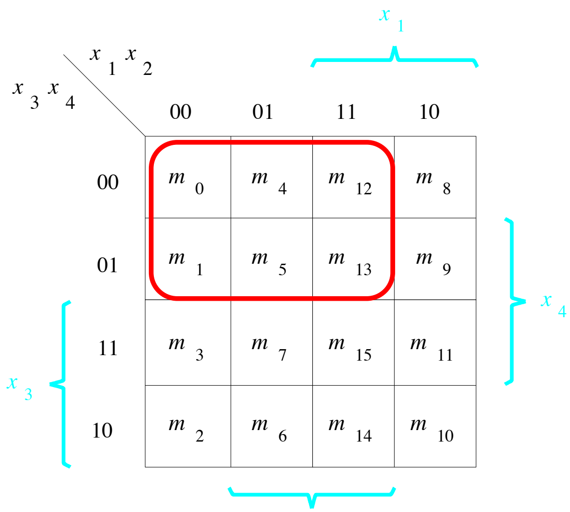


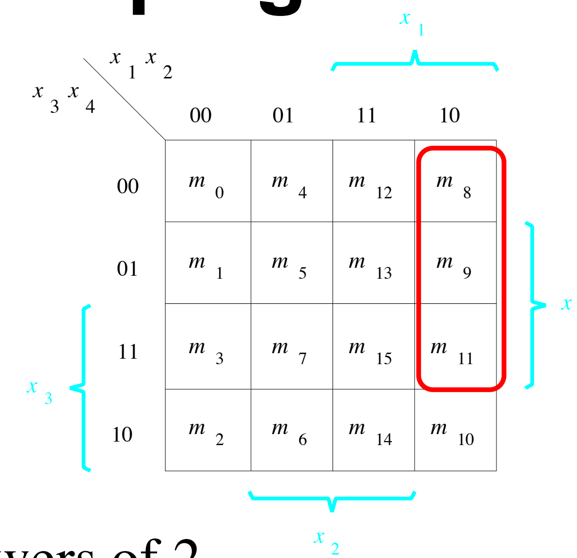



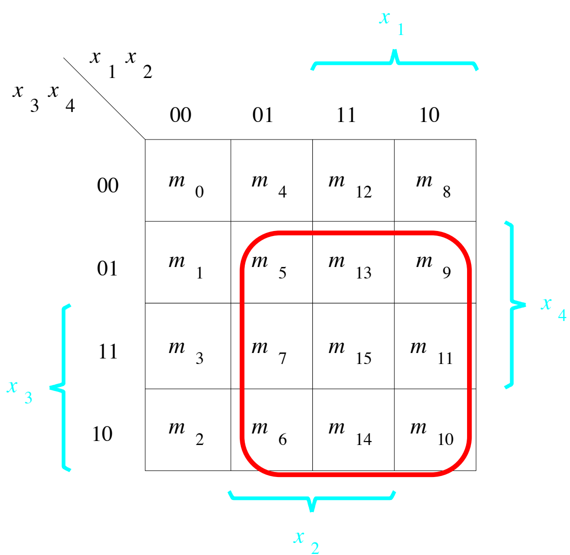



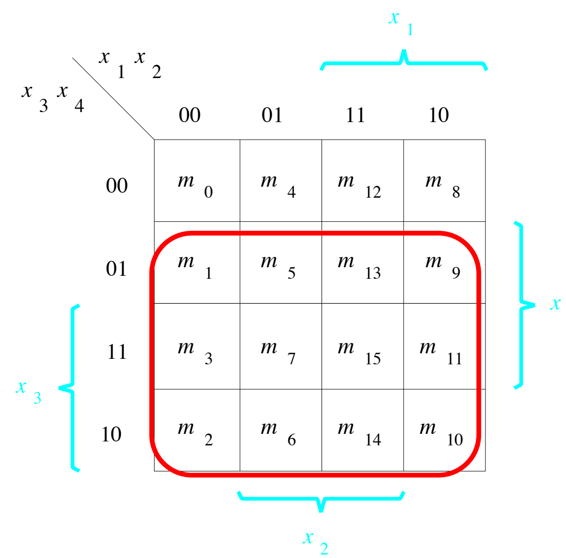
# Some Invalid Groupings



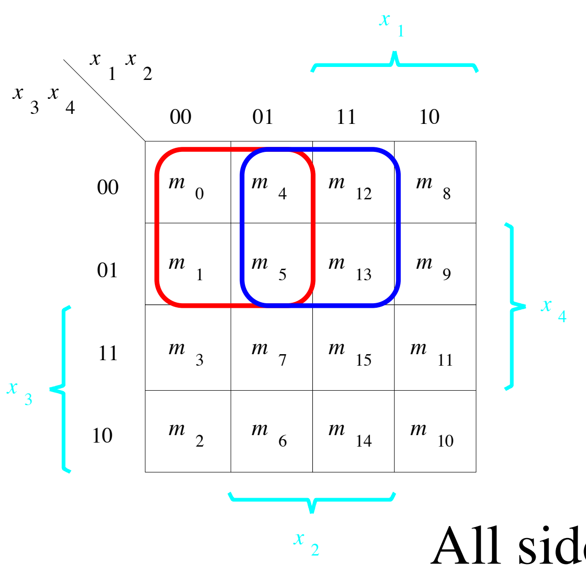



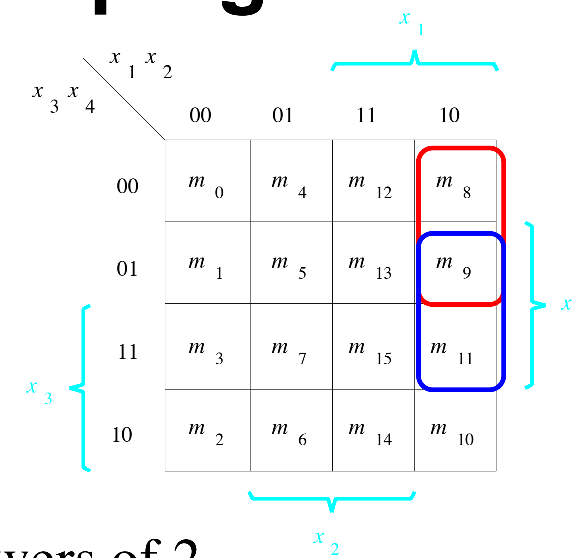

All sides must be powers of 2.



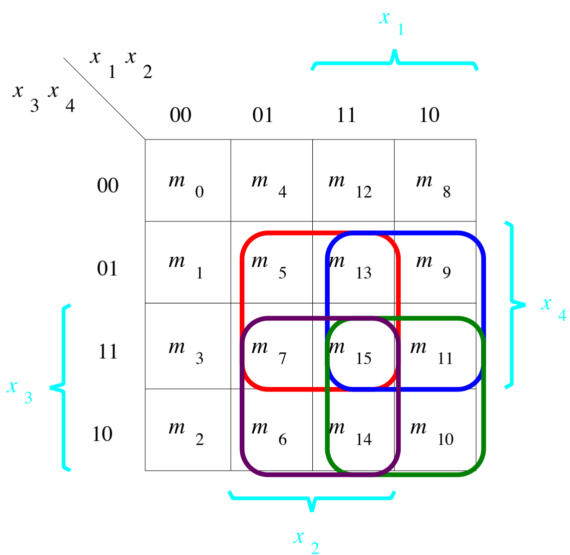



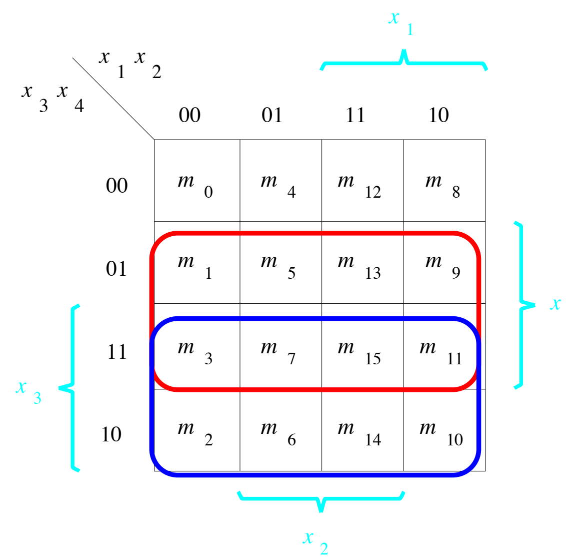
# Some valid Groupings



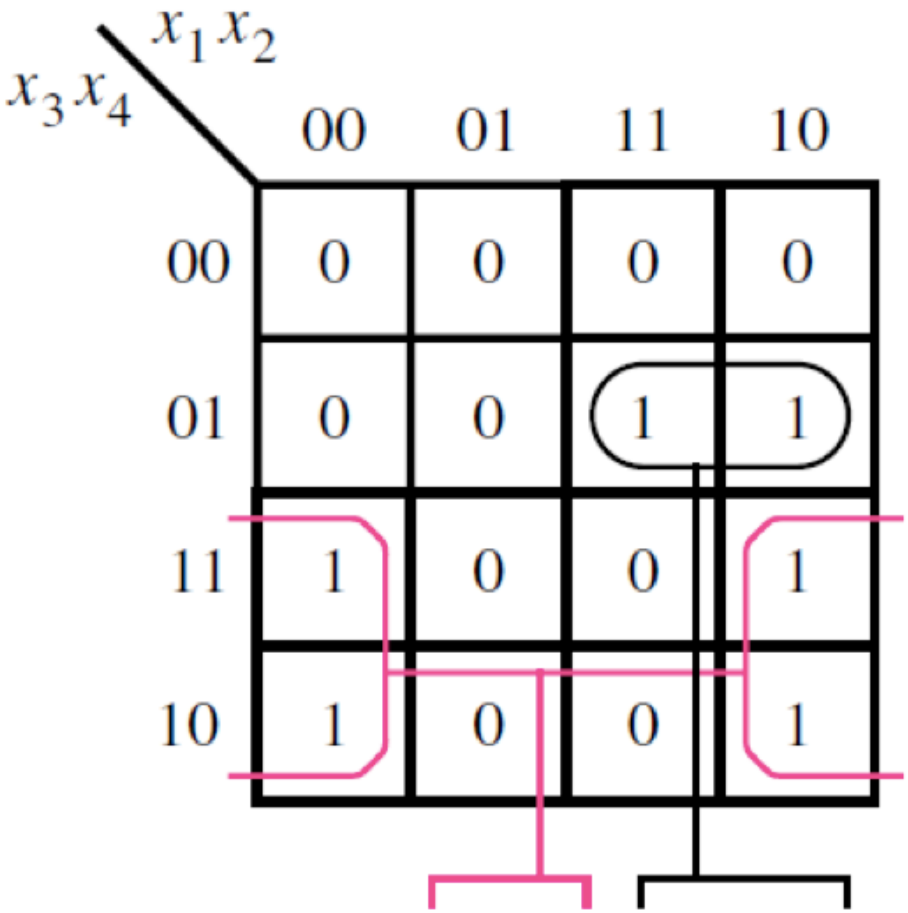

All sides must be powers of 2.



# Minimization Examples with 4-variable K-Maps

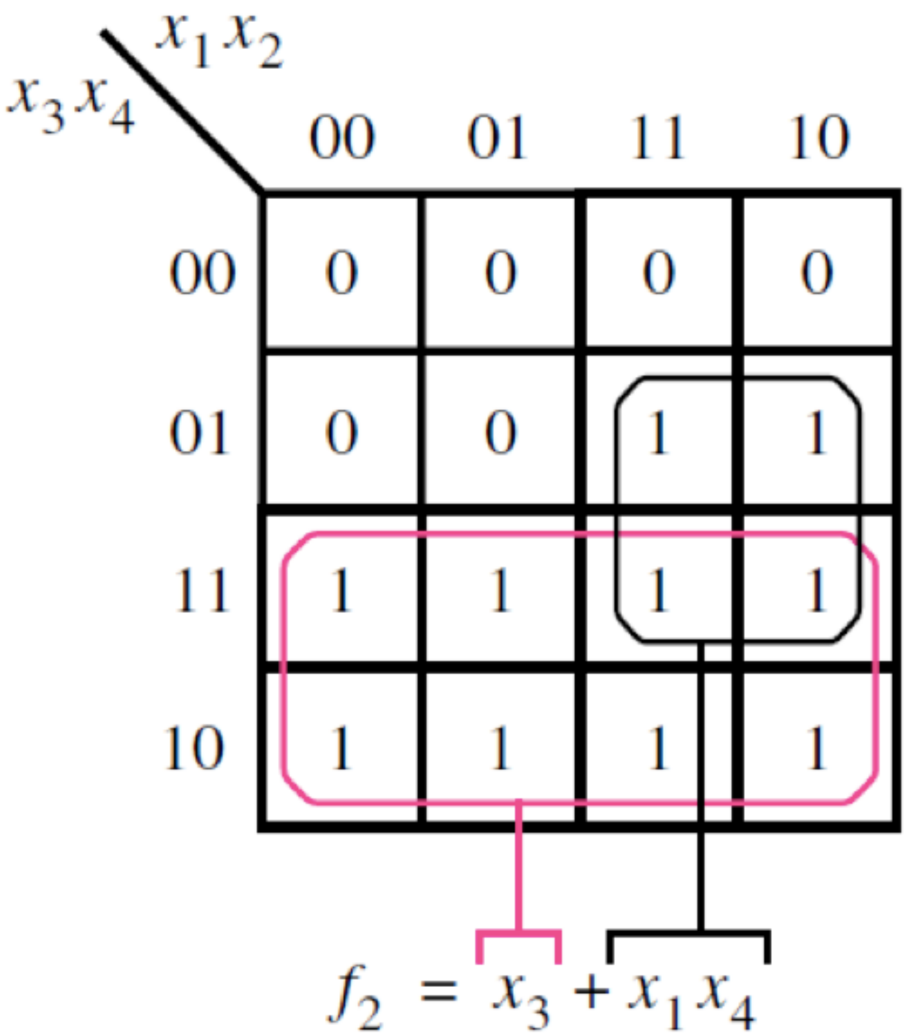
# Example of a four-variable Karnaugh map




$$f_1 = \bar{x}_2x_3 + x_1\bar{x}_3x_4$$

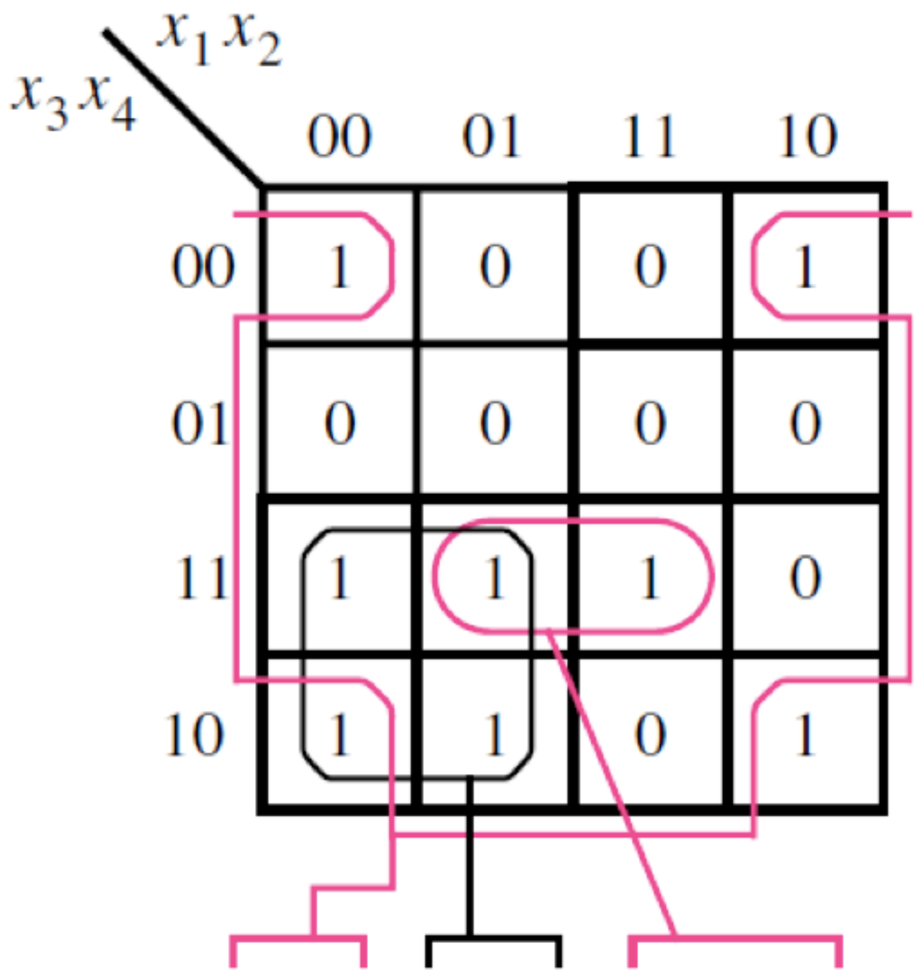
[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map




[ Figure 2.54 from the textbook ]

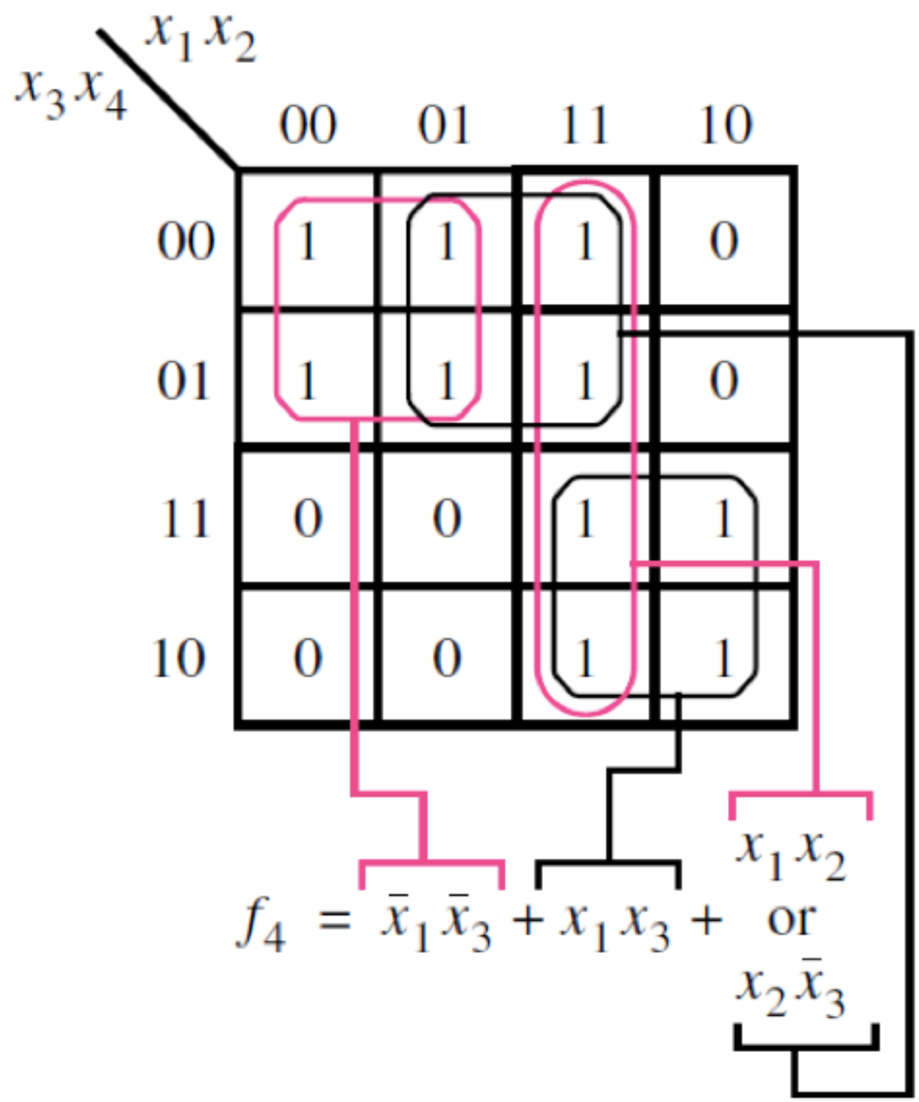
# Example of a four-variable Karnaugh map




$$f_3 = \bar{x}_2\bar{x}_4 + \bar{x}_1 x_3 + x_2 x_3 x_4$$

[ Figure 2.54 from the textbook ]

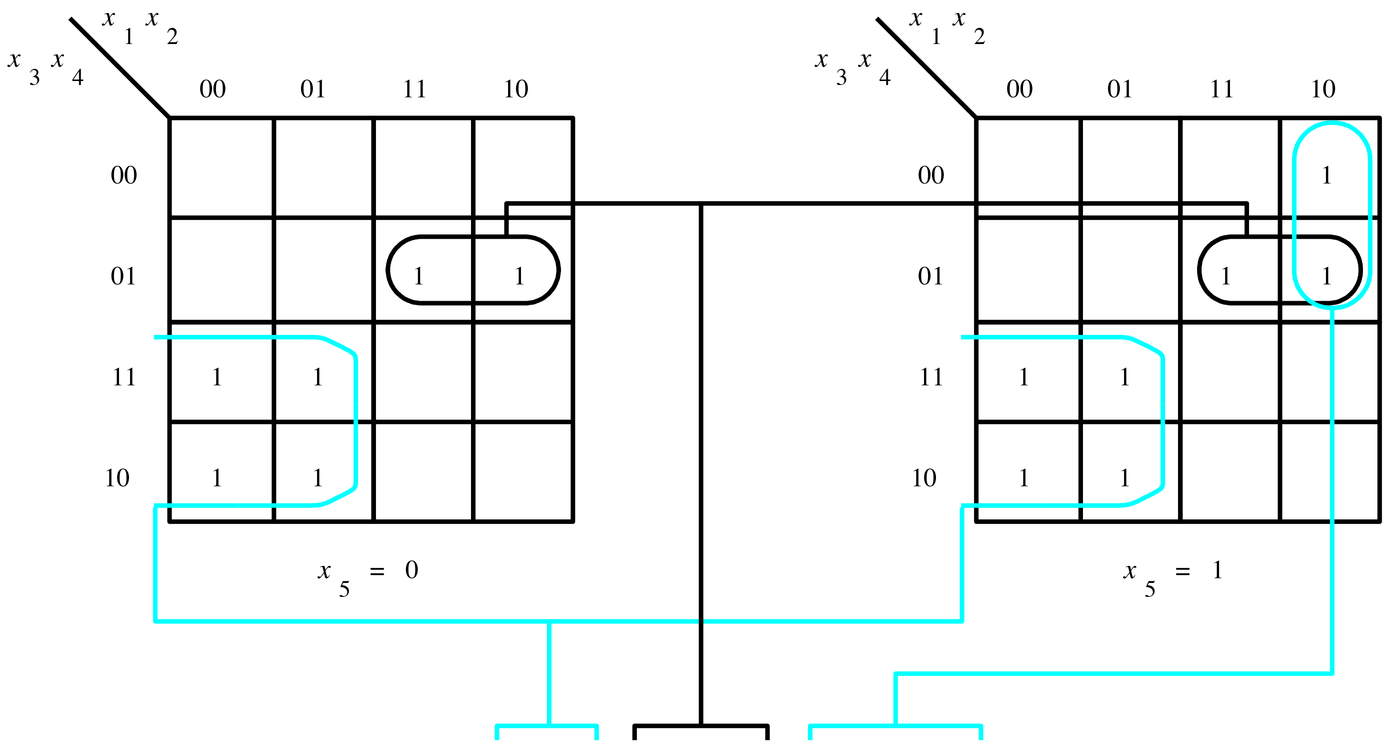
# Example of a four-variable Karnaugh map




[ Figure 2.54 from the textbook ]

# Five-Variable K-Map

# A five-variable Karnaugh map

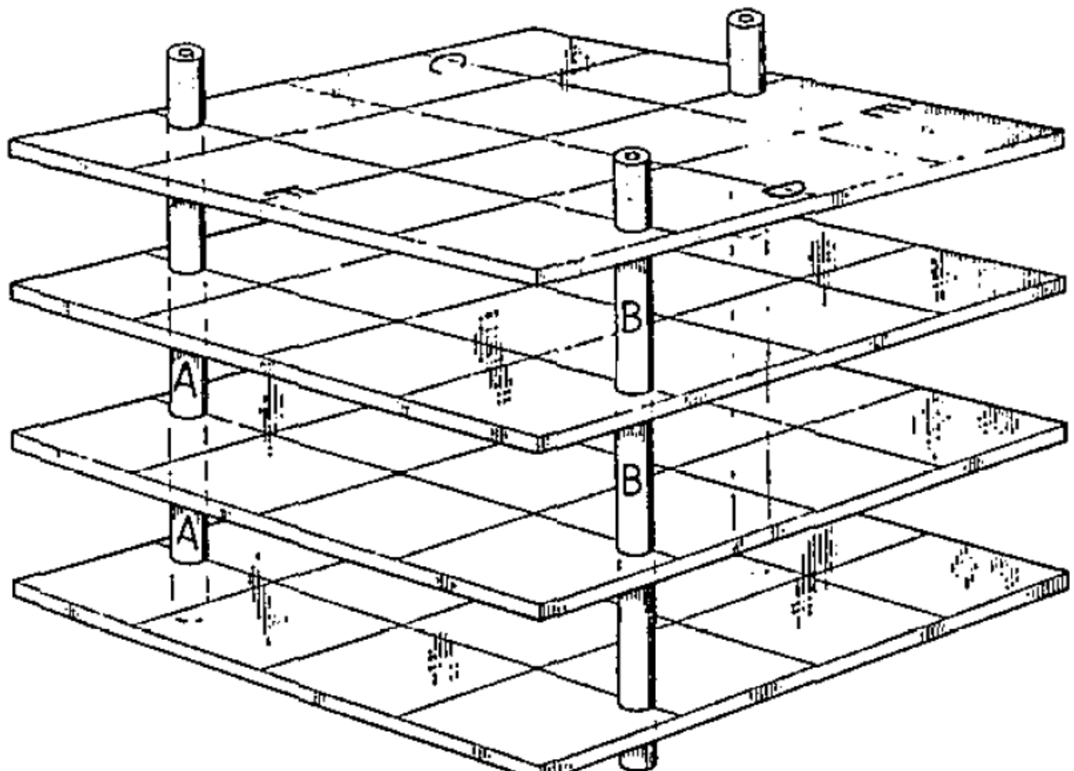



$$f_1 = \bar{x}_1 x_3 + x_1 \bar{x}_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 x_5$$

[ Figure 2.55 from the textbook ]

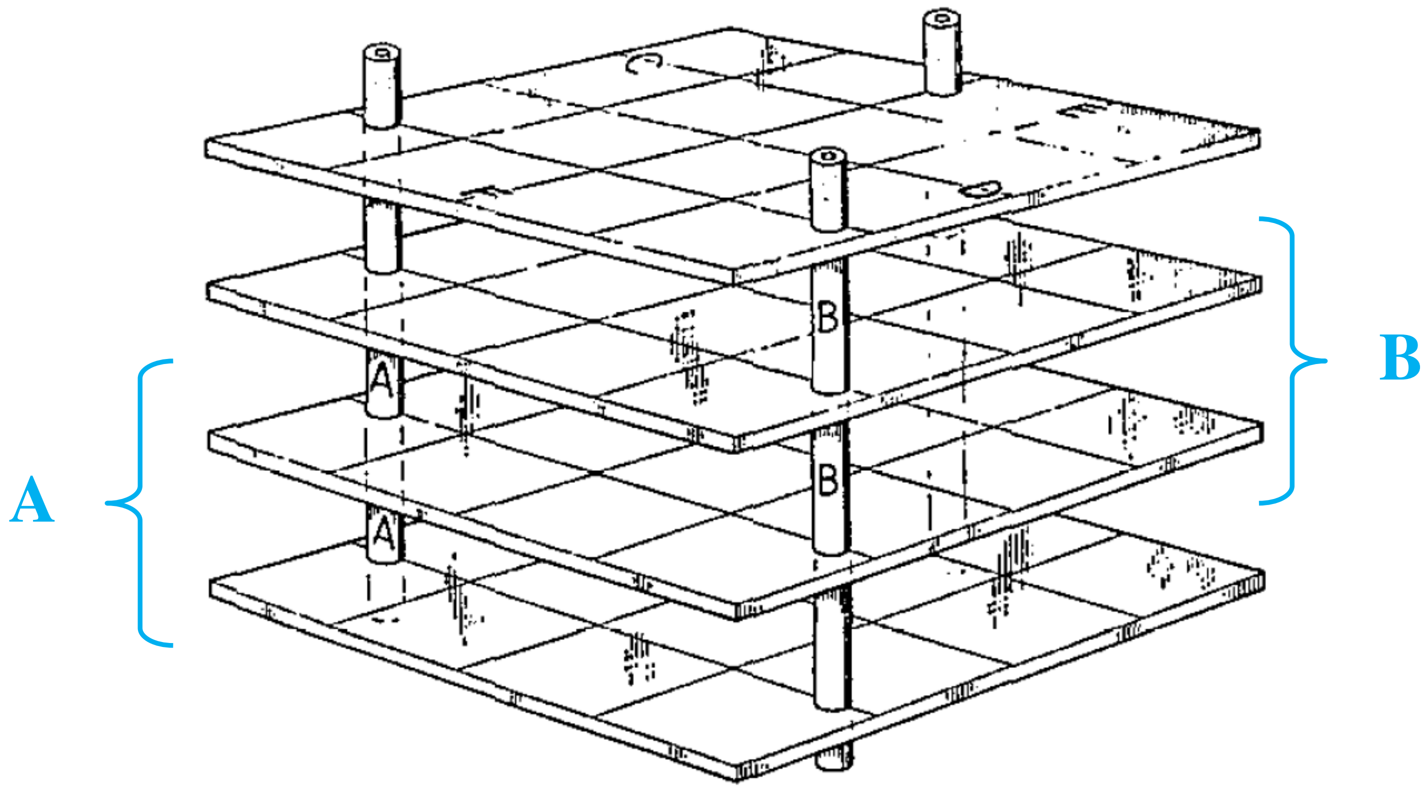
# Six-Variable K-Map

# A six-variable Karnaugh map



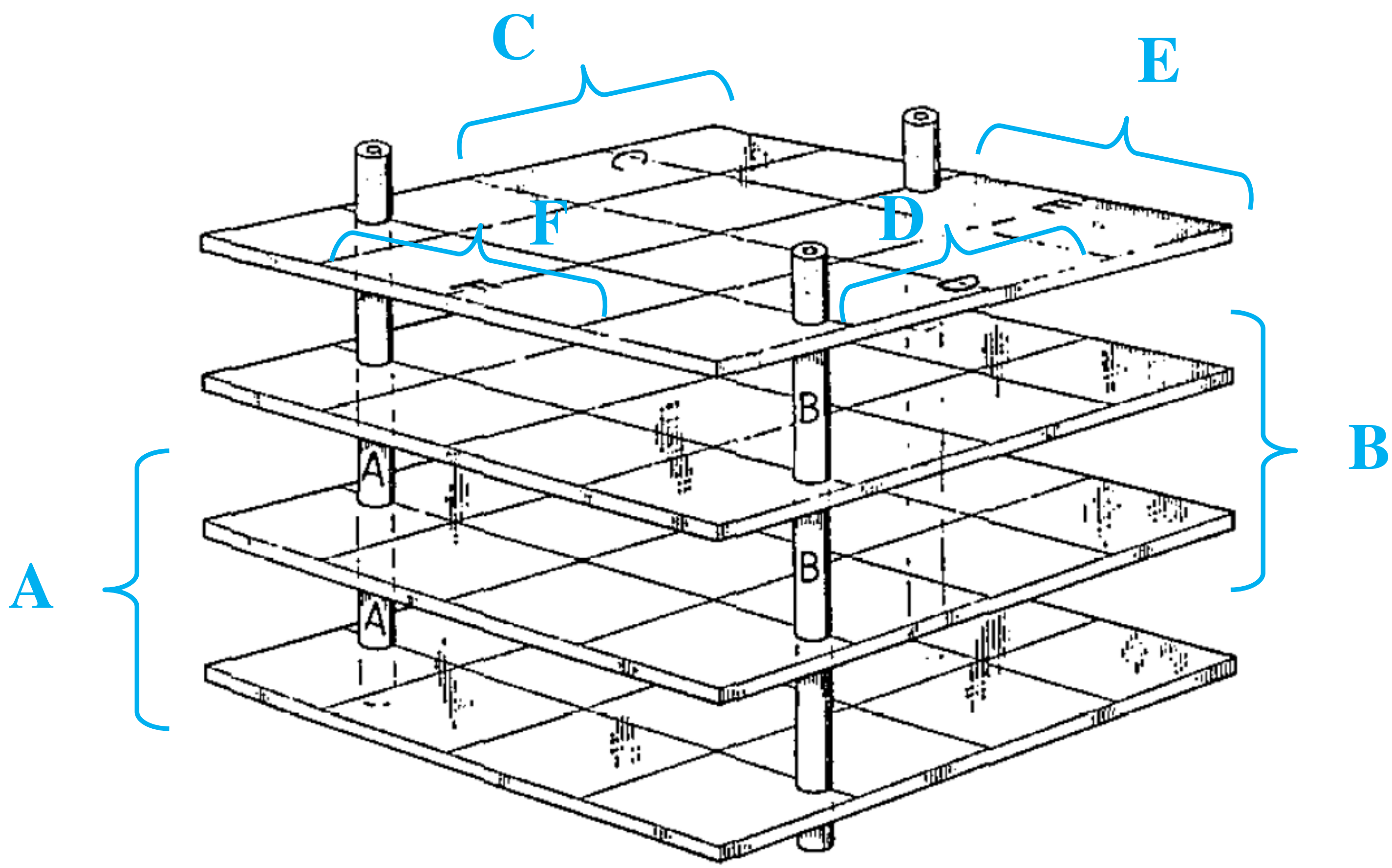

[ Figure 16, in Karnaugh 1953]

# A six-variable Karnaugh map




[ Figure 16, in Karnaugh 1953]

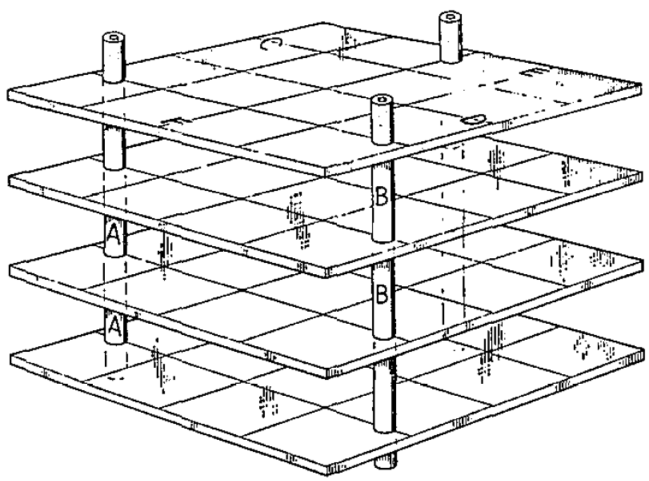
# A six-variable Karnaugh map

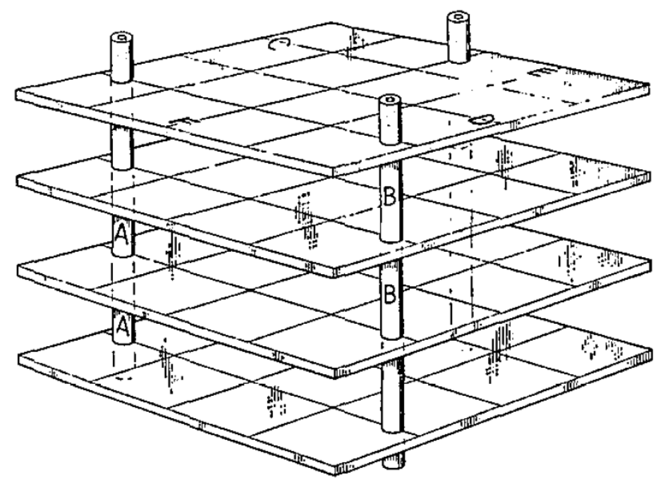



[ Figure 16, in Karnaugh 1953]

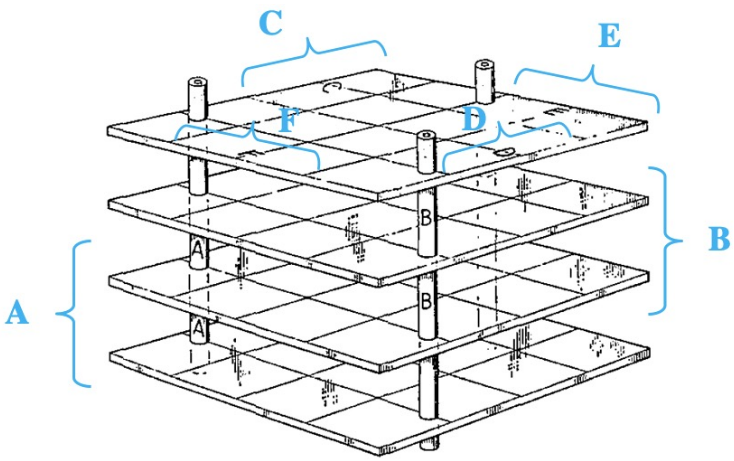
# Seven-Variable K-Map

# A seven-variable Karnaugh map

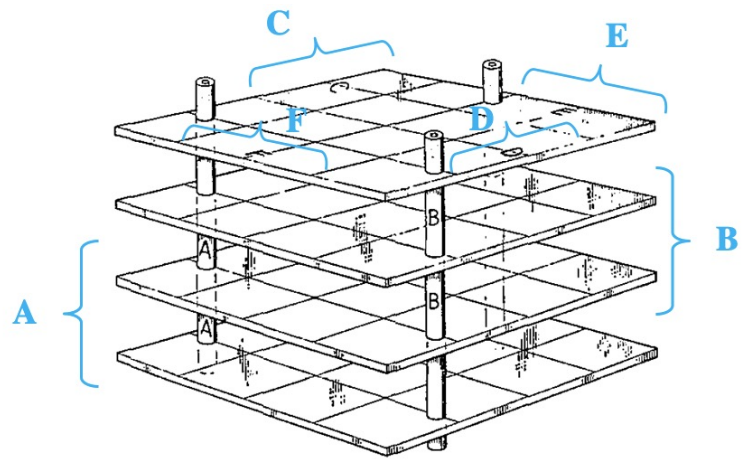


# A seven-variable Karnaugh map




G = 0




G = 1

[Suggested in Karnaugh 1953]

# Questions?

**THE END**