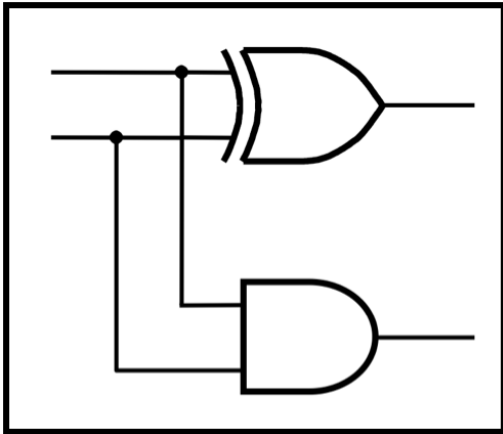




CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Karnaugh Maps

*CprE 2810: Digital Logic
Iowa State University, Ames, IA
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Administrative Stuff

- **HW3 is due today**

Administrative Stuff

- **HW4 is out**
- **It is due on Monday Sep 22 @ 10 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 26.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes
(you can bring up to 3 pages of handwritten notes).**
- **Sample exams are posted on the class web page.**
- **More details to follow.**

Quick Review

Do You Still Remember This Boolean Algebra Theorem?

14a. $x \cdot y + x \cdot \bar{y} = x$

14b. $(x + y) \cdot (x + \bar{y}) = x$

Combining

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \overline{y} = x}$
0	0	
0	1	
1	0	
1	1	

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \overline{y} = x}$
0	0	0
0	1	0
1	0	0
1	1	1

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$	
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	0

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$		
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	1	0

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$				
0	0	0	0	0		0
0	1	0	0	0		0
1	0	0	1	1		1
1	1	1	1	0		1

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$				
0	0	0	0	0	0	0
0	1	0	0	0	0	0
1	0	0	1	1	1	1
1	1	1	1	0	0	1

They are equal.

Motivation

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

An approach for simplifying logic expressions.

How do we guarantee that we have reached the minimum SOP/POS representation?

This method was described in 1953

M. Karnaugh, “The map method for synthesis of combinational logic circuits”

**Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics
(pages 593 – 599, Volume: 72, Issue: 5, Nov. 1953)**

<https://ieeexplore.ieee.org/document/6371932>

Two-Variable K-Map

Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

(a) Truth table

		x_1	
		0	1
x_2	0	0	0
	1	1	1

(b) Karnaugh map

Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table

		x_1	
		0	1
x_2	0	m_0	m_2
	1	m_1	m_3

(b) Karnaugh map

Minterms

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

x_1	x_2	m_0	m_1	m_2	m_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

$$\overline{x_1}x_2 + x_1x_2 = x_2$$

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

		x_1	
		0	1
x_2	0	0	0
	1	1	1

$$\overline{x}_1 x_2 + x_1 x_2 = x_2$$

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

		x_1	
		0	1
x_2	0	0	0
	1	1	1

x_2

$$\overline{x}_1x_2 + x_1x_2 = x_2$$

Another Grouping Example

		x_1	
		0	1
x_2	0	1	0
	1	0	0

m_0

		x_1	
		0	1
x_2	0	0	0
	1	1	0

m_1

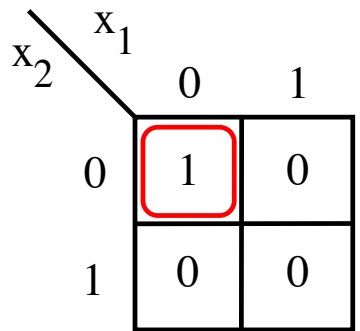
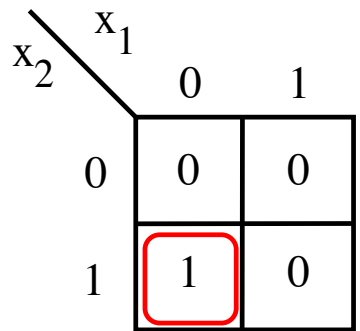
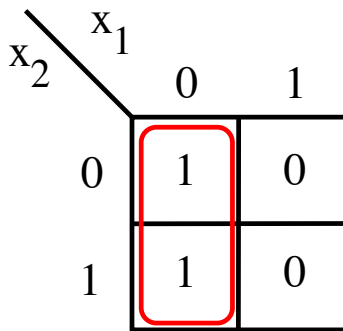
Another Grouping Example

$x_2 \swarrow \quad \nearrow x_1$ <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> </table> + $x_2 \swarrow \quad \nearrow x_1$ <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </table> = $x_2 \swarrow \quad \nearrow x_1$ <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </table>		0	1	0	1	0	1	0	0		0	1	0	0	0	1	1	0		0	1	0	1	0	1	1	0	m_0 + m_1 = $m_0 + m_1$
	0	1																										
0	1	0																										
1	0	0																										
	0	1																										
0	0	0																										
1	1	0																										
	0	1																										
0	1	0																										
1	1	0																										

Another Grouping Example

x_2 x_1 <table border="1" style="border-collapse: collapse; text-align: center;"><tr><td></td><td>0</td><td>1</td></tr><tr><td>0</td><td style="border: 2px solid red;">1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr></table>		0	1	0	1	0	1	0	0	+	x_2 x_1 <table border="1" style="border-collapse: collapse; text-align: center;"><tr><td></td><td>0</td><td>1</td></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>1</td><td style="border: 2px solid red;">1</td><td>0</td></tr></table>		0	1	0	0	0	1	1	0	=	x_2 x_1 <table border="1" style="border-collapse: collapse; text-align: center;"><tr><td></td><td>0</td><td>1</td></tr><tr><td>0</td><td style="border: 2px solid red;">1</td><td>0</td></tr><tr><td>1</td><td style="border: 2px solid red;">1</td><td>0</td></tr></table>		0	1	0	1	0	1	1	0
	0	1																													
0	1	0																													
1	0	0																													
	0	1																													
0	0	0																													
1	1	0																													
	0	1																													
0	1	0																													
1	1	0																													
m_0	+	m_1	=	$m_0 + m_1$																											

Another Grouping Example

	+		=	
m_0	+	m_1	=	$m_0 + m_1$
$\bar{x}_1\bar{x}_2$	+	$\bar{x}_1x_2$	=	$\bar{x}_1$

Property 14a (Combining)

Grouping Rules

- **Group “1”s with rectangles**
- **Both sides must be a power of 2:**
 - 1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4
- **Can use/cover the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
 - Try to use as few groups as possible to cover all “1”s.
 - For each group, try to make it as large as you can (i.e., if you can use a 2x2, don’t use a 2x1 even if that is enough).

Two-Variable K-map

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table

		x_1	
		0	1
x_2	0	m_0	m_2
	1	m_1	m_3

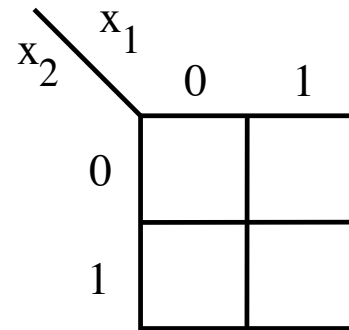
(b) Karnaugh map

Step-By-Step Example

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

1. Draw The Map

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1



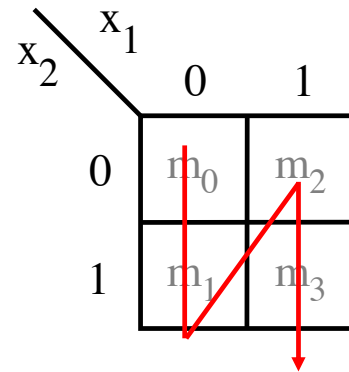
2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

		x_1	
		0	1
x_2	0	m_0	m_2
	1	m_1	m_3

2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1



2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

		x_1	
		0	1
x_2	0	1	0
	1	1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

		x_1	
		0	1
x_2	0	1	0
	1	1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

		x_1	
		0	1
x_2	0	1	0
	1	1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

		x_1	
		0	1
x_2	0	1	0
	1	1	1

4. Write The Expression

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

		x_1	
x_2		0	1
0	1	1	0
1	1	1	1

4. Write The Expression

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

		x_1	
x_2		0	1
0	1	1	0
1	1	1	1

$$\bar{x}_1 + x_2$$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	1	0
	1	1	0

$$\overline{x_1}$$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	0	1
	1	0	1

x_1

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	1	1
	1	0	0

$\overline{x_2}$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	0	0
	1	1	1

x_2

These are all valid groupings

$x_2 \backslash x_1$	0	1
	0	1
0	1	0
1	0	0

$\bar{x}_1 \bar{x}_2$

$x_2 \backslash x_1$	0	1
	0	1
0	0	0
1	1	0

$\bar{x}_1 x_2$

$x_2 \backslash x_1$	0	1
	0	1
0	0	1
1	0	0

$x_1 \bar{x}_2$

$x_2 \backslash x_1$	0	1
	0	1
0	0	0
1	0	1

$x_1 x_2$

These are all valid groupings

x_2	x_1	
	0	1
0	1	0
1	1	0

$\bar{x}_1$

x_2	x_1	
	0	1
0	0	1
1	0	1

x_1

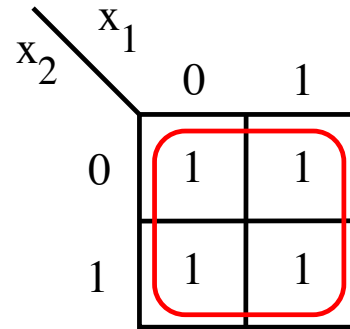
x_2	x_1	
	0	1
0	1	1
1	0	0

$\bar{x}_2$

x_2	x_1	
	0	1
0	0	0
1	1	1

x_2

This one is valid too



A Karnaugh map for a 2-variable function with variables x_1 and x_2 . The map is a 2x2 grid where all four cells contain the value 1. A red rounded rectangle encloses the entire 2x2 grid, indicating a single group that covers all possible input combinations. The variables x_1 and x_2 are labeled on the top and left sides of the grid, respectively, with their binary values 0 and 1.

$x_2 \backslash x_1$	0	1
0	1	1
1	1	1

In this case the result is the constant function 1.

Invalid Groupings

A Karnaugh map for variables x_1 and x_2 . The map is a 2x2 grid. The columns are labeled x_1 with values 0 and 1. The rows are labeled x_2 with values 0 and 1. The cells contain the following values:

$x_2 \backslash x_1$	0	1
0	1	0
1	0	1

A red line is drawn around all four cells, indicating an invalid grouping of four 1s. This is invalid because the four cells do not share a common adjacent path (they are not connected in a single continuous line).

A Karnaugh map for variables x_1 and x_2 . The map is a 2x2 grid. The columns are labeled x_1 with values 0 and 1. The rows are labeled x_2 with values 0 and 1. The cells contain the following values:

$x_2 \backslash x_1$	0	1
0	0	1
1	1	0

A red line is drawn around all four cells, indicating an invalid grouping of four 1s. This is invalid because the four cells do not share a common adjacent path (they are not connected in a single continuous line).

Can't group diagonally. Why?

		x_1	
		0	1
x_2	0	1	0
	1	0	0

m_0

		x_1	
		0	1
x_2	0	0	0
	1	0	1

m_3

Can't group diagonally. Why?

x_2 x_1	0	1
	0	1
0	1	0
1	0	0

m_0

+

+

x_2 x_1	0	1
	0	1
0	0	0
1	0	1

m_3

=

=

x_2 x_1	0	1
	0	1
0	1	0
1	0	1

$m_0 + m_3$

Can't group diagonally. Why?

$x_2 \diagdown$ <table border="1" style="border-collapse: collapse; text-align: center;"><tr><td></td><td>x_1</td><td>0</td><td>1</td></tr><tr><td>0</td><td></td><td style="border: 2px solid red;">1</td><td>0</td></tr><tr><td>1</td><td></td><td>0</td><td>0</td></tr></table> m_0		x_1	0	1	0		1	0	1		0	0	$+$	$x_2 \diagdown$ <table border="1" style="border-collapse: collapse; text-align: center;"><tr><td></td><td>x_1</td><td>0</td><td>1</td></tr><tr><td>0</td><td></td><td>0</td><td>0</td></tr><tr><td>1</td><td></td><td>0</td><td style="border: 2px solid red;">1</td></tr></table> m_3		x_1	0	1	0		0	0	1		0	1	$=$	$x_2 \diagdown$ <table border="1" style="border-collapse: collapse; text-align: center;"><tr><td></td><td>x_1</td><td>0</td><td>1</td></tr><tr><td>0</td><td></td><td style="border: 2px solid red;">1</td><td>0</td></tr><tr><td>1</td><td></td><td>0</td><td style="border: 2px solid red;">1</td></tr></table> $m_0 + m_3$		x_1	0	1	0		1	0	1		0	1
	x_1	0	1																																					
0		1	0																																					
1		0	0																																					
	x_1	0	1																																					
0		0	0																																					
1		0	1																																					
	x_1	0	1																																					
0		1	0																																					
1		0	1																																					

Can't group diagonally. Why?

$ \begin{array}{c cc} & x_1 & \\ x_2 & 0 & 1 \\ \hline 0 & 1 & 0 \\ 1 & 0 & 0 \end{array} $	+	$ \begin{array}{c cc} & x_1 & \\ x_2 & 0 & 1 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 1 \end{array} $	=	$ \begin{array}{c cc} & x_1 & \\ x_2 & 0 & 1 \\ \hline 0 & 1 & 0 \\ 1 & 0 & 1 \end{array} $
m_0	+	m_3	=	$m_0 + m_3$
$\overline{x_1}\overline{x_2}$	+	x_1x_2	=	$\overline{x_1}\overline{x_2} + x_1x_2$

We can't use Property 14a here. This can't be simplified.

Three-Variable K-Map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 \ x_2$			
x_3		x_3			
		00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

(b) Karnaugh map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 \ x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

	x_1	x_2		
x_3	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

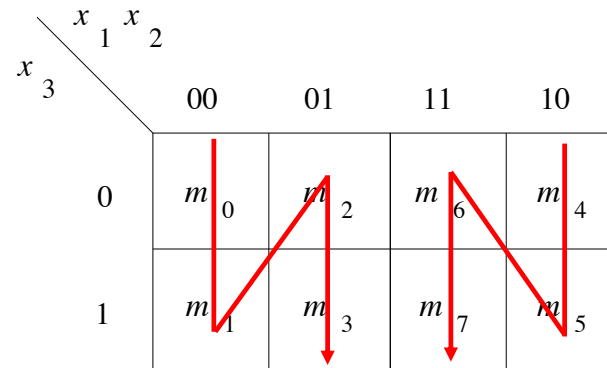
Notice the placement of

- Variables
- Binary pair values
- Minterms

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table



(b) Karnaugh map

Notice the placement of

- Variables
- Binary pair values
- Minterms

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

00

01

11

10

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000
001
011
010
110
111
101
100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000
001
011
010
110
111
101
100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000

001

011

010

110

111

101

100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000

001

011

010

110

111

101

100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000

001

011

010

110

111

101

100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000
001
011
010
110
111
101
100

Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000

001

011

010

110

111

101

100

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
x_2		00	01	11	10
	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
x_2		00	01	11	10
	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
x_2		00	01	11	10
	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_I$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

Adjacency Rules

x_1x_2					
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns



Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
x_2		00	01	11	10
	0	000	010	110	100
1		001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

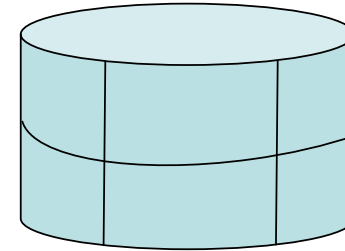
These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

Adjacency Rules

x_1x_2					
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns

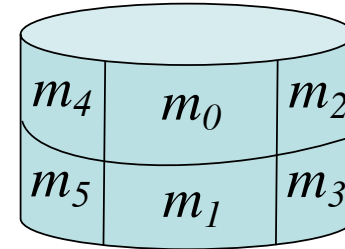


As if the K-map were
drawn on a cylinder

Adjacency Rules

x_1x_2					
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns

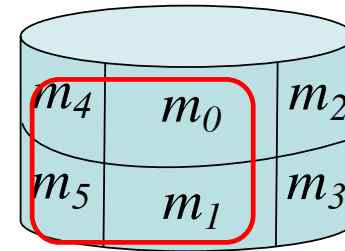


As if the K-map were
drawn on a cylinder

Adjacency Rules

x_1x_2		x_3			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns

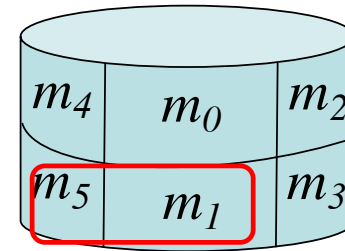


As if the K-map were
drawn on a cylinder

Adjacency Rules

$x_1 x_2$		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5	

adjacent
columns



As if the K-map were
drawn on a cylinder

Examples of Valid Groupings

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	0
	1	0	0	0	0

$\overline{x_1} \overline{x_2} \overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	0	0
	1	0	0	0	0

$\overline{x_1} x_2 \overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	0	0	0

$x_1 x_2 \overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	1
	1	0	0	0	0

$x_1 \overline{x_2} \overline{x_3}$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	0	0	0

$$\overline{x_1} \overline{x_2} x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	0	0

$$\overline{x_1} x_2 \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	1	0

$$x_1 x_2 x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	0	1

$$\overline{x_1} \overline{x_2} x_3$$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	0
	1	1	0	0	0

$\overline{x_1} \overline{x_2}$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	0	0
	1	0	1	0	0

$\overline{x_1} x_2$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	0	1	0

$x_1 x_2$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	1
	1	0	0	0	1

$x_1 \overline{x_2}$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	1	0	0
	1	0	0	0	0

$\overline{x_1} \overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	0
	1	0	0	0	0

$x_2 \overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	1
	1	0	0	0	0

$x_1 \overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	1
	1	0	0	0	0

$\overline{x_2} \overline{x_3}$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	1	0	0

$\overline{x_1} x_3$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	1	0

$x_2 x_3$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	1	1

$x_1 x_3$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	0	0	1

$\overline{x_2} x_3$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	1	0	0
	1	1	1	0	0

$\overline{x_1}$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	0
	1	0	1	1	0

x_2

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	1
	1	0	0	1	1

x_1

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	1
	1	1	0	0	1

$\overline{x_2}$

Groupings and Expressions

x_1x_2		00	01	11	10
x_3	0	1	1	1	1
	1	0	0	0	0

$\overline{x_3}$

x_1x_2		00	01	11	10
x_3	0	0	0	0	0
	1	1	1	1	1

x_3

Groupings and Expressions

$x_3 \backslash x_1x_2$		00	01	11	10
0	0	0	0	0	0
1	0	0	0	0	0

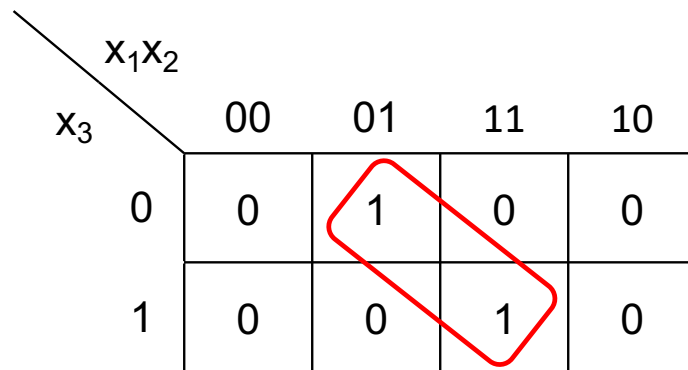
0

$x_3 \backslash x_1x_2$		00	01	11	10
0	1	1	1	1	1
1	1	1	1	1	1

1

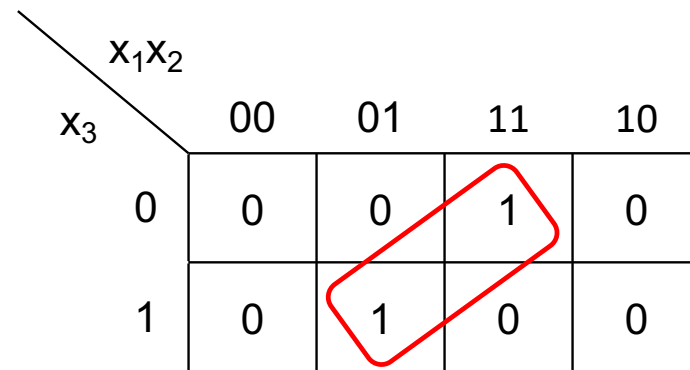
Examples of **Invalid** Groupings

Some **Invalid** Groupings



A Karnaugh map for three variables x_1, x_2, x_3 . The vertical axis is x_3 with values 0 and 1. The horizontal axis is x_1x_2 with values 00, 01, 11, and 10. The map contains 1s at positions (0, 01) and (1, 11). A red line connects these two 1s diagonally, representing an invalid grouping because they do not share a common value for any single variable.

$x_3 \backslash x_1x_2$	00	01	11	10
0	0	1	0	0
1	0	0	1	0



A Karnaugh map for three variables x_1, x_2, x_3 . The vertical axis is x_3 with values 0 and 1. The horizontal axis is x_1x_2 with values 00, 01, 11, and 10. The map contains 1s at positions (0, 11) and (1, 01). A red line connects these two 1s diagonally, representing an invalid grouping because they do not share a common value for any single variable.

$x_3 \backslash x_1x_2$	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Can't group diagonally.

Some **Invalid** Groupings

x_1x_2		00	01	11	10
x_3	0	1	1	1	0
	1	0	0	0	0

x_1x_2		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	1	1

Can't group three in a row.
Each side must be a power of 2.

Some **Invalid** Groupings

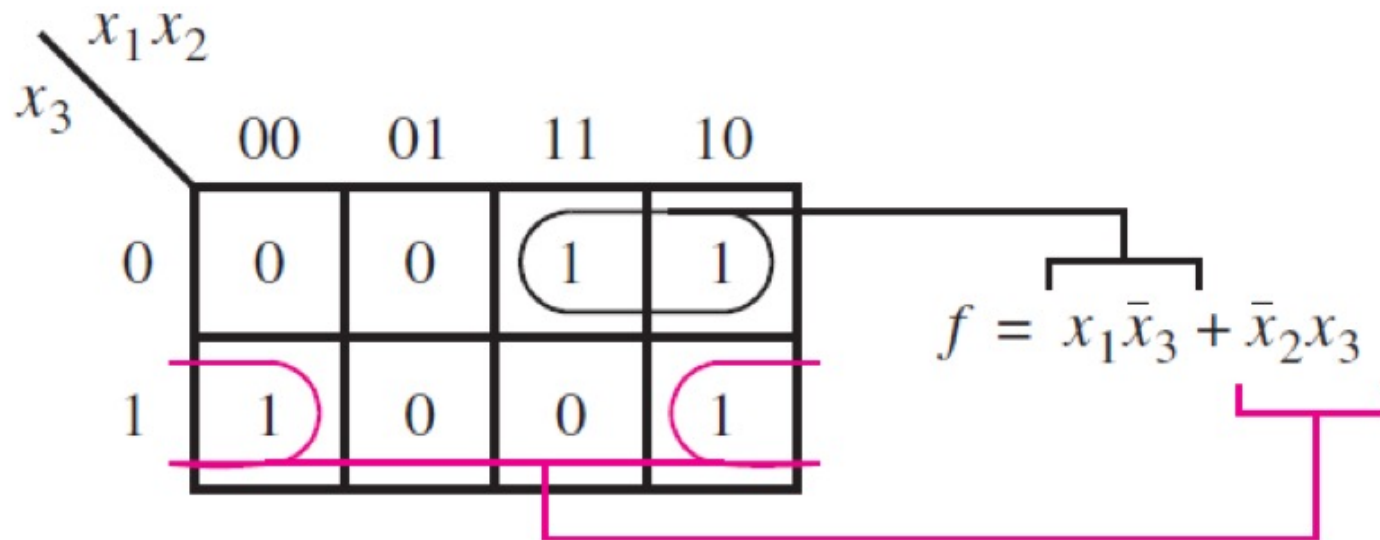
x_1x_2		00	01	11	10
x_3	0	1	0	1	1
	1	0	0	0	0

x_1x_2		00	01	11	10
x_3	0	0	0	1	0
	1	0	1	1	0

Can't group zeros and ones together.

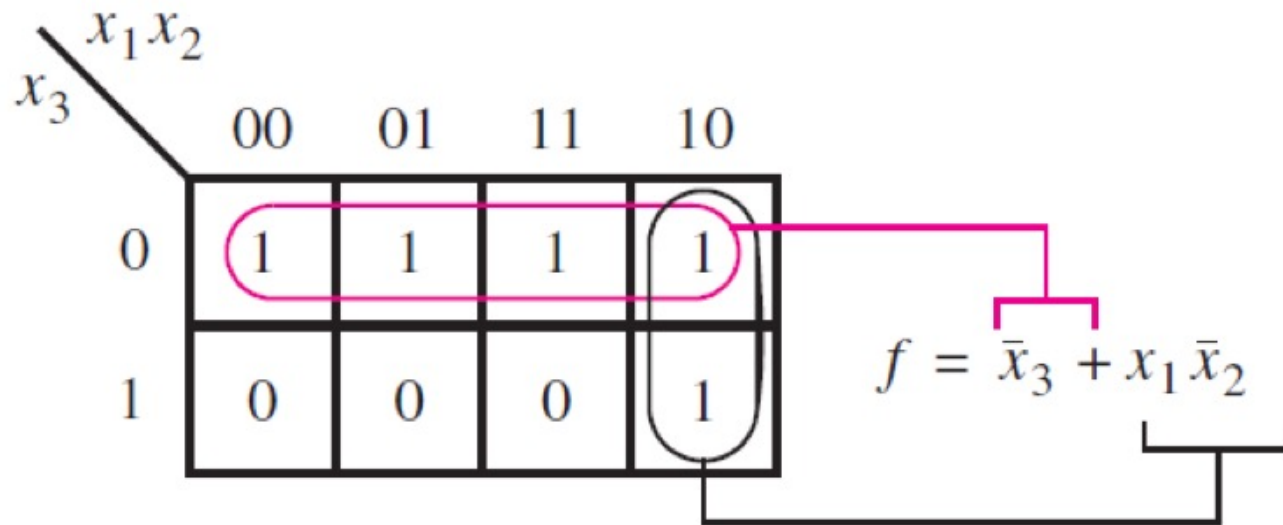
Minimization Examples with 3-variable K-Maps

Examples of three-variable Karnaugh maps



[Figure 2.52a from the textbook]

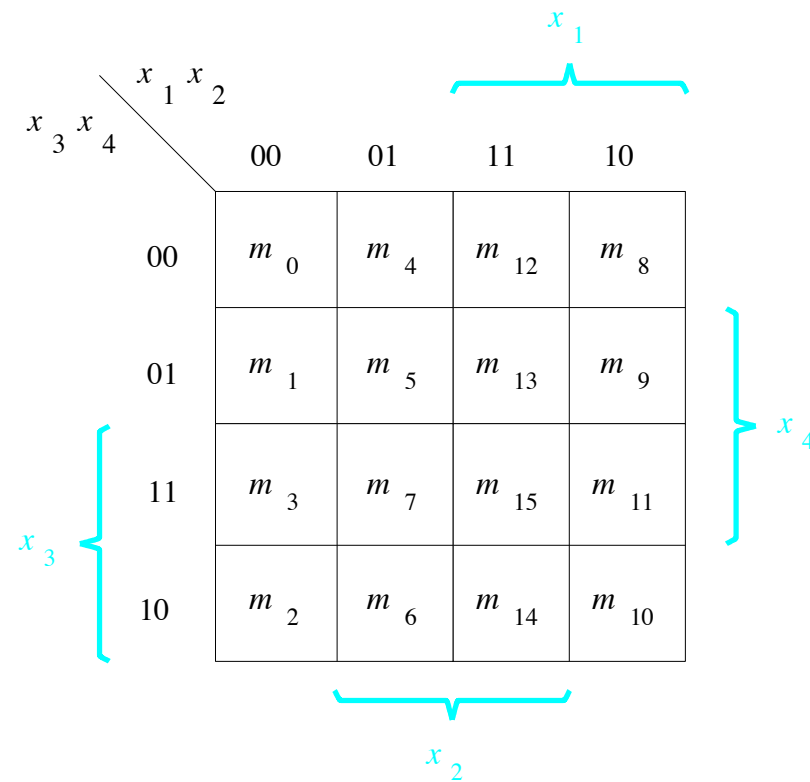
Examples of three-variable Karnaugh maps



[Figure 2.52b from the textbook]

Four-Variable K-Map

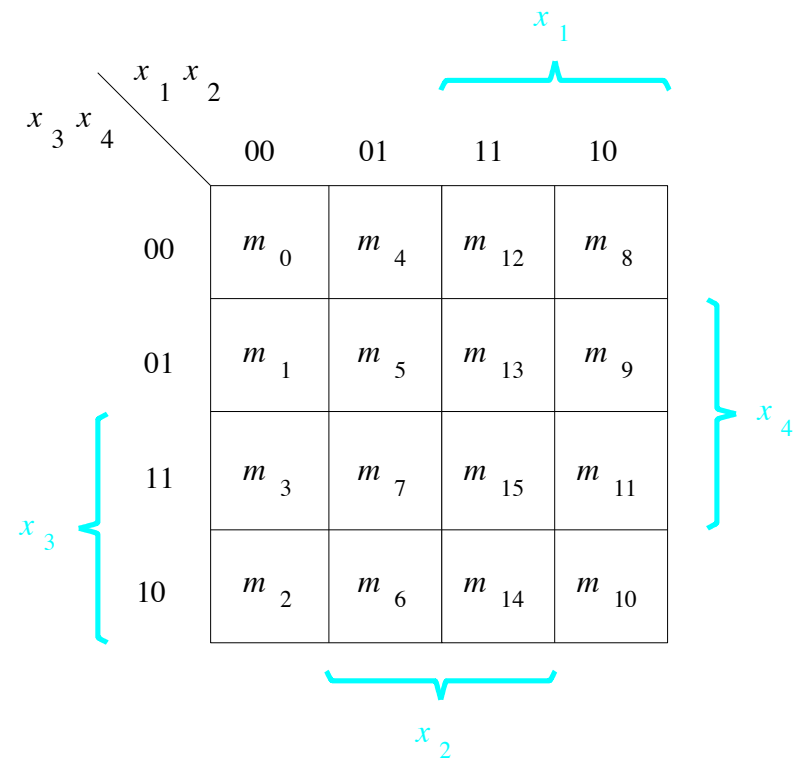
A four-variable Karnaugh map



[Figure 2.53 from the textbook]

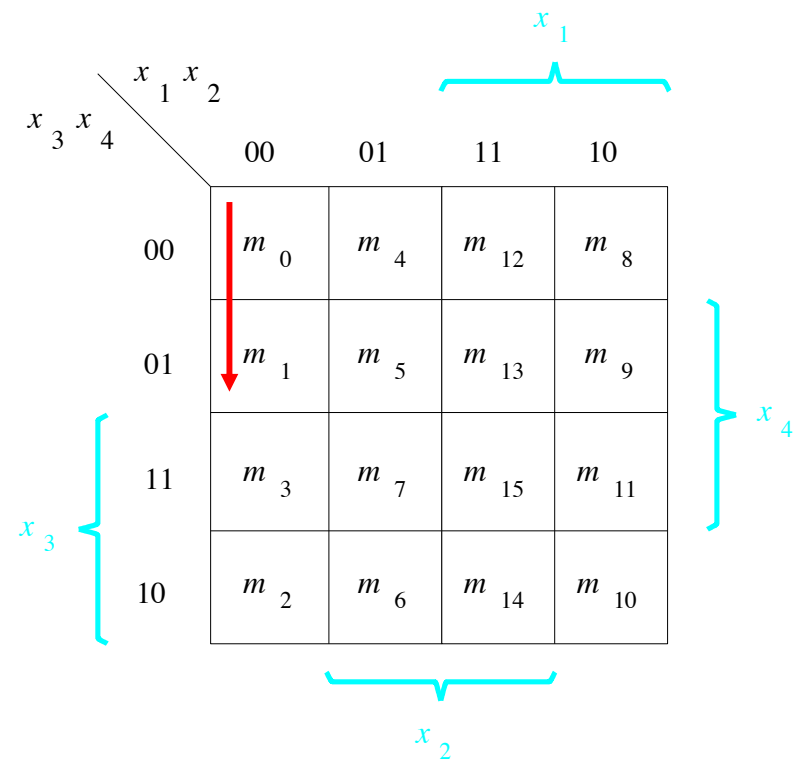
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



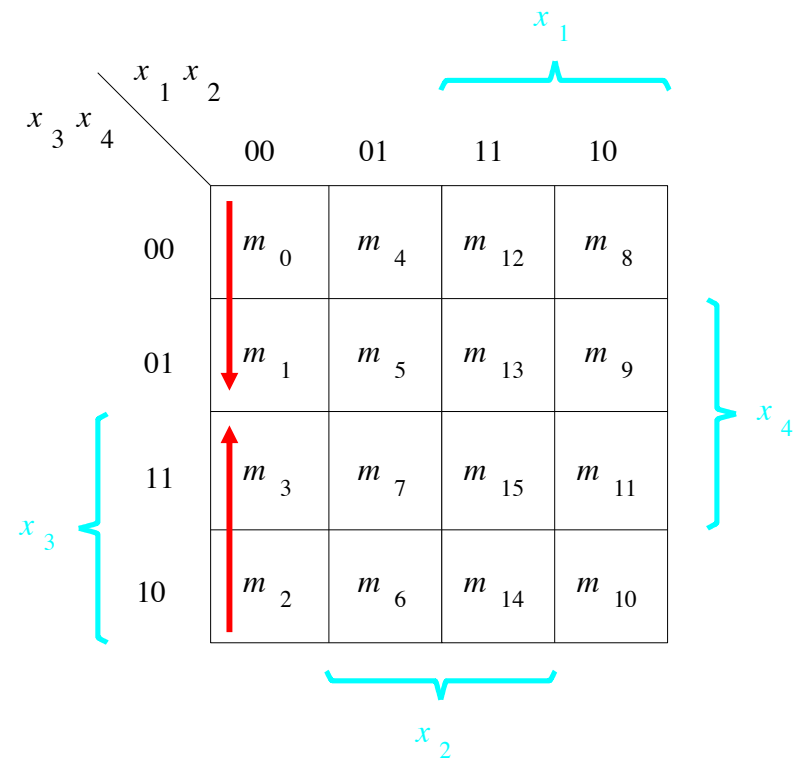
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



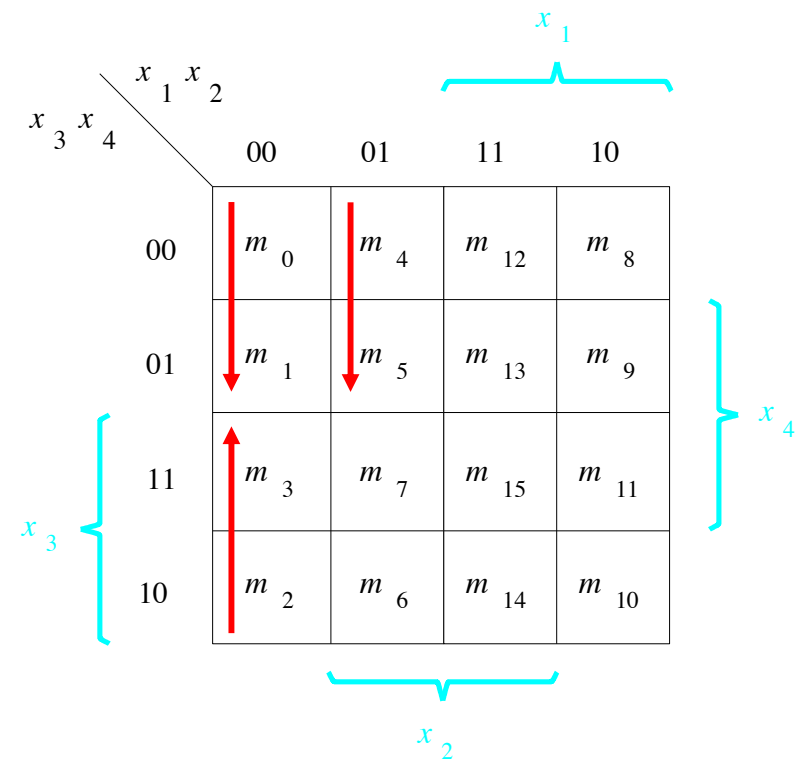
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



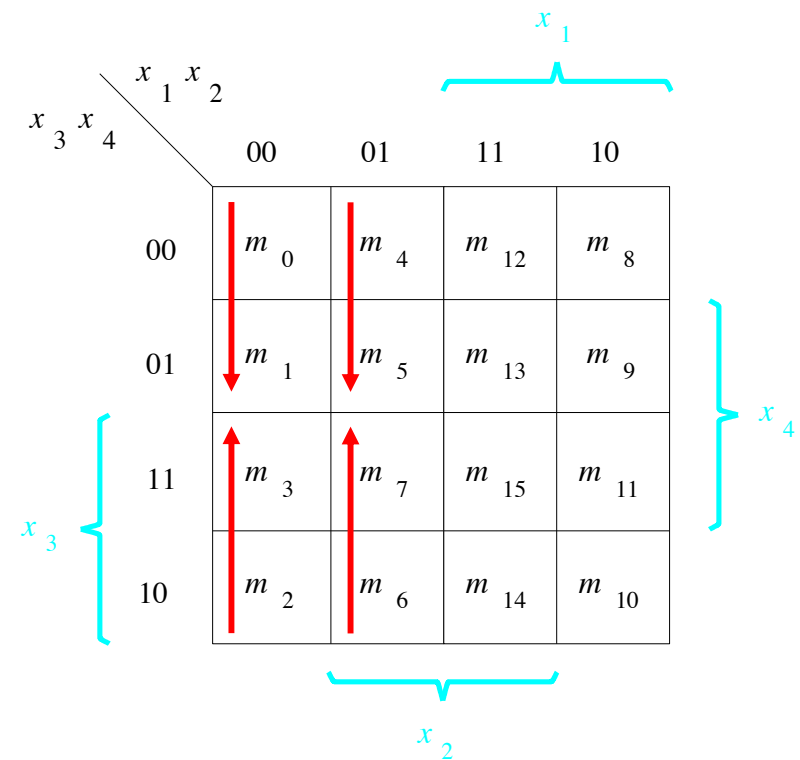
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



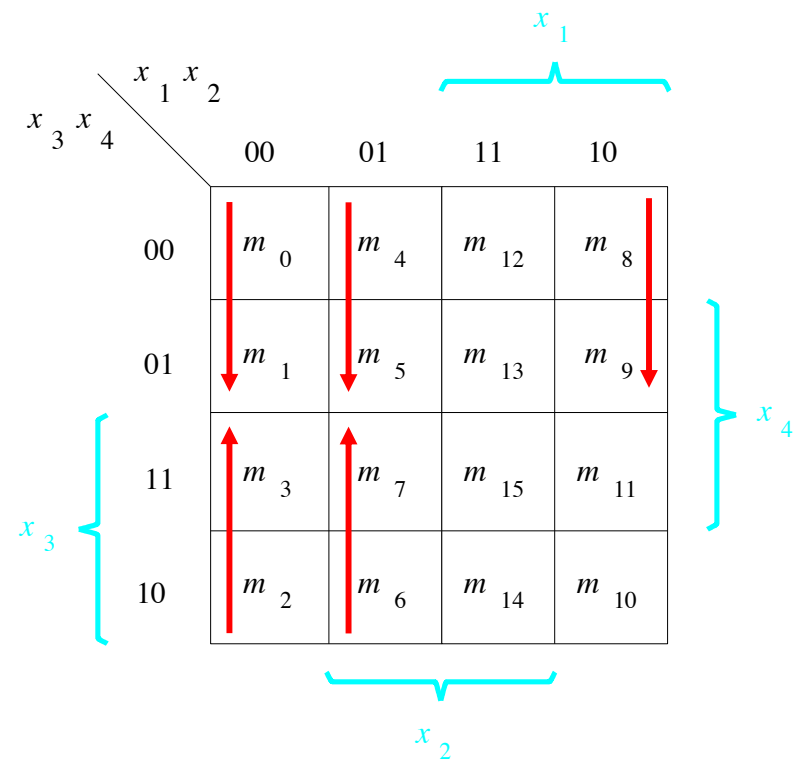
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



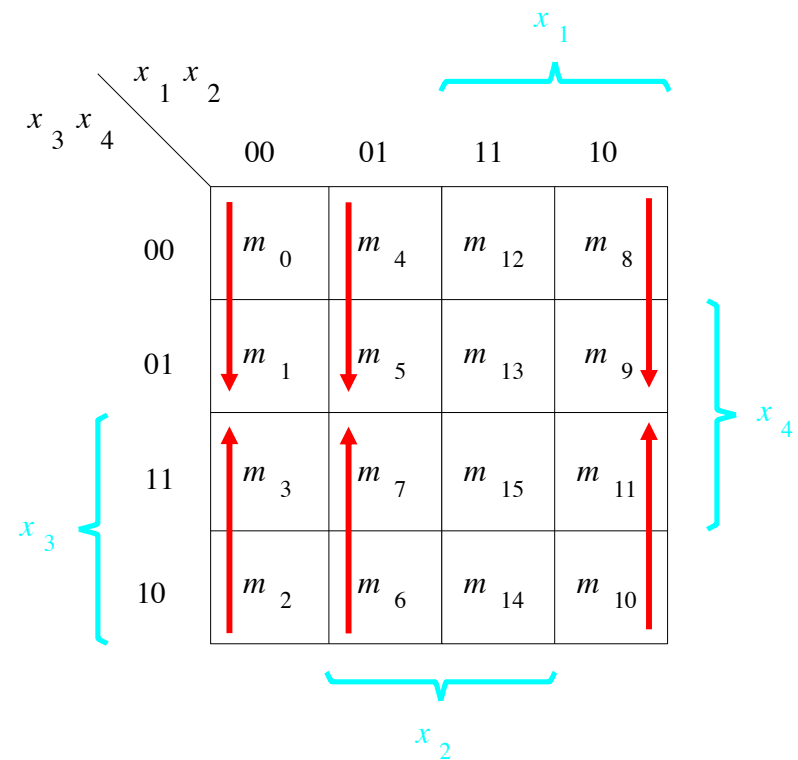
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



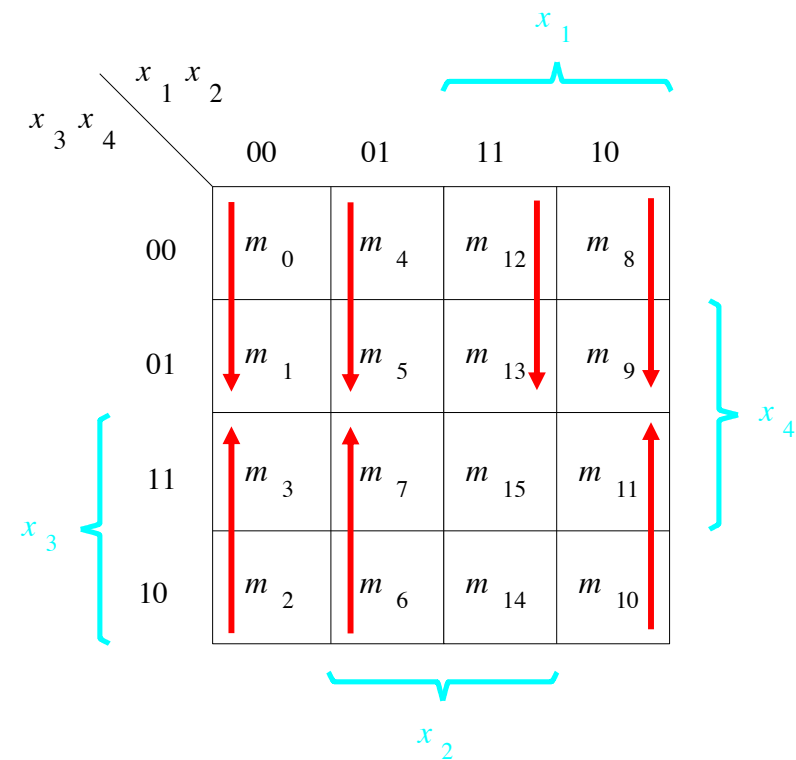
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



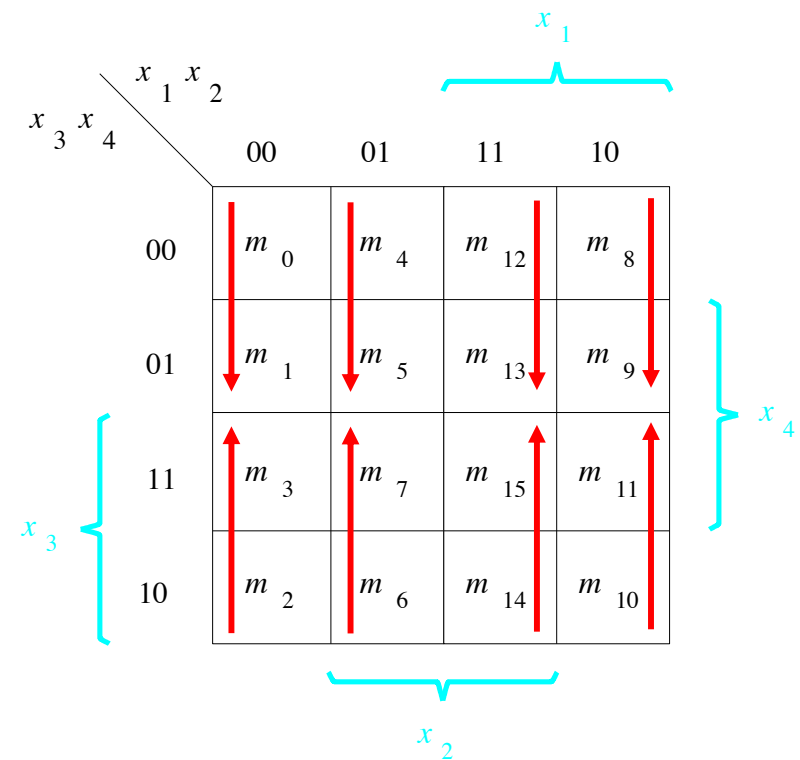
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules

$x_3 \backslash x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent
columns

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent
columns

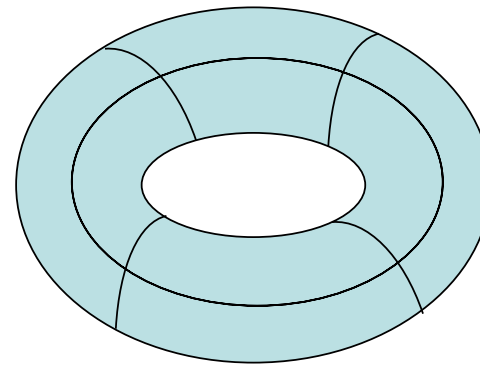
adjacent
rows

Adjacency Rules

x_1x_2					
x_3x_4		00	01	11	10
	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

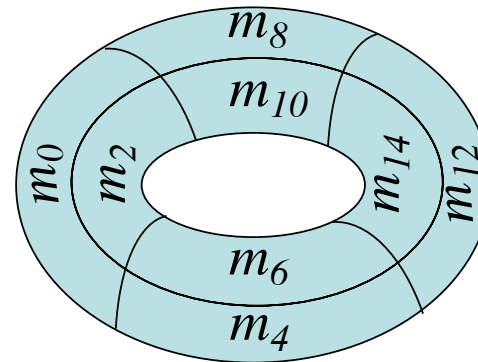
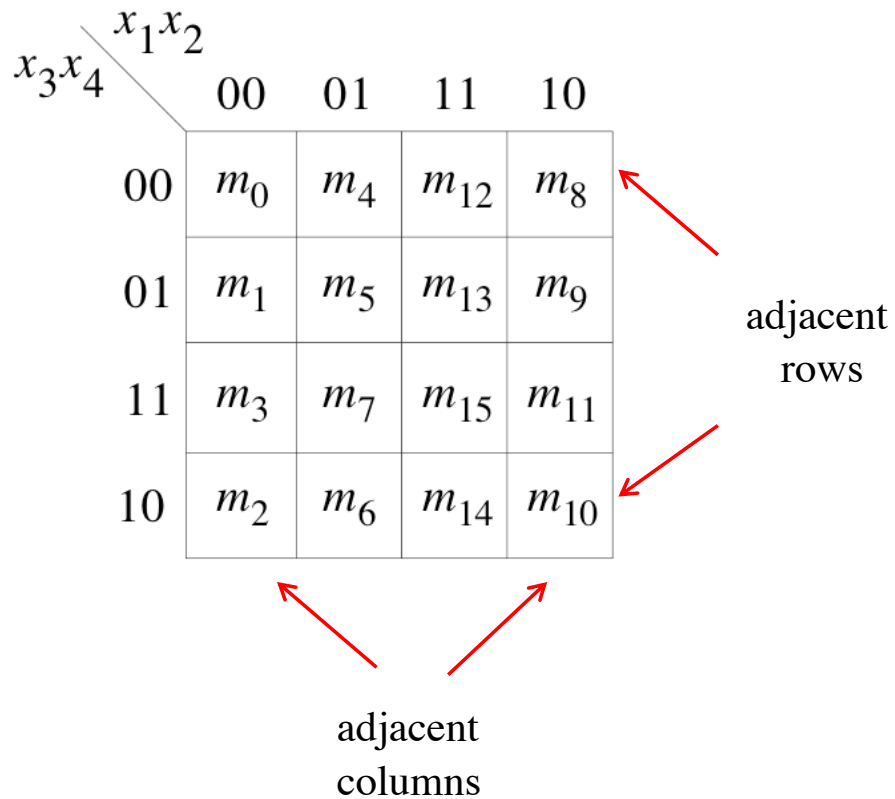
adjacent rows

adjacent columns



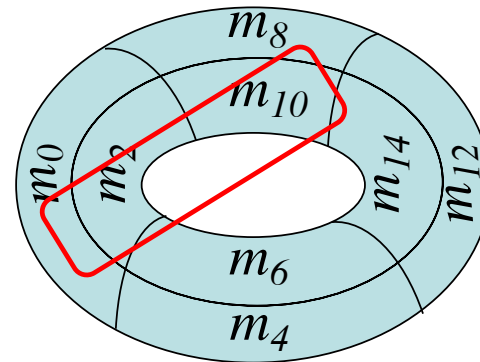
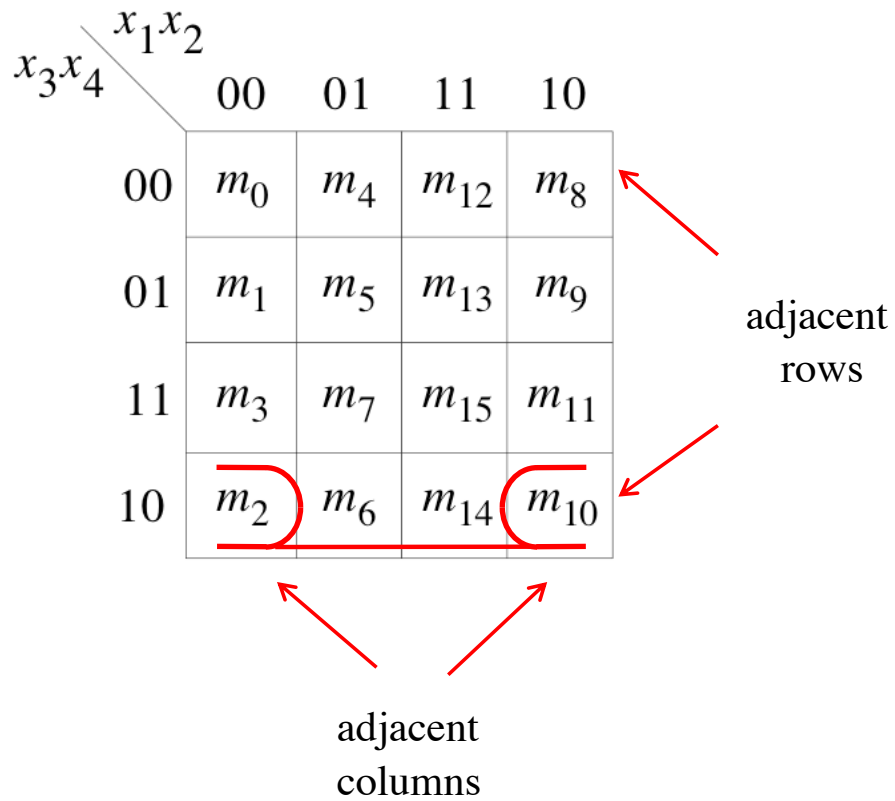
As if the K-map were drawn on a torus

Adjacency Rules



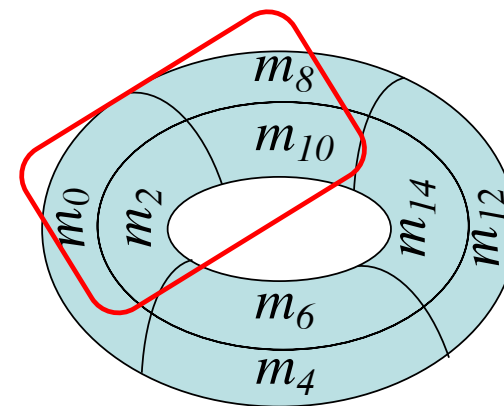
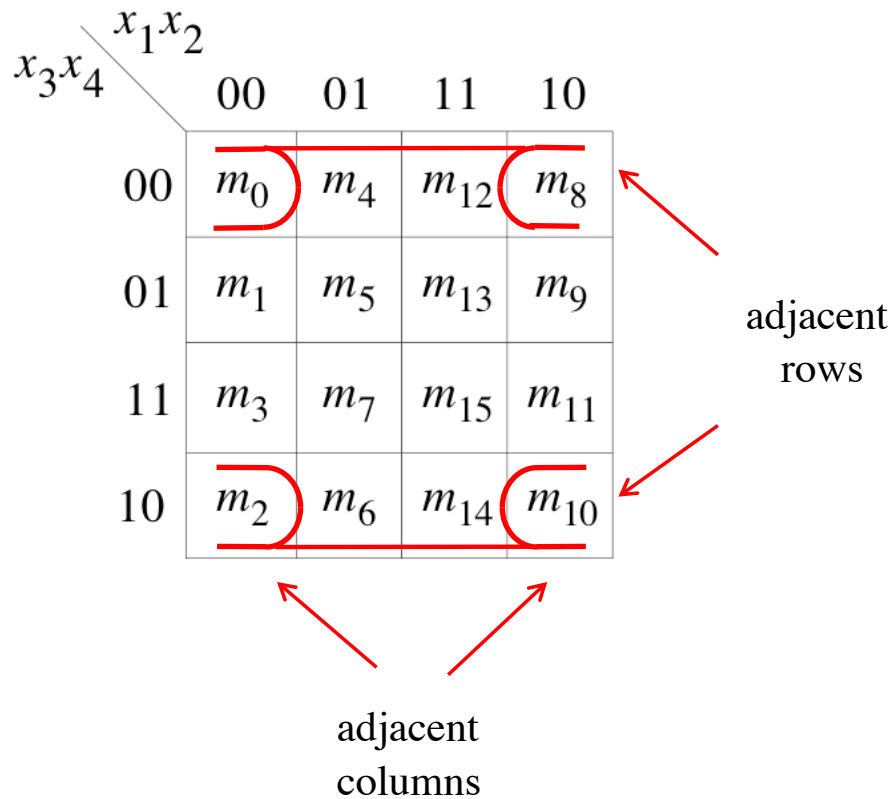
As if the K-map were drawn on a torus

Adjacency Rules



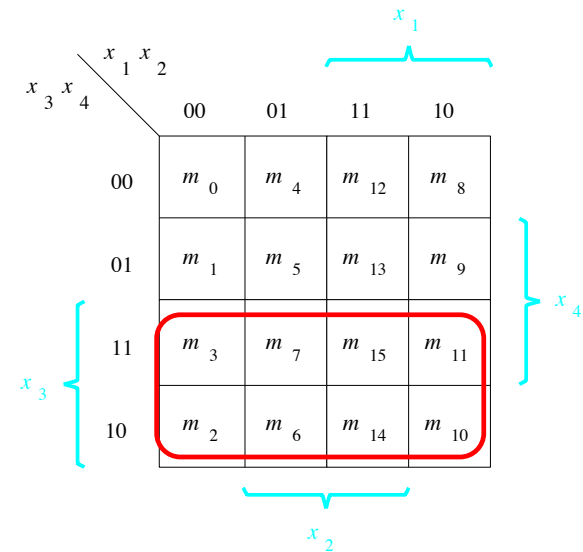
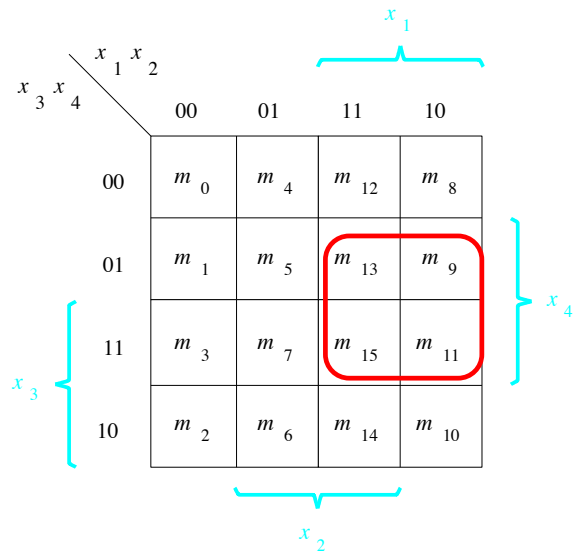
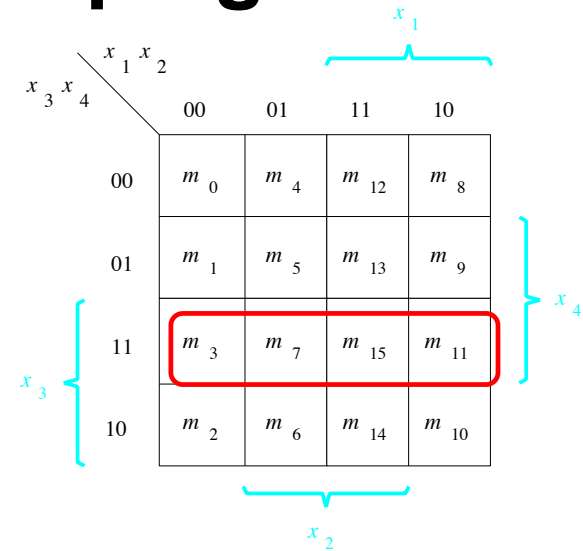
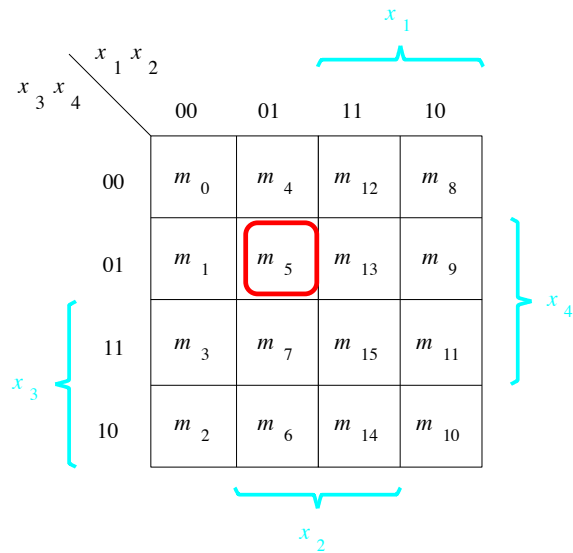
As if the K-map were drawn on a torus

Adjacency Rules



As if the K-map were drawn on a torus

Some Valid Groupings



Some Valid Groupings

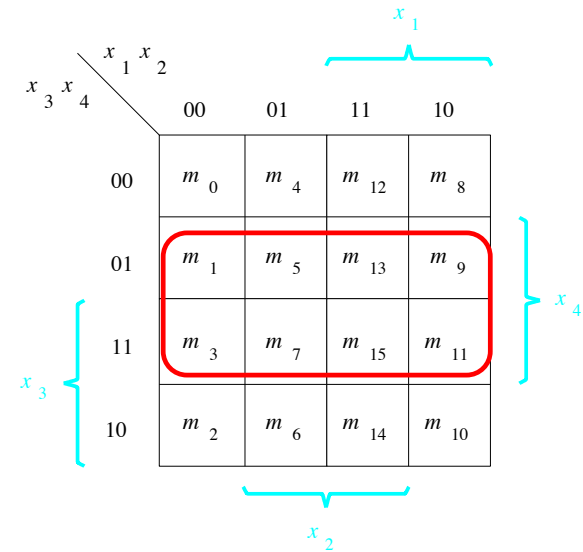
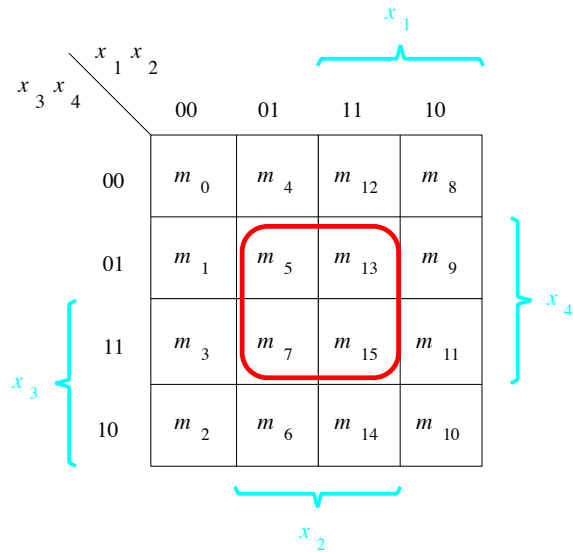
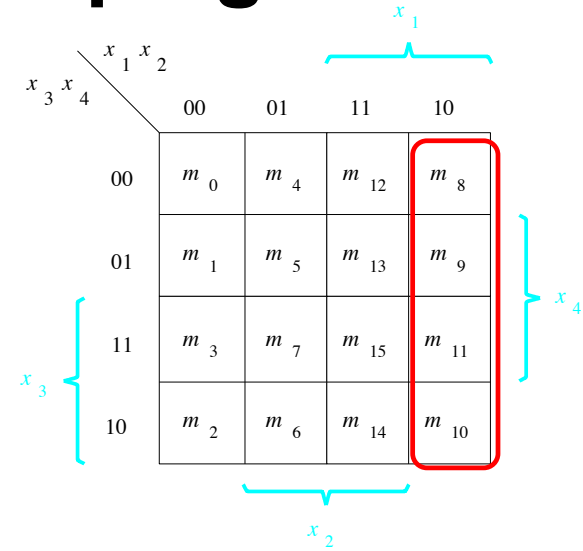
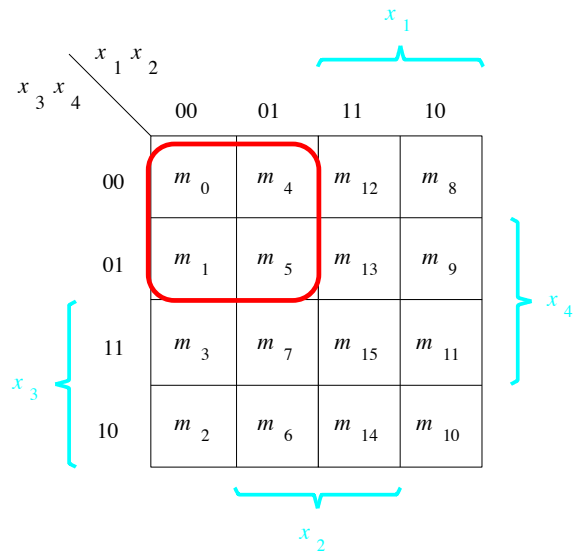
		$x_1 \ x_2$			
$x_3 \ x_4$		00	01	11	10
	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

		$x_1 \ x_2$			
$x_3 \ x_4$		00	01	11	10
	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

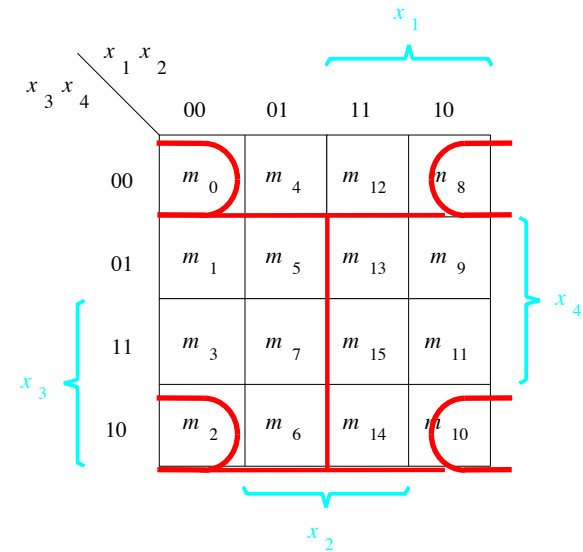
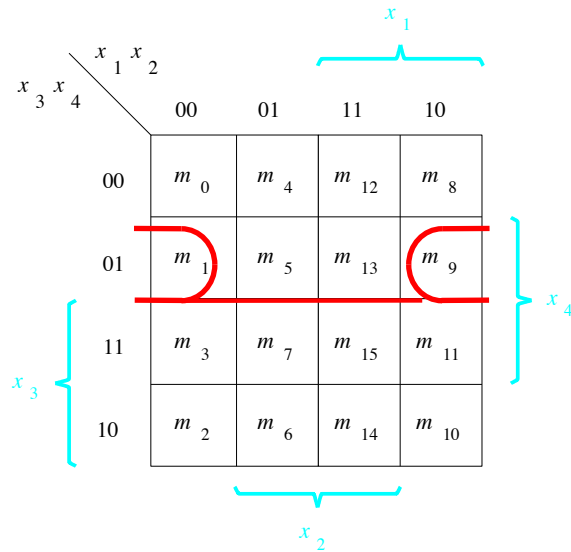
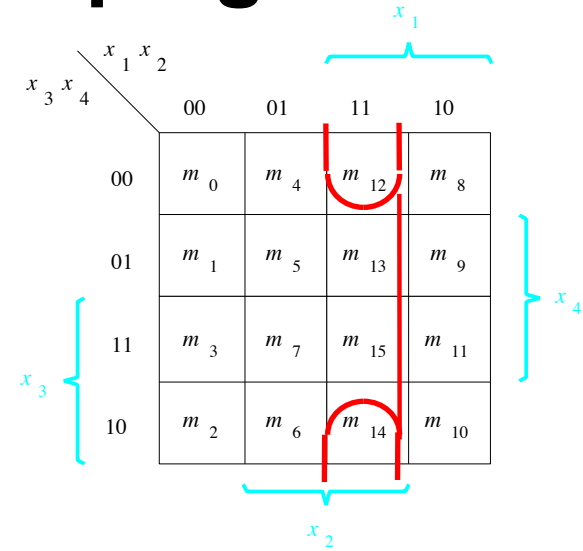
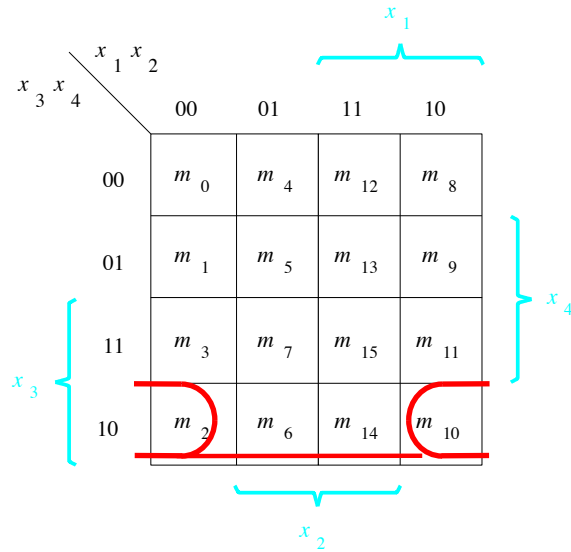
		$x_1 \ x_2$			
$x_3 \ x_4$		00	01	11	10
	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

		$x_1 \ x_2$			
$x_3 \ x_4$		00	01	11	10
	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

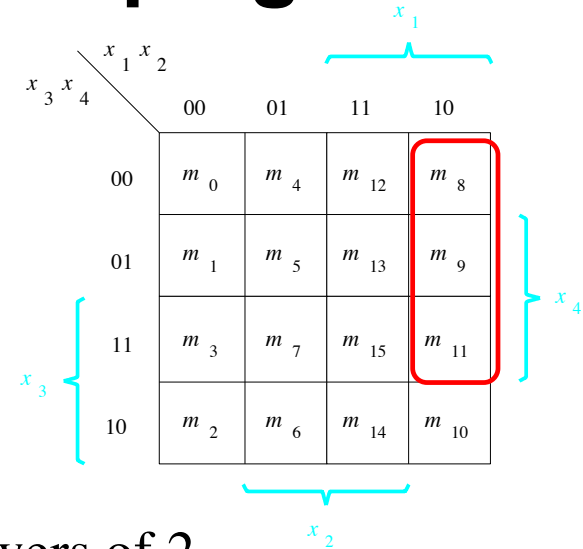
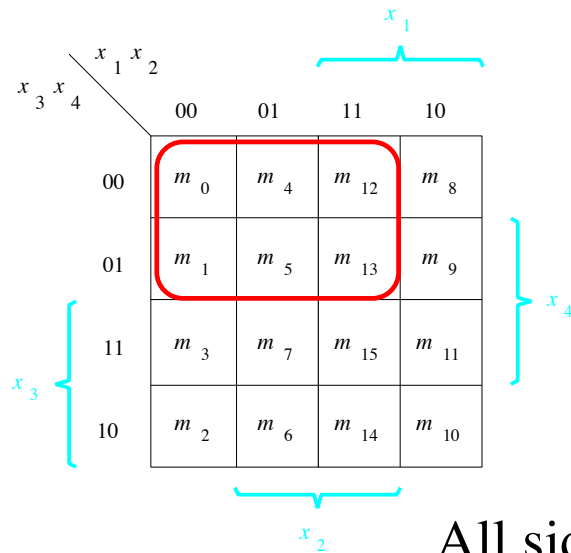
Some Valid Groupings



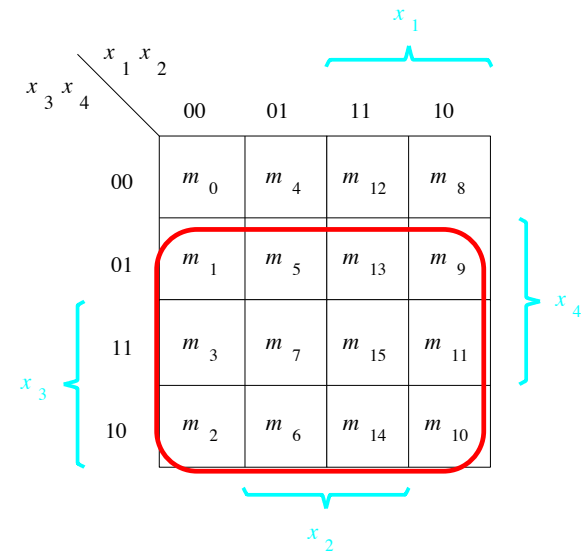
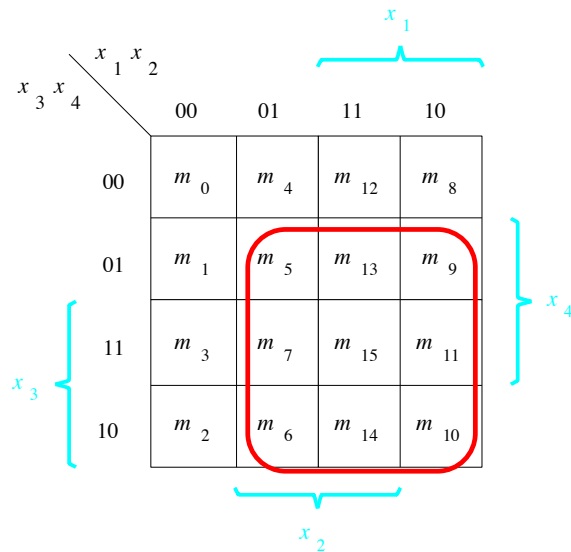
Some Valid Groupings



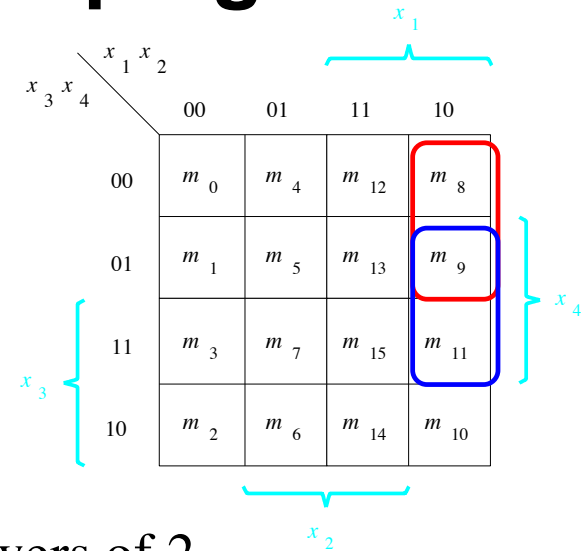
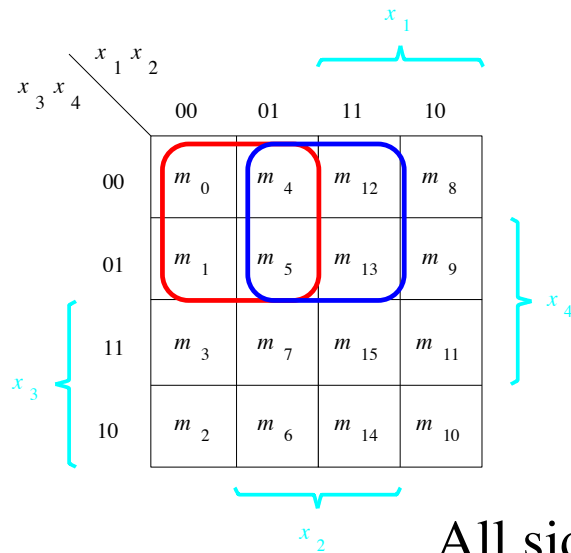
Some Invalid Groupings



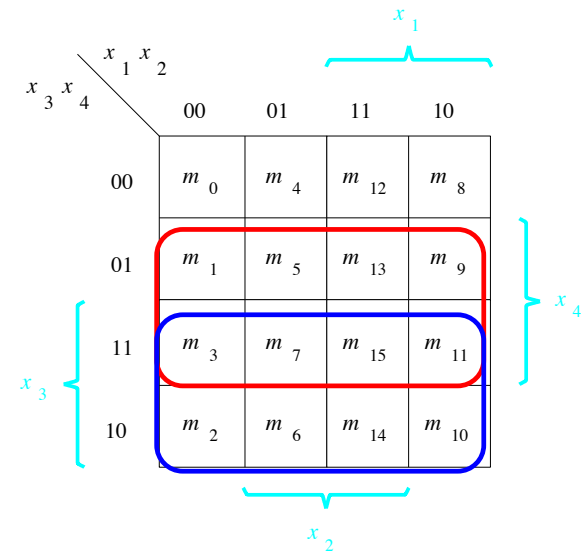
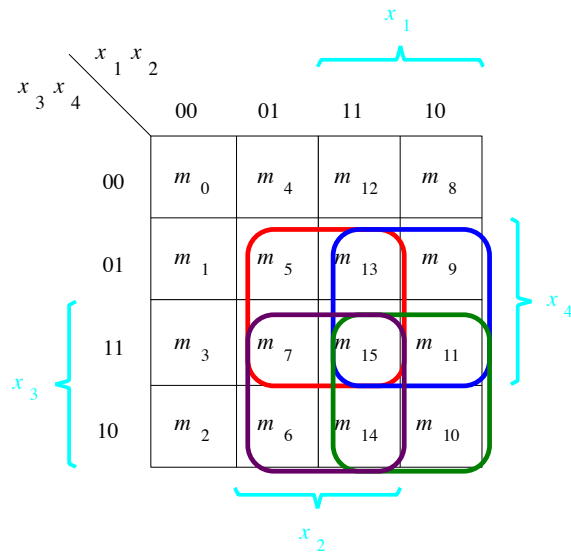
All sides must be powers of 2.



Some **valid** Groupings

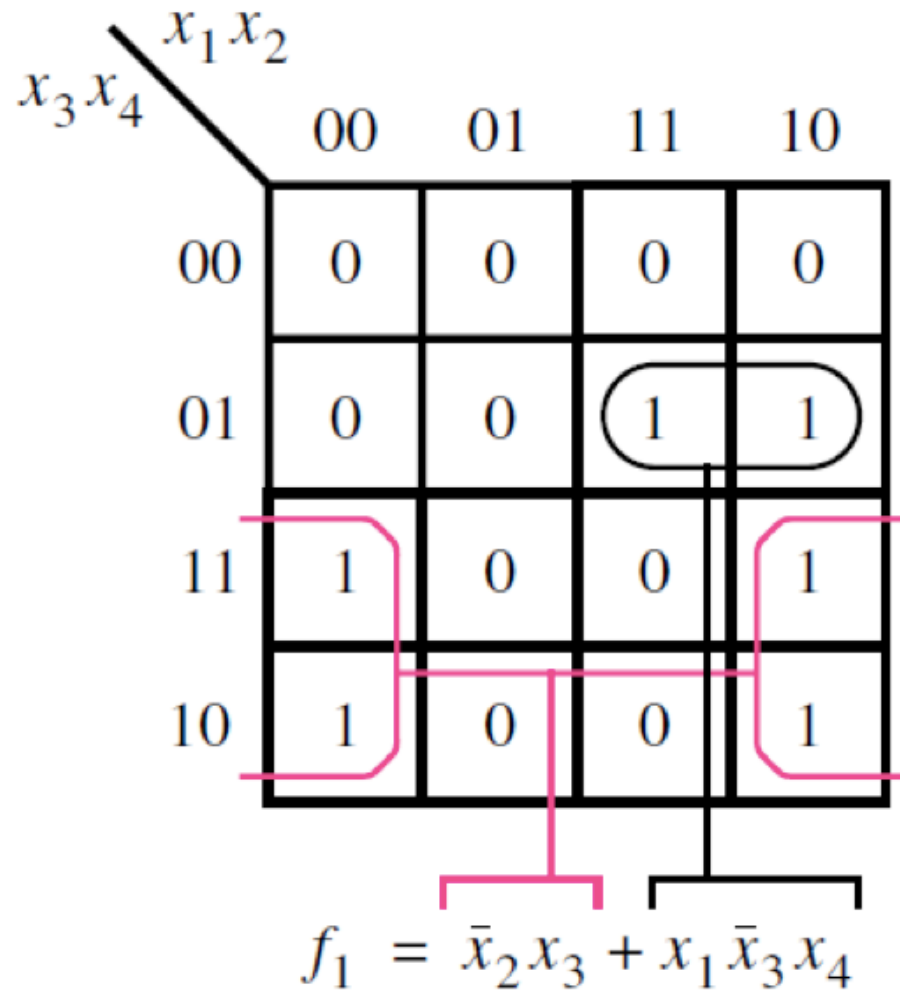


All sides must be powers of 2.



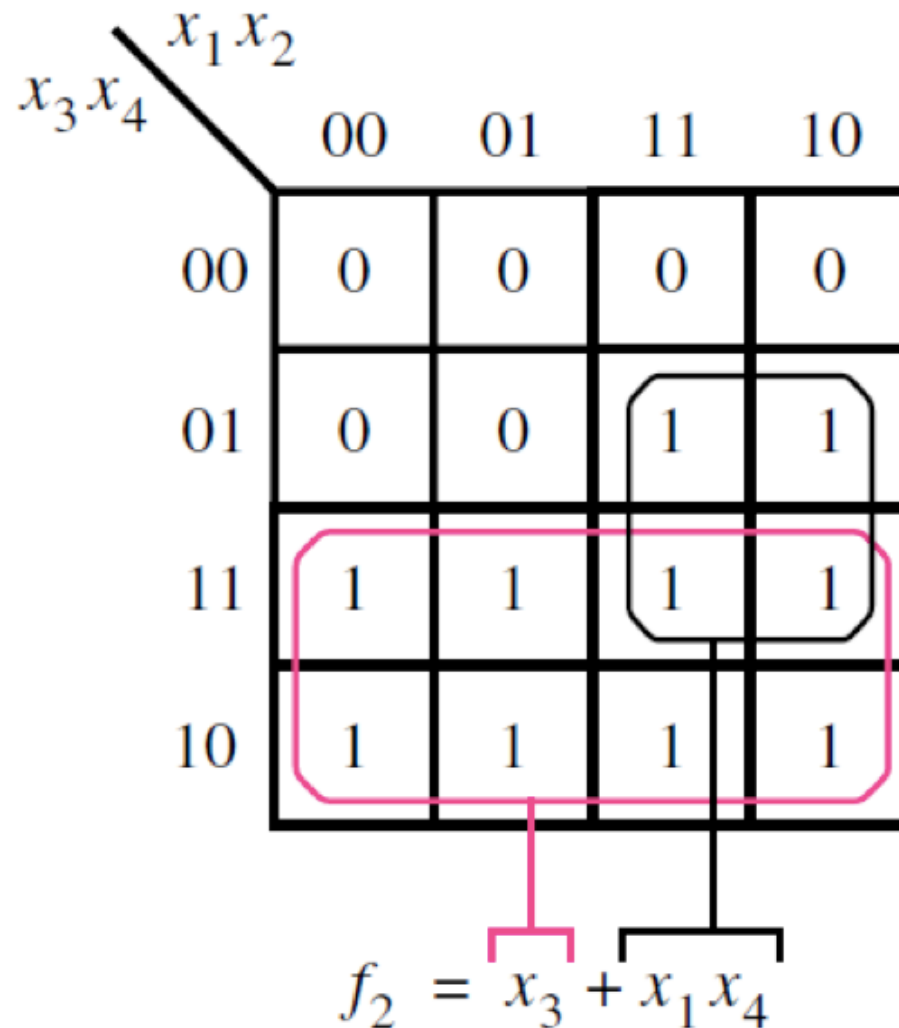
Minimization Examples with 4-variable K-Maps

Example of a four-variable Karnaugh map



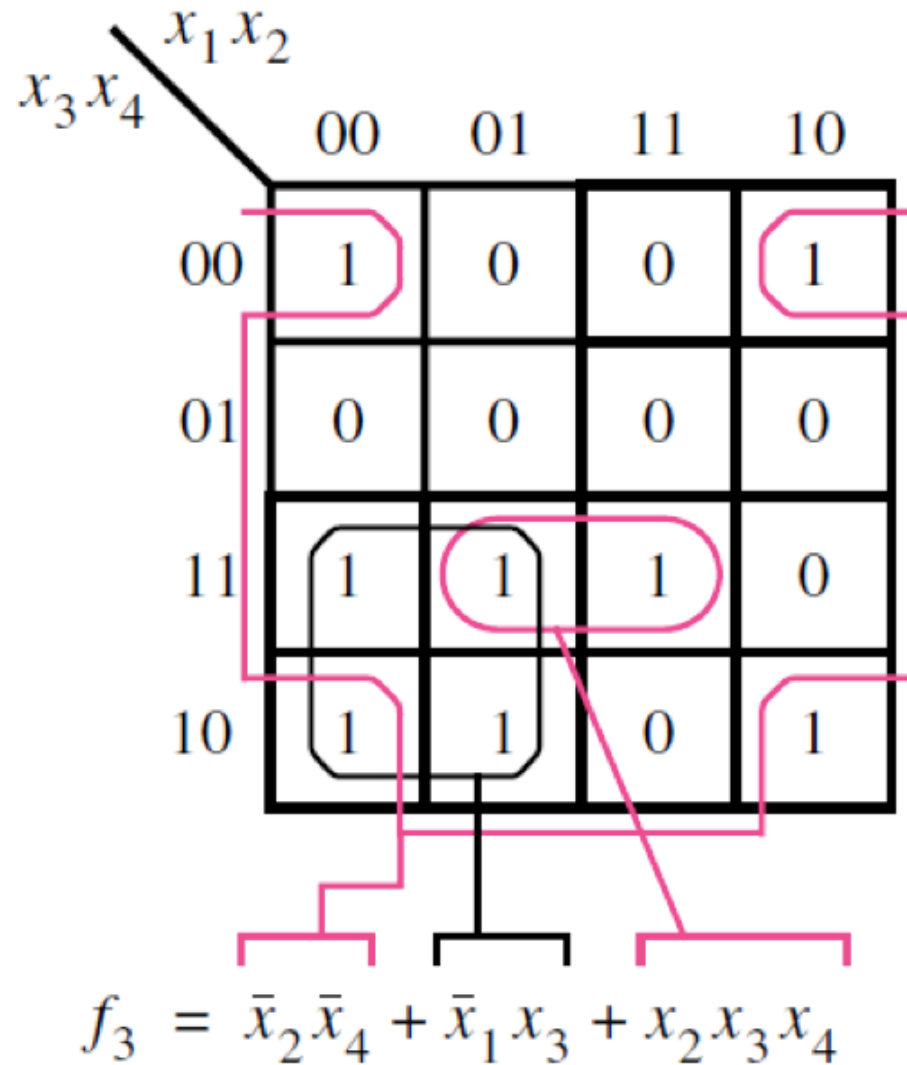
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



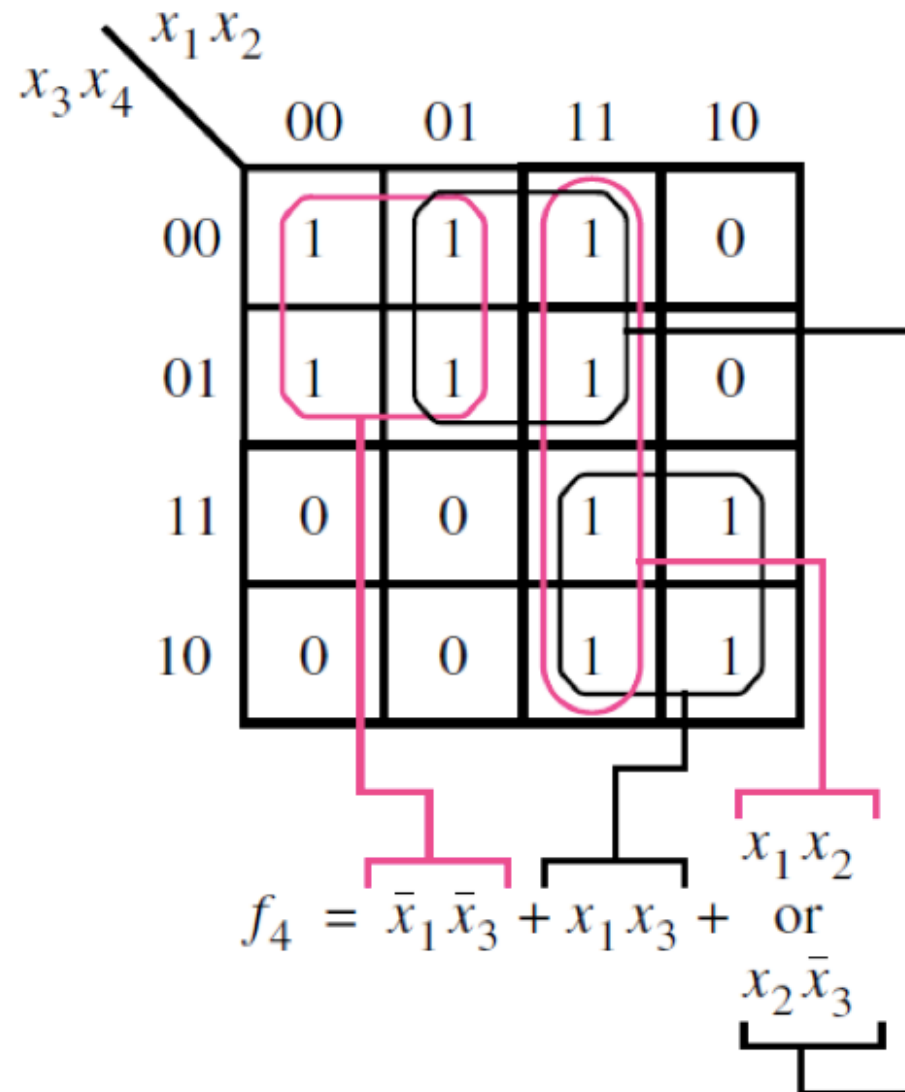
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

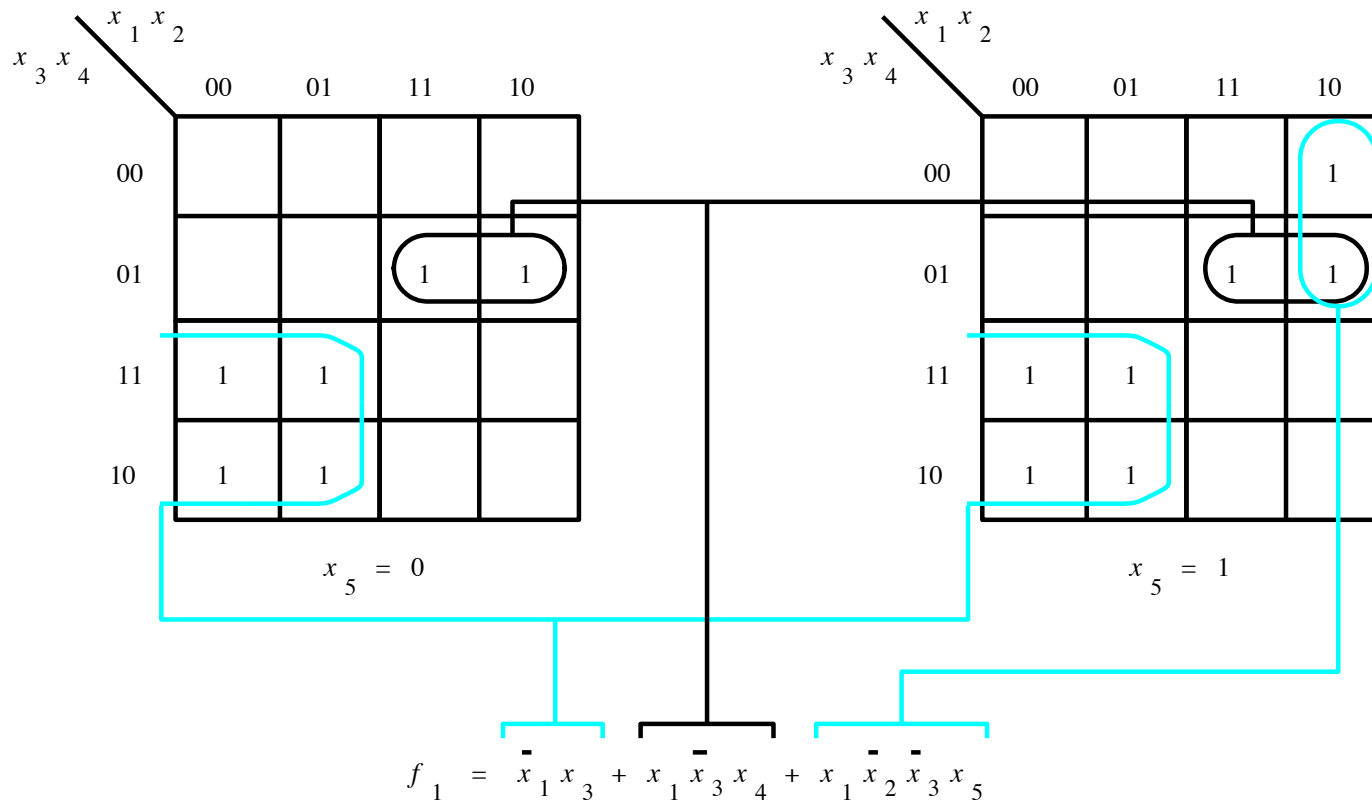
Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Five-Variable K-Map

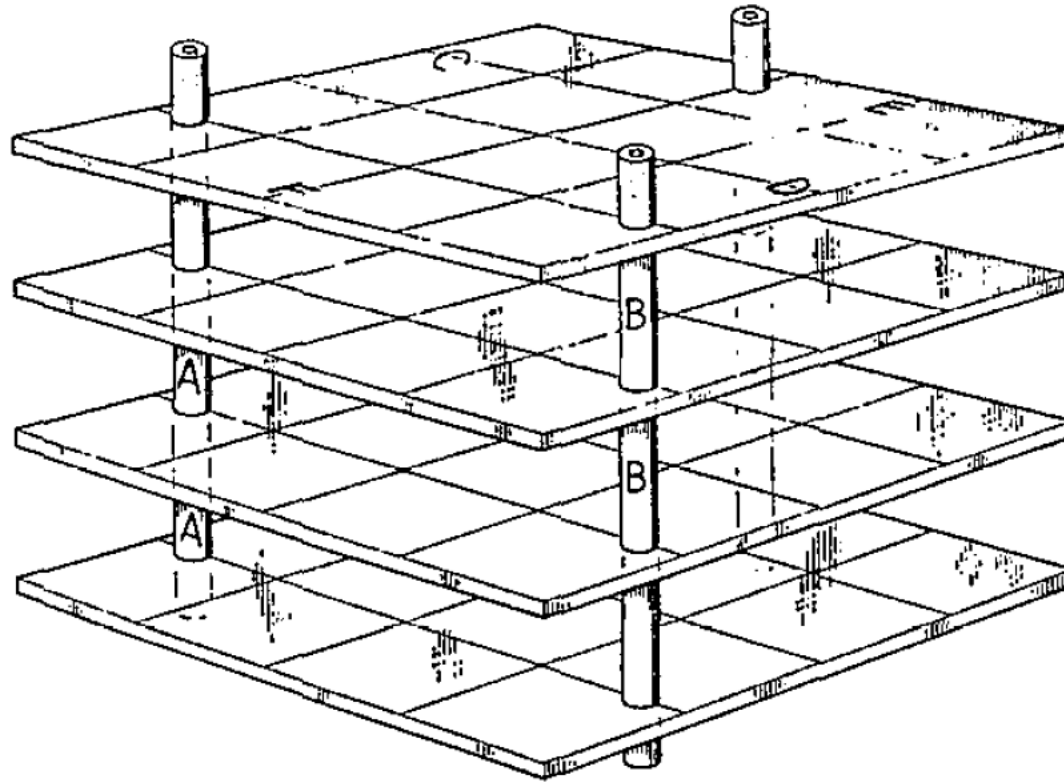
A five-variable Karnaugh map



[Figure 2.55 from the textbook]

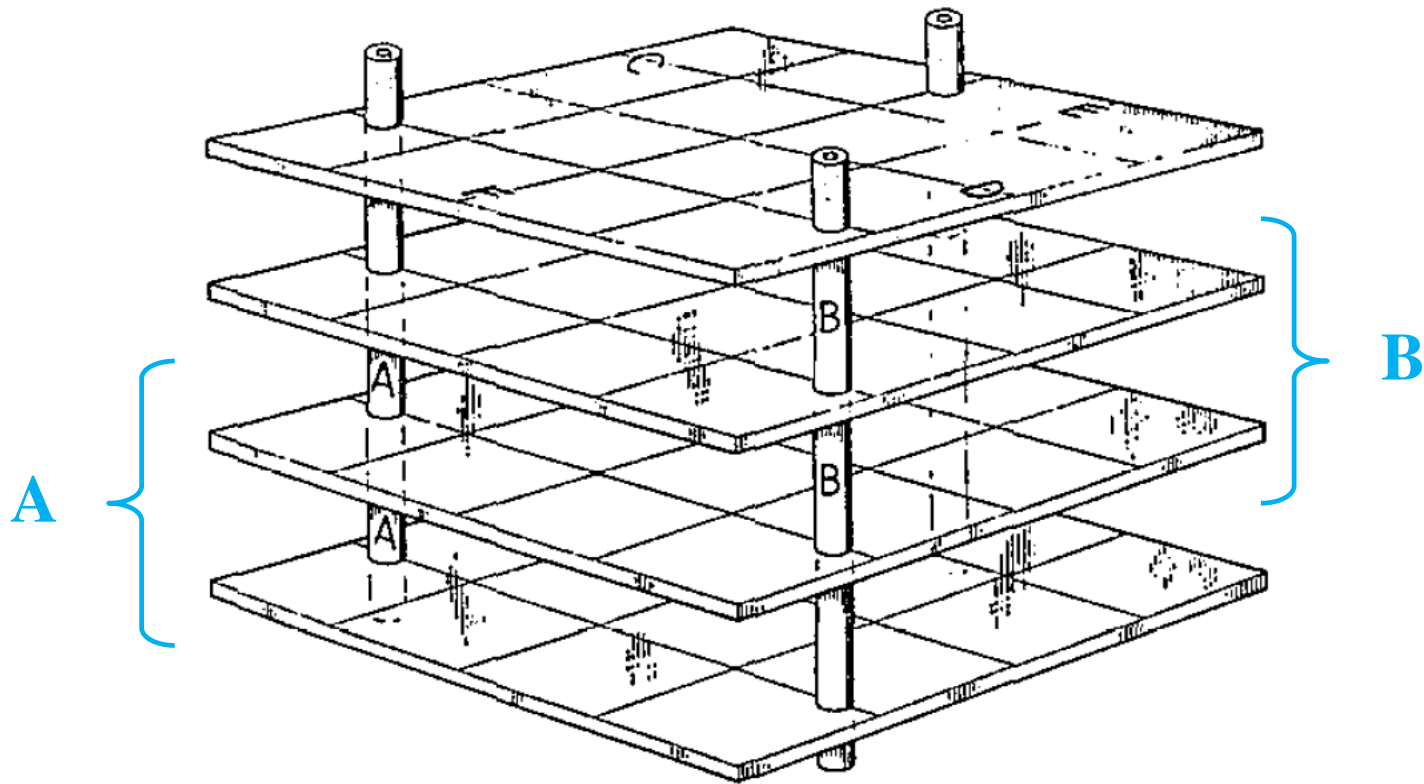
Six-Variable K-Map

A six-variable Karnaugh map



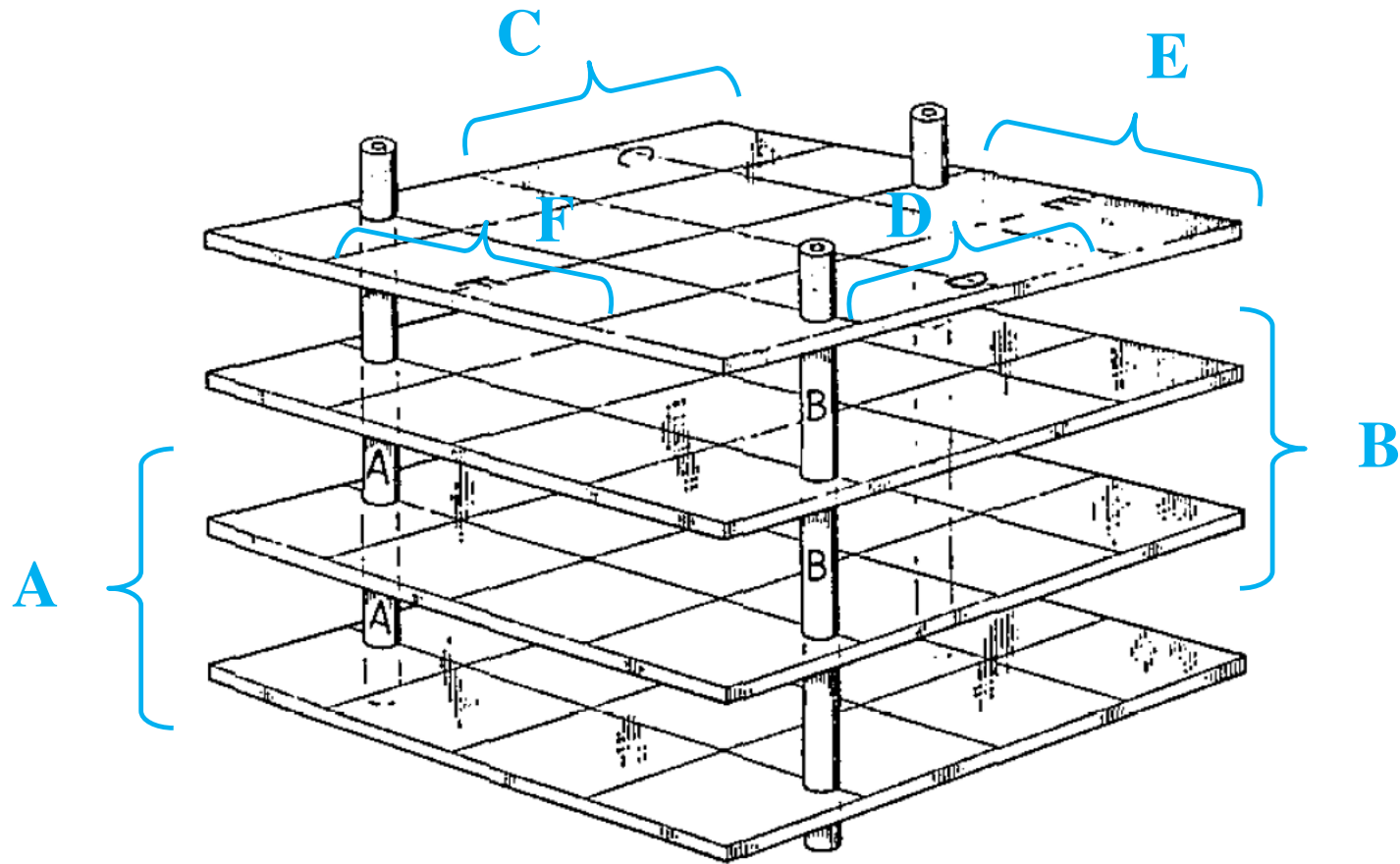
[Figure 16, in Karnaugh 1953]

A six-variable Karnaugh map



[Figure 16, in Karnaugh 1953]

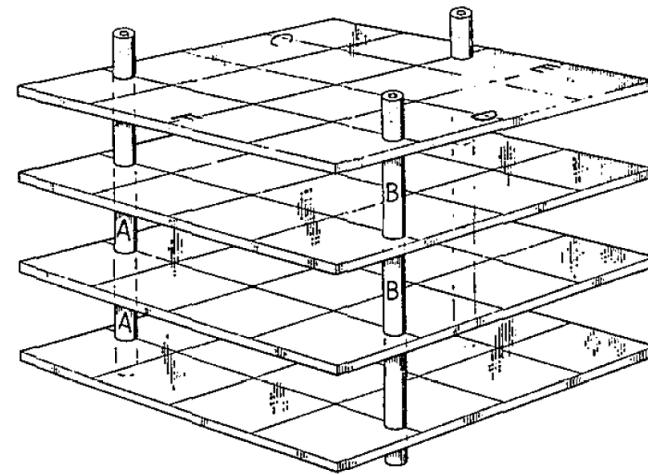
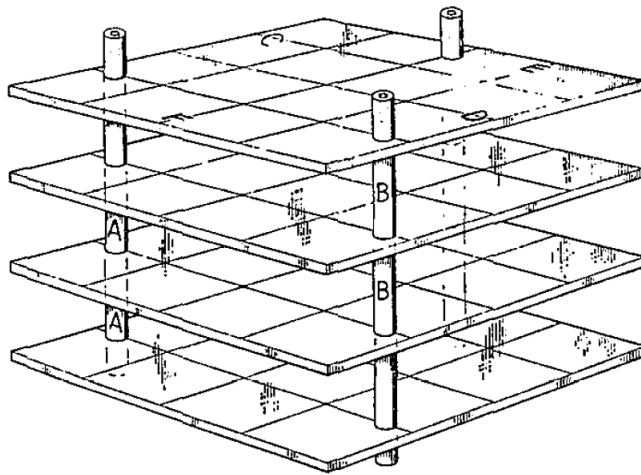
A six-variable Karnaugh map



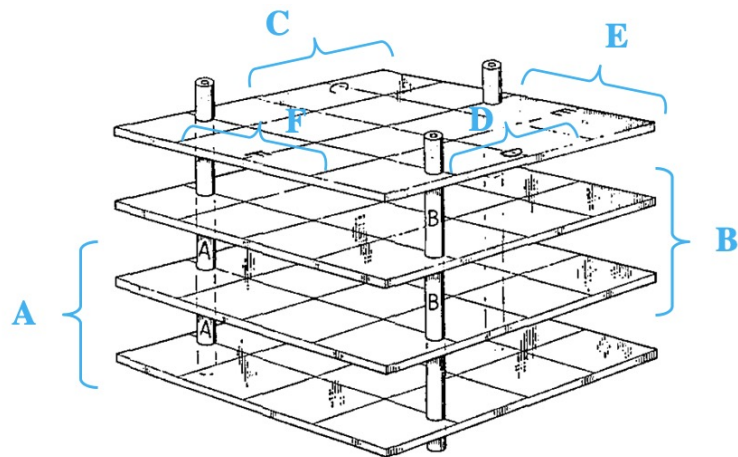
[Figure 16, in Karnaugh 1953]

Seven-Variable K-Map

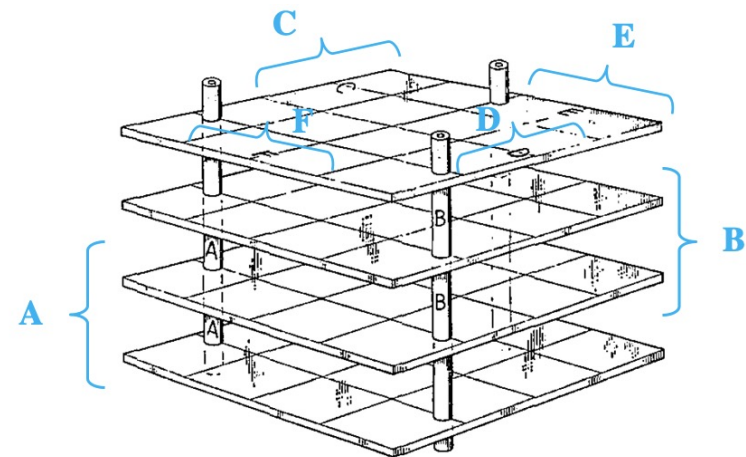
A seven-variable Karnaugh map



A seven-variable Karnaugh map



G = 0



G = 1

[Suggested in Karnaugh 1953]

Questions?

THE END