P1. (10 points) Design the simplest SOP circuit that implements the following function (show all steps): $\quad \mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=\sum m(1,3,4,6,7)$.
Then draw the circuit diagram using only NAND gates.
P2. (10 points) Realize each of the following Boolean functions using:

1) a minimum number of NAND only,
2) a minimum number of NOR gates only.
A) $\left(A^{\prime} B\right)+\left(A B^{\prime}\right)$
B) $(\mathrm{AB})+\left(\mathrm{A}^{\prime} \mathrm{B}^{\prime}\right)$

P3. (20 points) A full adder is a circuit that adds three bits $X, Y, Z$ together and returns two bits $C$ and $S$ to represent the total as a 2-bit binary number $C$. $C$ is the MSB and $S$ is the LSB. For example, if $X=1, Y=1, Z=0$, the total should be 2 , or $10_{2}$ in binary. Hence $C=1$ and $S=0$.
A) Write the truth tables for the functions $C$ and $S$.
B) Write the functions C and S in short hand notation using (i) minterms and (ii) maxterms.
C) Write the functions C and S in canonical sum-of-products (SOP) and canonical product-of-sums (POS) forms.
D) Obtain the simplest SOP expressions for the functions C and S and draw their respective circuit diagrams.

P4. (20 points) A logic circuit has four inputs (A, B, C, D) and one output (F).
The output is a logic $\mathbf{0}$ if and only if three or four of the inputs are 0 ; the output is a logic 1 otherwise.
A) Find and draw the truth table.
B) Write the maxterms expression for $F(A, B, C, D)$.
C) Using AND and OR gates, design a minimum logic circuit to realize F.
D) What is the cost of this circuit in terms of the number of gates and inputs?

P5. (15 points) A logic circuit has three inputs $P, Q, R$ and one output $S$. $S$ is high (Logic 1 ) whenever $\mathrm{P}=0$ or whenever $\mathrm{Q}=\mathrm{R}=1$.
A) Derive the truth table for the above.
B) Use Boolean algebra to derive the simplified expression from the canonical SOP form.
C) Derive the logic circuit for the simplified Boolean expression.

P6. (10 points) Consider the truth table below, which has threee inputs $(X, Y, Z)$ and two outputs ( $\mathrm{M}, \mathrm{N}$ ):

| X | Y | Z | M | N |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

A) Write the functions M and N in short-hand notation using minterms and maxterms (4 points).
B) Write the functions M and N in canonical SOP and canonical POS forms then obtain the simplest SOP expressions for both M and N ( 6 points).

P7. (10 points) Show how to implement a NOT function using: (a) 2-input NAND gates only, (b) 2-input NOR gates only. For part (a) and part (b), you should use a different way from what has been shown in class (connecting both terminals to the input signal). Hint: you are allowed to connect constant voltages (i.e., logic values 0 or 1) to the inputs of the logic gates.

P8. (5 points) Rewirte the following function in SOP form:

$$
F(a, b, c, d, e)=\left(a+c^{\prime}\right)(a+d)\left(a b^{\prime} c+e\right)
$$

