Title: FUNDAMENTAL LIMITATIONS ON COMMUNICATION CHANNELS WITH NOISY FEEDBACK: INFORMATION FLOW, CAPACITY AND BOUNDS

Abstract:

Since the success of obtaining the capacity (i.e. the maximal achievable transmission rate under which the message can be recovered with arbitrarily small probability of error) for non-feedback point-to-point communication channels by C. Shannon (in 1948), Information Theory has been proved to be a powerful tool to derive fundamental limitations in communication systems. During the last decade, motivated by the emerging of networked systems, information theorists have turned lots of their attention to communication channels with feedback (through another channel from receiver to transmitter). Under the assumption that the feedback channel is noiseless, a large body of notable results has been derived, although much work still needs to be done. However, when this ideal assumption is removed, i.e., the feedback channel is noisy, only few valuable results can be found in the literature and many challenging problems are still open.

This thesis aims to address some of these long-standing noisy feedback problems, with concentration on the channel capacity. First of all, we analyze the fundamental information flow in noisy feedback channels. We introduce a new notion, the residual directed information, to characterize the noisy feedback channel capacity where the standard directed information fails. As an illustration, finite-alphabet noisy feedback channels have been studied in details. Next, we provide an information flow decomposition equality which serves as a foundation of other novel results in this thesis.

With the results of information flow in hand, we next investigate time-varying Gaussian channels with additive Gaussian noise feedback. Following the notable Cover-Pombra results in 1989, we define the n-block noisy feedback capacity and derive a pair of n-block upper and lower bounds on the n-block noisy feedback capacity. These bounds can be obtained by efficiently solving convex optimization problems. Under the assumption of stationarity on the additive Gaussian noises, we show that the limits of these n-block bounds can be characterized in a power spectral optimization form. In addition, two computable lower bounds are derived for the Shannon capacity.

Next, we consider a class of channels where feedback could not increase the capacity and thus the noisy feedback capacity equals to the non-feedback capacity. We derive a necessary condition (characterized by the directed information) for the capacity-achieving channel codes.

Finally, we introduce a new framework of communication channels with noisy feedback where the feedback information received by the transmitter is also available to the decoder with some finite delays. We investigate the capacity and linear coding schemes for this extended noisy feedback channels.

To summarize, this thesis firstly provides a foundation (i.e. information flow analysis) for analyzing communications channels with noisy feedback. In light of this analysis, we next present a sequence of novel results, e.g. channel coding theorem, capacity bounds, etc., which result in a significant step forward to address the long-standing noisy feedback problem.