Randomized Smoothing Networks

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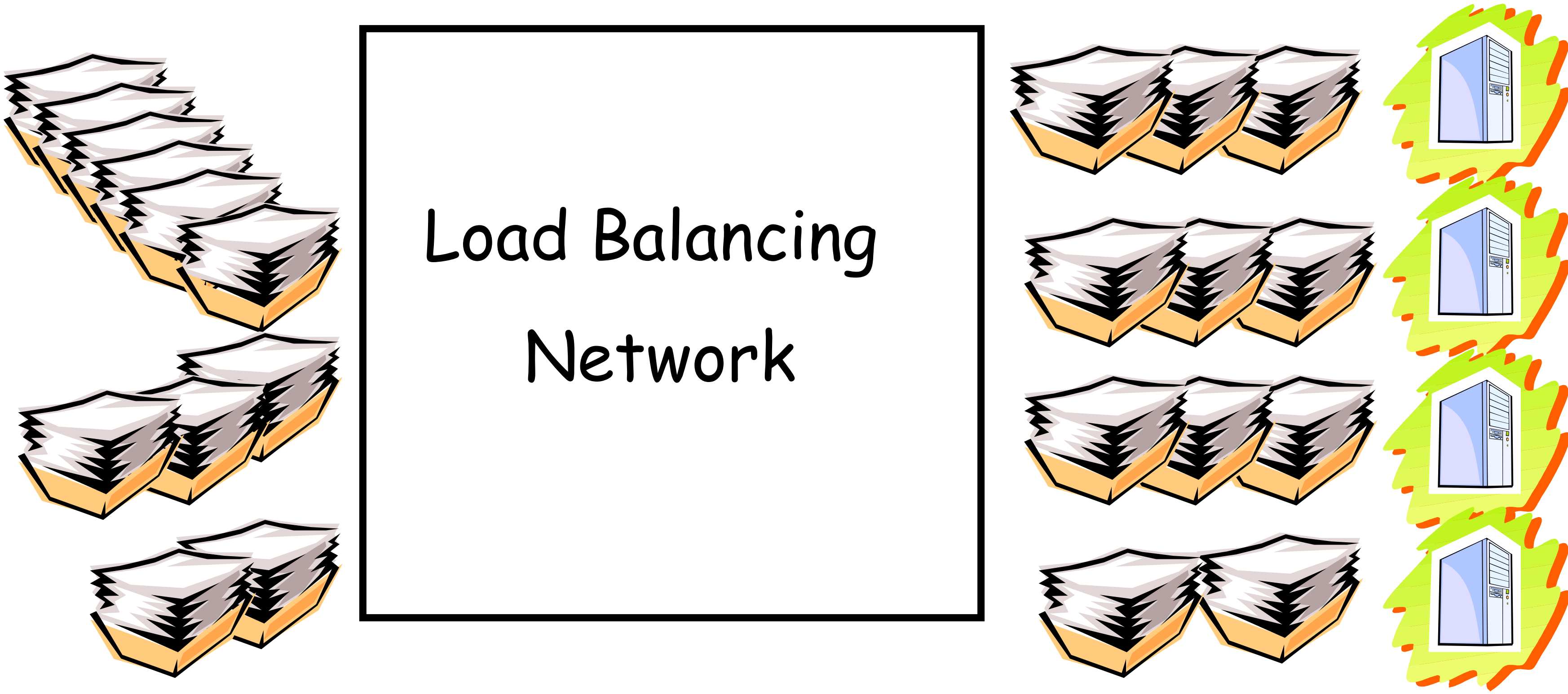
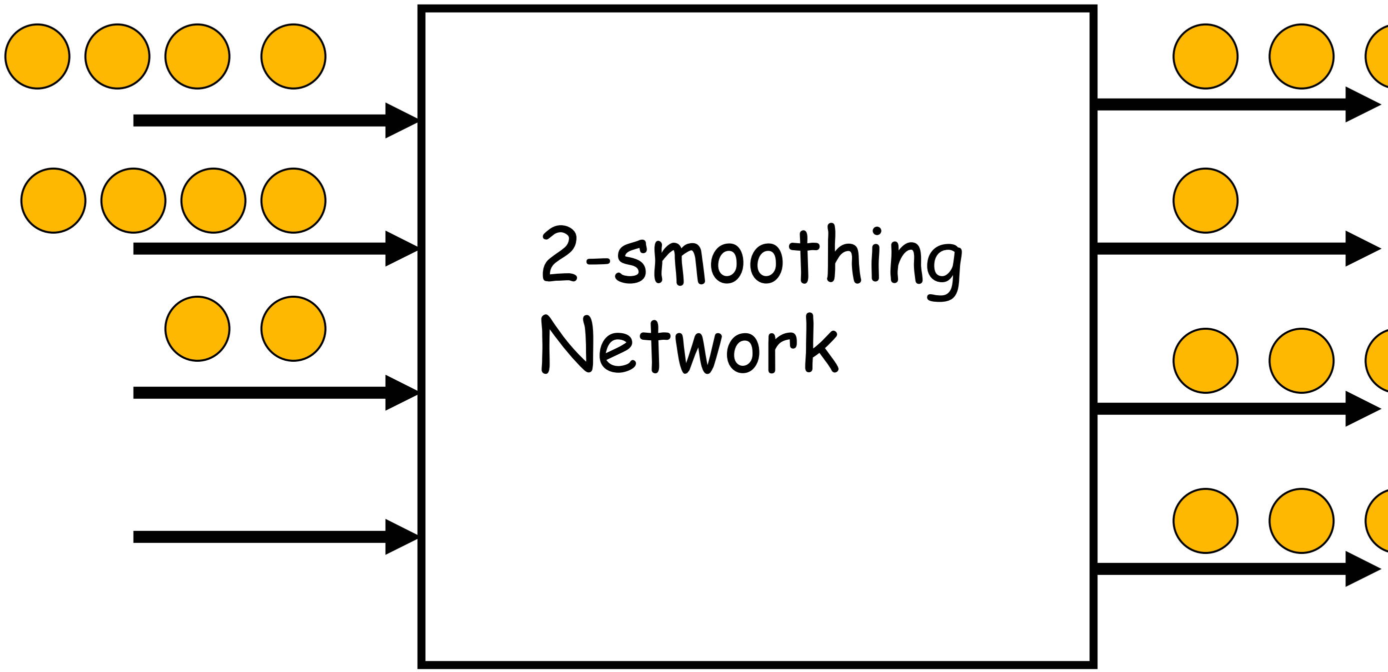
Distributed Load Balancing



Example: Routing Tasks to Processors on a Grid

Smoothing Networks



In a k-smoothing Network, the numbers of Tokens on different output wires differ by at most k

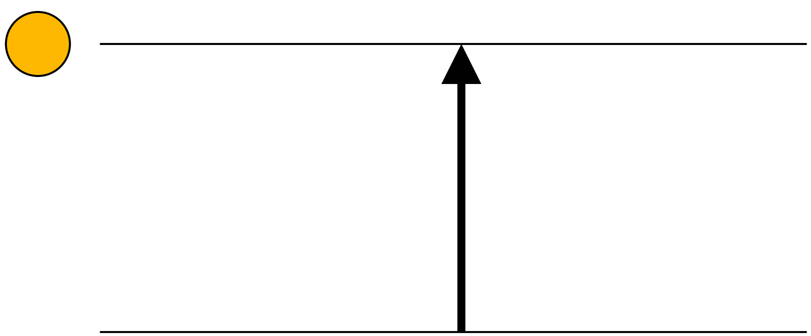
How to Build Smoothing Data Structures?

- Centralized Solution
- Random routing
  - Distributed and local
  - The smoothness can diverge without bound as more tokens enter the network
- $\boxed{\text{Balancing Networks}}$

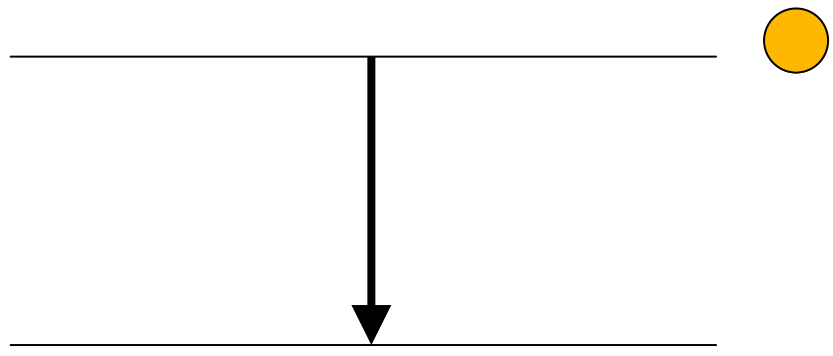
Overview of Talk

- Describe Smoothing Networks built out of balancers
- Our Result: Randomized Smoothing Networks better than Deterministic Ones
- Analysis of the Randomized Butterfly (or Block) network

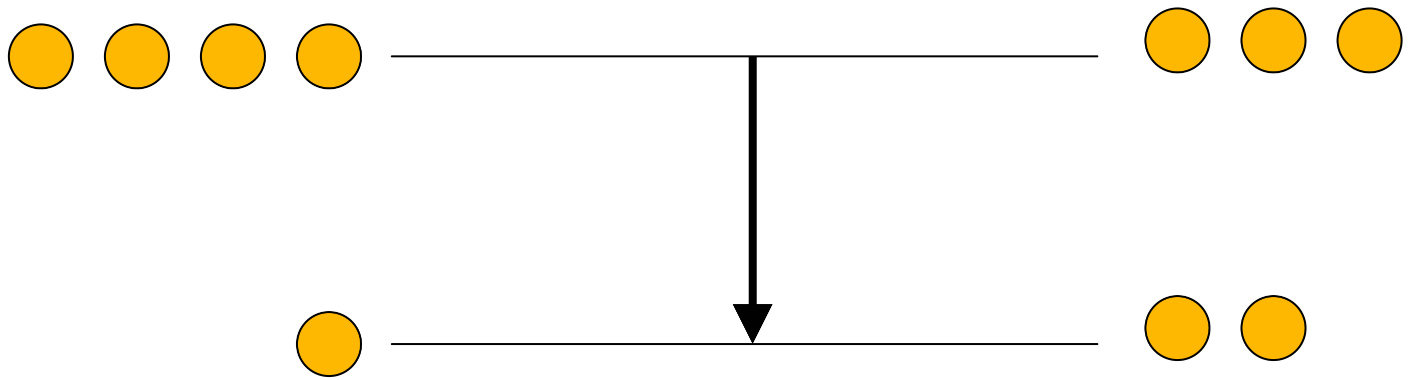
Basic Component: Balancer



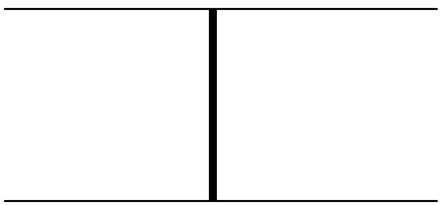
Balancer



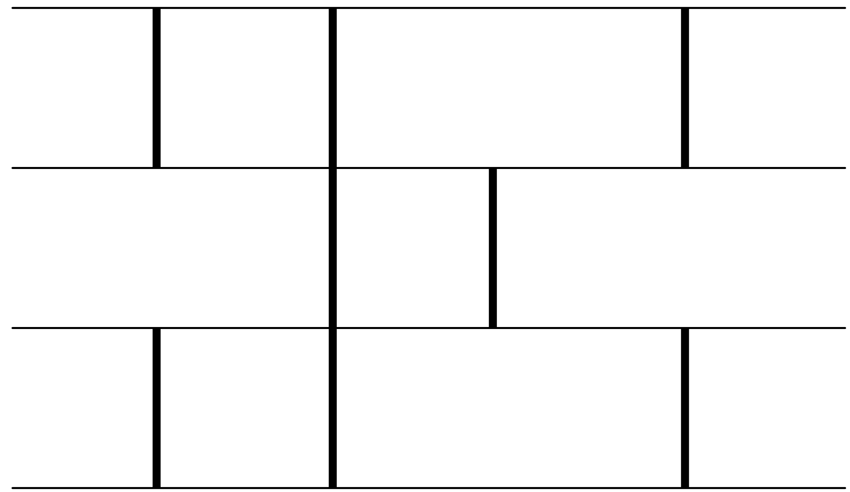
Balancer



Smoothing Networks:
Scalable Distributed Data Structures



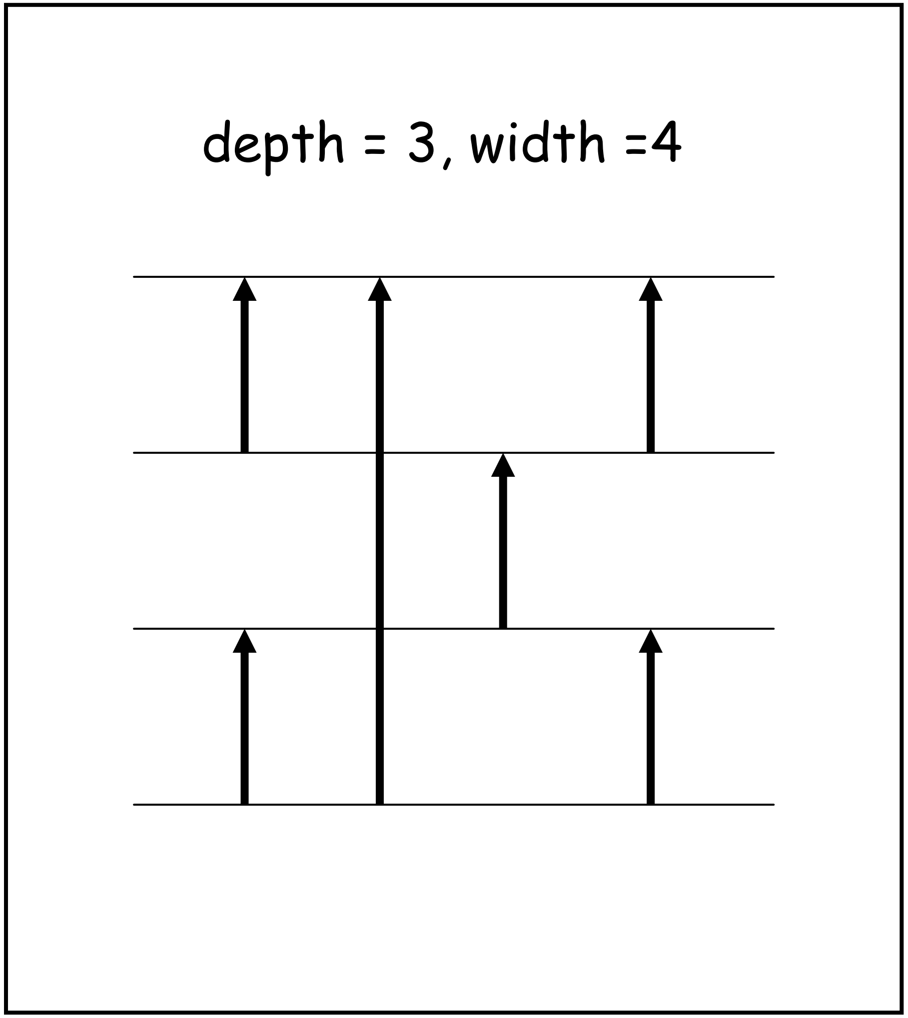
Bitonic[2]



Bitonic[4]

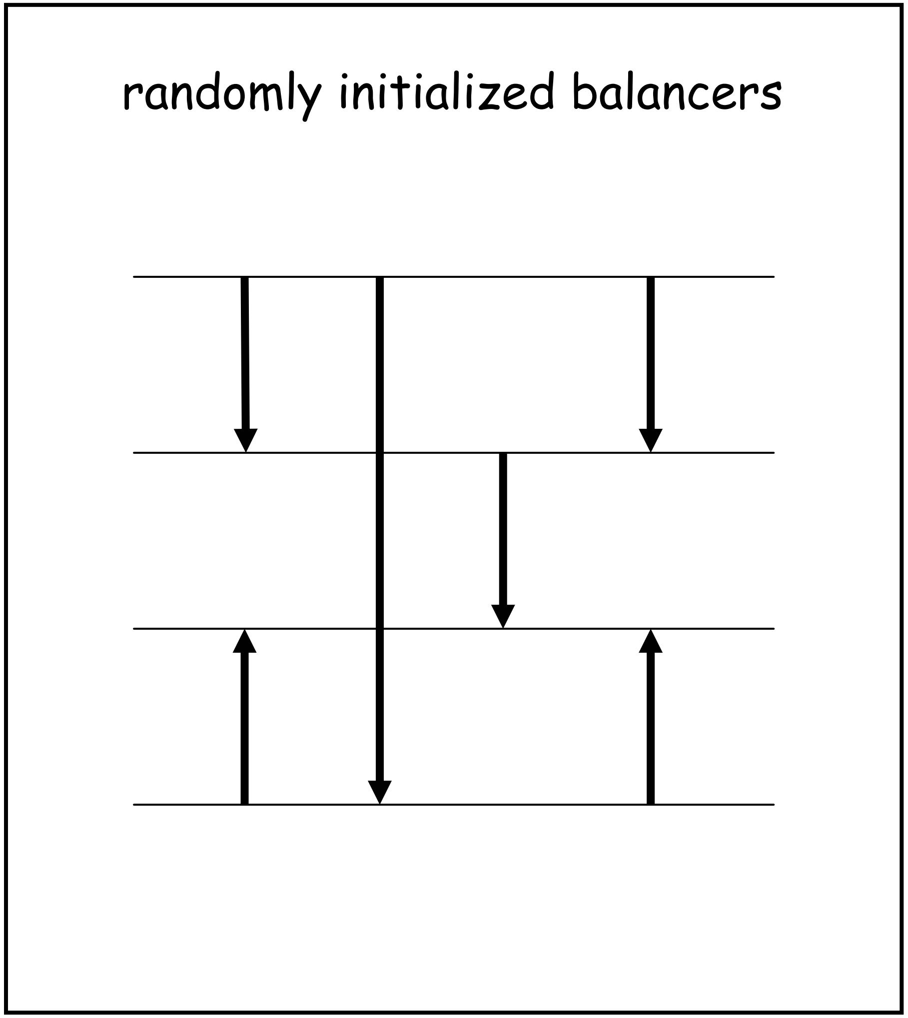
Prior Work: Deterministic Smoothing Networks

- The Network Balancers are Initialized in a very specific way
- (Usually) All balancers are pointing up



Our Work: Randomized Smoothing Networks

- Random balancers – initialized as follows
  - With prob = ½, up
  - With prob = ½, down
- Resulting Initial Network State also random
- How good are the smoothing properties of randomized networks?



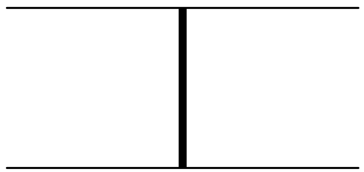
Benefits of Randomized Networks

1. Easy Initialization
   - Each balancer sets itself to a random state
   - Completely Local Actions

2. Easy to recover from faults
   - Global reconfiguration unnecessary

3. Better Smoothing Properties

Butterfly Network: Inductive Construction



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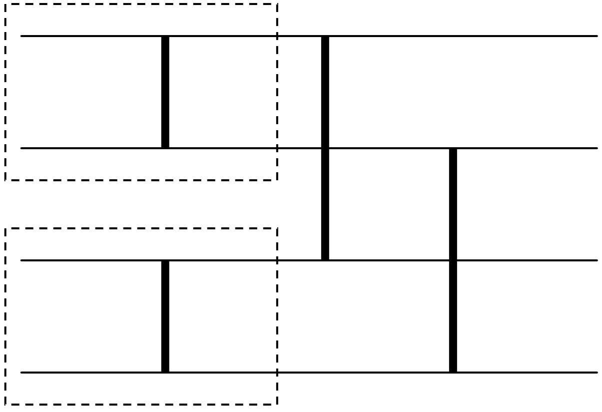

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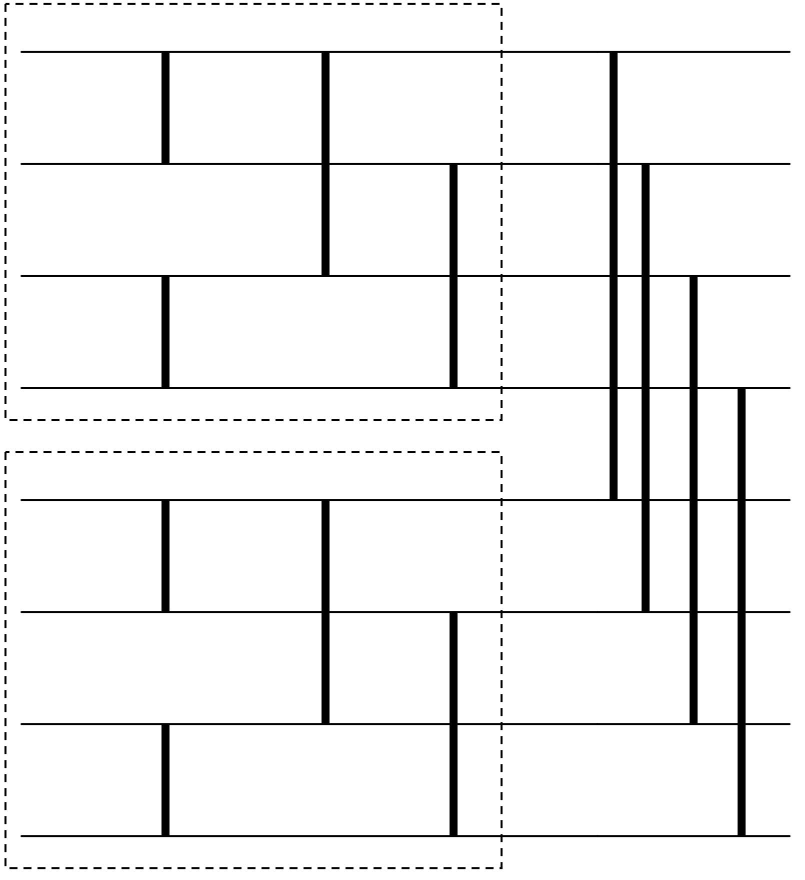

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Our Main Result

- Many Randomized Networks produce much smoother outputs than Regular Deterministically Initialized Networks

- For a Butterfly network of width w:
  - The output of a randomized network has smoothness $2.83\sqrt{\log w}$ with high probability
  - For any deterministic initialization, worst case output smoothness is no better than $O(\log w)$

Implications

- $2.83\sqrt{\log w}$ nearly constant for practical purposes
  - If w =10^9, this is 14
  - Smoothness independent of the number of tokens entering the network
- Network is very simple and easy to construct
- No further constants hidden in O(…)

Previous Work

- Aspnes Herlihy and Shavit, 1991
  - Bitonic and Periodic Counting networks (which are also 1-smoothing)
  - Isomorphic to sorting networks
  - Width w, depth $O(\log^2 w)$
- Klugerman and Plaxton, 1992
  - Better depth constructions of counting networks
  - Width w, depth $\approx O(\log w)$
  - Constants are very high, not practical

Previous Work (2)

- Aiello, Venkatesan and Yung, 1993
  - Counting and Smoothing Networks using random and deterministic balancers
  - An alternate definition of a randomized balancer
    - Our results apply to their definition of random balancers also
  - In contrast, we use only randomized balancers

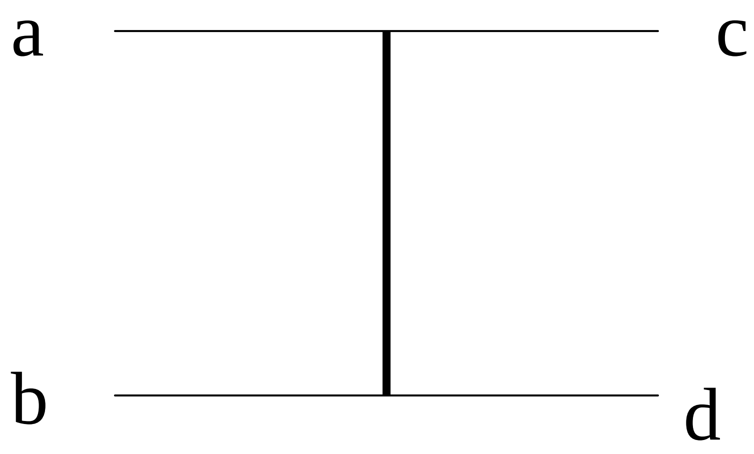
- Herlihy and Tirthapura, 2003
  - Worst case smoothness of deterministic Butterfly, Periodic and Bitonic networks
  - Matching upper and lower bounds
  - Self-stabilizing constructions

Analysis: Random balancer

a, b = numbers of tokens entering the balancer
c, d = numbers of tokens exiting the balancer

$r = +\frac{1}{2}$ with probability $\frac{1}{2}$
$\quad = -\frac{1}{2}$ with probability $\frac{1}{2}$

$c = (a+b)/2 + D(a+b)\, r$
$d = (a+b)/2 - D(a+b)\, r$



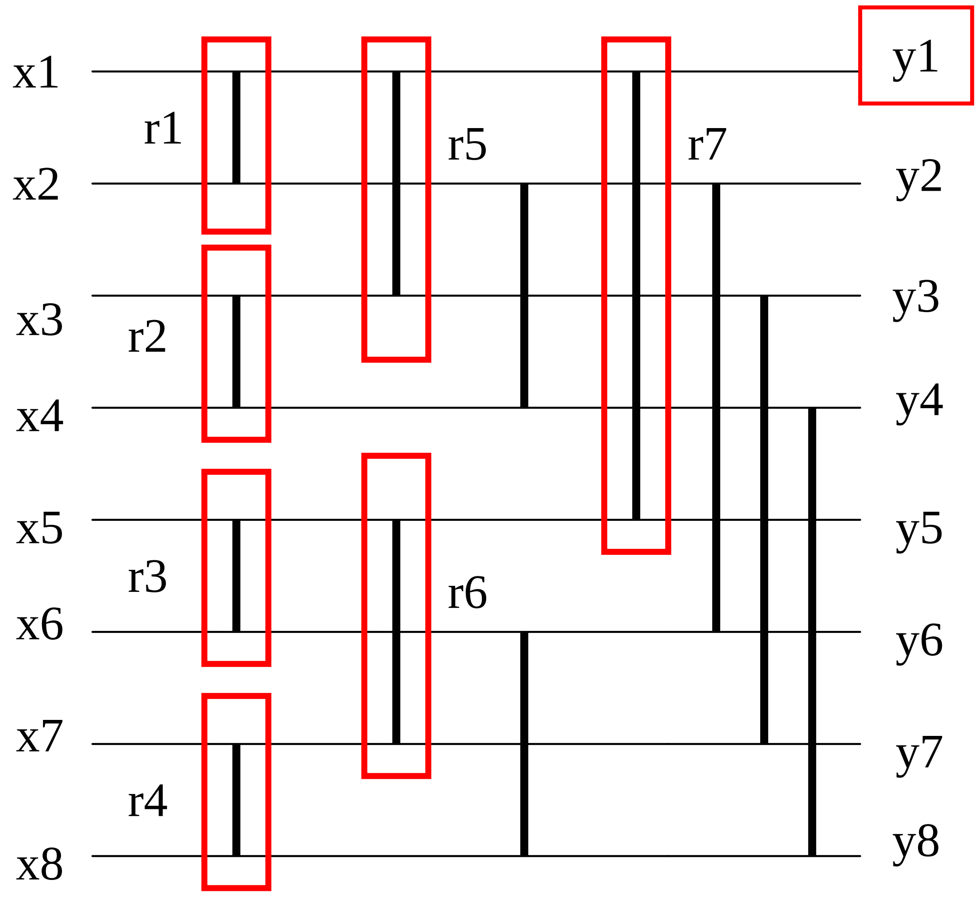
D = odd characteristic function

Analysis (2): Butterfly[8]

y1 depends only on
the balancer decisions
r1…r7 and on x1..x8

y1 = f(x1,x2,x3,..x8,
r1,r2,………,r7)

E[y1] = (x1+x2+…+x8)/8



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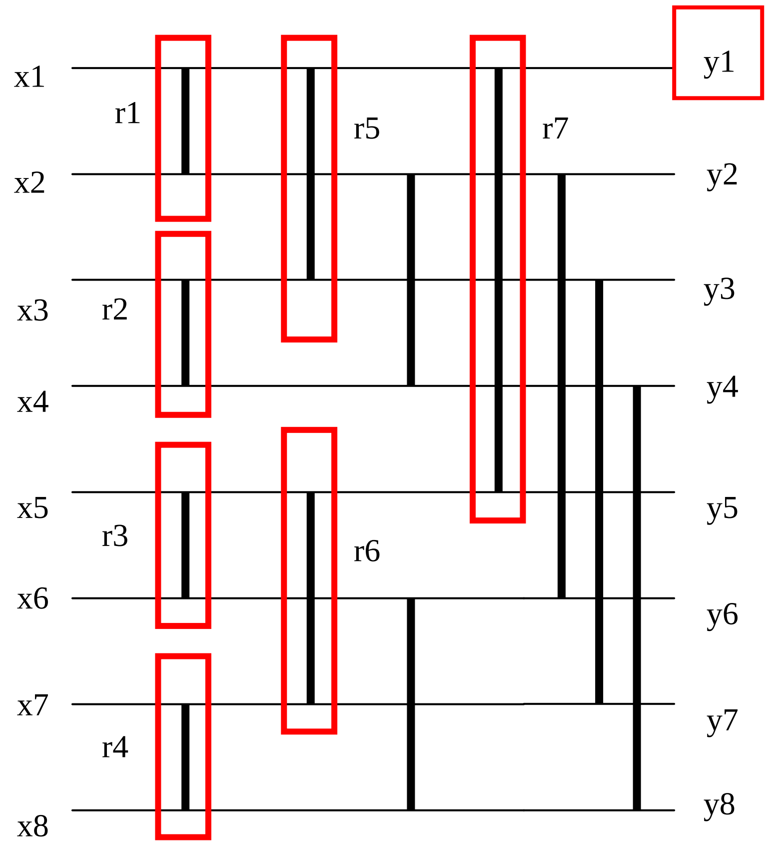
Analysis (3)

Random variable y1 is hard to work with directly

Define Alternate Random Variable z1:

z1 = (r1+r2+r3+r4)/4
+ (r5+r6)/2
+ r7
+ (x1+…+x8)/8



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Analysis (4)

- Random Variable z1 is easier to handle, since it is a linear function of r1, r2, … r7

$$\Pr[y1 > \delta] \le 2\Pr[z1 > \delta]$$

- Use Hoeffding's inequality to bound the tail of z1, and hence y1

Theorem

The output of Butterfly[w] with random balancers is $2.83\sqrt{\log w}$ smooth with probability at least *(1-4/w)* , irrespective of the input sequence

Contrast: For any deterministic initialization, there exist inputs which lead to *(log w)* smooth outputs

Conclusions

- Simple network (butterfly) with small depth and very good smoothness
- Easy, local reconfiguration after faults

Open Questions

- Do there exist simple, log-depth networks with constant output smoothness (with high probability)?
- Can the upper bound on the butterfly network be improved?
- Matching Lower bounds?
- Smoothing networks for tokens of unequal weights?