**Applications of fixed point theory to distributed optimization, robust convex optimization, and stability of stochastic systems**

Large-scale multi-agent networked systems are becoming more and more popular due to applications in robotics and signal processing. Although distributed algorithms have been proposed for efficient computations rather than centralized computations for large data optimization, existing algorithms are still suffering from some disadvantages such as distribution dependency or B-connectivity assumption of random communication graphs. This study applies fixed point theory to analyze distributed optimization problems and to overcome existing difficulties such as distribution dependency or B-connectivity assumption of random communication graphs. A new mathematical terminology and a new mathematical optimization problem are defined. It is shown that the optimization problem includes centralized optimization and distributed optimization problems over random networks. Centralized robust convex optimization is defined on Hilbert spaces which is included in the defined optimization problem. An algorithm using diminishing step size is proposed to solve the optimization problem under suitable assumptions. Consequently, as a special case, it results in an asynchronous algorithm for solving distributed optimization over random networks without distribution dependency or B-connectivity assumption of random communication graphs. It is shown that the random Picard iteration or the random Krasnoselskii-Mann iteration may be used for solving the feasibility problem of the defined optimization. Consequently, as special cases, they result in asynchronous algorithms for solving linear algebraic equations and average consensus over random networks without distribution dependency or B-connectivity assumption of random communication graphs. As a generalization of the proposed algorithm for solving distributed optimization over random networks, an algorithm is proposed for solving distributed optimization with state-dependent interactions and time-varying topologies without B-connectivity assumption on communication graphs. So far these algorithms are special cases of stochastic discrete-time systems. It is shown via an example that fixed point theory may analyze stability of stochastic discrete-time systems better than classical Linear System Theory, Lyapunov's approach, and LaSalle's Invariance Principle. This results in a motivation of analyzing stability of stochastic nonlinear discrete-time systems by means of fixed point theory. In this regard, suitable definitions of stability are provided. It is shown that difficulties such as independency and identical distribution of random variable sequence which arise in using Lyapunov's and LaSalle's methods may be overcome by means of fixed point theory.