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## Analysis and Synthesis Methods for Uncertain Nonlinear Network Systems

## Abstract:

The last decade has seen a surge in intermingling ideas from control and communication and their application to biological systems, power systems giving rise to new research areas like Networked Control Systems (NCS), Cyber-Physical Systems (CPS), Gene Regulatory Networks (GRN) to name a few. This has led researchers to study control systems with practical constraints imposed on them. Control over communication channels is one such widely researched area where the effect of unreliable interconnection on the stability performance of the system has been studied. Most of the current research for control over communication channels have focused on LTI plant dynamics. Furthermore the results involving nonlinear plant dynamics have reverted to local linearization techniques. It is wellknown that for nonlinear systems, results based on local linearization at an equilibrium point will be local in nature and does not account for the global dynamics of the nonlinear system. For the proposed applications of network control systems to electric power grid and biological networks it is essential to develop results for the analysis of nonlinear systems over networks.

In this work, we are primarily interested in the interaction of nonlinear systems and controllers over unreliable interconnections modelled as a stochastic multiplicative uncertainty. We provide analysis and synthesis methods for the control and observation of uncertain nonlinear network controlled systems. Our analysis methods indicate fundamental limitations arise in the stabilization and observation of nonlinear systems over uncertain channels. Our main result provides limitation for the observation of nonlinear system over erasure channel expressed in terms of the probability of erasure and positive Lyapunov exponents of the open loop nonlinear plant. The positive Lyapunov exponents are measure of dynamical complexity and comparing our results with existing results for LTI systems, we show that Lyapunov exponents emerge as a natural generalization of eigenvalues from linear to nonlinear systems.

Entropy is another measure of dynamical complexity. Using results from ergodic theory of dynamical systems we also relate the limitation for stabilization and observation with the entropy corresponding to the invariant measure capturing the global dynamics of the nonlinear systems. Existing Bode-like fundamental limitation results for nonlinear systems relate limitation for stabilization with the entropy corresponding to the invariant measure at the equilibrium point. Our results are the first to connect the limitation for stabilization with the entropy corresponding to invariant measure other than the one associated with equilibrium point.

Our synthesis methods for the design of robust controller and observer against uncertain channels revolves around special class of nonlinear systems -Lure systems. These systems are essentially linear

systems with sector-bounded nonlinearity in the feedback loop. For this special class of nonlinear systems, we delve into the theoretical tools of absolute stability to obtain some synthesis methods which provide design criteria for nonlinear systems over unreliable interconnections. Stability of Lur'e systems is a special case of the stability of interconnected passive systems. Thus we can characterize the unreliability of the interconnection, that guarantees the desired performance for Lur'e systems, in terms of the passivity of the linear system. Passivity theory is a rich theory with wide spread applications to nonlinear controller design and observation, which extends ideas of system stability to input-output systems using the ideas of dissipativity. Our synthesis methods developed for Lure systems with input and output stochastic channel uncertainties provide natural extension of the powerful passivity based synthesis tools developed for deterministic Lure systems. In particular, our results help understand the trade-off between passivity and stochastic uncertainty in feedback control systems.