

Convex-Set–Constrained Sparse Signal Recovery: Theory and Applications

Abstract

I will describe a projected Nesterov’s proximal-gradient (PNPG) approach for sparse signal reconstruction. The objective function that we wish to minimize is a sum of a convex differentiable data-fidelity (negative log-likelihood (NLL)) term and a convex regularization term. We apply sparse signal regularization where the signal belongs to a closed convex set within the closure of the domain of the NLL; the convex-set constraint facilitates flexible NLL domains and accurate signal recovery. Signal sparsity is imposed using the ℓ_1 -norm penalty on the signal’s linear transform coefficients. The PNPG approach employs a projected Nesterov’s acceleration step with restart and a duality-based inner iteration to compute the proximal mapping. We propose an *adaptive* step-size selection scheme to obtain a good local majorizing function of the NLL and reduce the time spent backtracking. Thanks to step-size adaptation, PNPG converges faster than the methods that do not adjust to the local curvature of the NLL. We present an integrated derivation of the momentum acceleration and proofs of $\mathcal{O}(k^{-2})$ objective function convergence rate and convergence of the iterates, which account for adaptive step size, inexactness of the iterative proximal mapping, and the convex-set constraint. The tuning of PNPG is largely application independent. Tomographic and compressed-sensing reconstruction experiments with Poisson generalized linear and Gaussian linear measurement models demonstrate the performance of the proposed approach.

I will then address the problem of upper-bounding the regularization constant for the convex-set–constrained sparse signal recovery problem behind the PNPG framework. This work is the first to upper-bound the regularization constant for total-variation (TV) regularizations.

Finally, I will show application of the PNPG framework to X-ray computed tomography (CT) and outline a method for sparse image reconstruction from Poisson-distributed polychromatic X-ray CT measurements under the blind scenario where the material of the inspected object and the incident energy spectrum are unknown. To obtain a parsimonious mean measurement-model parameterization, we first rewrite the measurement equation by changing the integral variable from photon energy to mass attenuation, which allows us to combine the variations brought by the unknown incident spectrum and mass attenuation into a single unknown mass-attenuation spectrum function; the resulting measurement equation has the Laplace integral form. We apply a block coordinate-descent algorithm that alternates between an NPG image reconstruction step and a limited-memory Broyden-Fletcher-Goldfarb-Shanno with box constraints (L-BFGS-B) iteration for updating mass-attenuation spectrum parameters. Our NPG-BFGS algorithm is the first physical-model based image reconstruction method for simultaneous blind sparse image reconstruction and mass-attenuation spectrum estimation from polychromatic measurements.