GS^3: Scalable Self-configuration and Self-healing in Wireless Sensor Networks*

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ABSTRACT

We present GS^3, a distributed algorithm for scalable self-configuration and self-healing in multi-hop wireless sensor networks. The algorithm enables network nodes in a 2D plane to configure themselves into a cellular hexagonal structure where cells have tightly bounded geographic radius and the overlap between neighboring cells is low. The structure is self-healing under various perturbations, such as node joins, leaves, deaths, movements, and state corruptions. For instance, it slides as a whole if nodes in many cells die at the same rate. Moreover, its configuration and healing are scalable in three respects: first, local knowledge enables each node to maintain only limited information with respect to a constant number of nearby nodes; second, local self-healing guarantees that all perturbations are contained within a tightly bounded region with respect to the perturbed area and dealt with in the time taken to diffuse a message across the region; third, only local coordination is needed in both configuration and self-healing.

Keywords

Multi-hop wireless sensor network, geography-aware self-configuration, local self-healing, locality, cellular hexagon

1. INTRODUCTION

As increasingly small network nodes are becoming available, many "sense-compute-actuate" networks are being realized. Several of these networks use unattended wireless sensor nodes [7, 11, 12], which communicate with one another via intermediate node relays due to limited transmission range or in order to save energy [15, 20]. The number of nodes is potentially large (thousands and millions of nodes are considered in earthquake relief and unmanned space vehicle scenarios, for instance) [7]. Thus, scalability is a key issue for large-scale multi-hop wireless sensor networks.

One way to achieve scalability is by "divide and conquer", or hierarchical control. Network nodes are first grouped into a set of clusters by some clustering criterion. A leader is elected in each cluster to represent the cluster at higher levels. The same clustering scheme may be

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iteratively applied to the cluster leaders to form a hierarchy. In this hierarchy, local control is applied at each level to achieve certain global objectives.

Most existing work on clustering in wireless networks [3, 18] treats a network as a geography-unaware graph. The clustering criteria adopted are, for instance, the number of nodes in a cluster and the logical — as opposed to geographic — diameter (in the number of hops) of clusters. However, geography-unaware clustering can be such that the communication links between a cluster leader and other nodes in its cluster are long, the geographic overlap between neighboring clusters is large, and routing traffic load is unbalanced across different clusters [8]. Therefore, more energy is consumed when a non-leader node communicates with its cluster leader by the only long link between them, simultaneous transmissions at neighboring clusters collide frequently, and energy dissipation is not balanced among clusters. Consequently, the lifetime of a network and the communication quality as well as efficiency in the network are reduced. Therefore, in order to save energy and improve communication quality as well as efficiency, the geographic radius of clusters should be taken into account in clustering algorithms.

Other reasons for considering the geographic radius of clusters (which we simply call *radius,* henceforth) in wireless sensor networks, especially large-scale, resource constrained multi-hop ones, include:

- Many multi-hop wireless sensor network applications, such as environment monitoring and temperature sensing, are inherently geography-aware, and so reflecting geography in the underlying network structure enables optimization of system performance.
- Cluster radius affects the efficiency of such local coordination as data aggregation and load balancing.
- Cluster radius affects the potential degree of frequency reuse in networks. The smaller the cluster radius, the more the frequency reuse.
- Cluster radius affects the scalability as well as availability of networks, since it affects the number of clusters and the number of nodes in each cluster (the more the nodes in a cluster, the more available the cluster is).
- Given that nodes are approximately uniformly distributed and such fidelity control mechanisms as that in [4] adapt the number of active nodes in each area of a network, guaranteed radius of a cluster also guarantee the number of nodes in the cluster.

Challenges and opportunities While accounting for geography in clustering, it is desirable that the radius of clusters be bounded from above as well as from below. The tightness of the bound impacts load balancing as well as the uniformity of energy dissipation in a network (and hence the lifetime of the network). It is also intuitively desirable that the tightness of the bound reflect such intrinsic network properties as node distribution density.

Given that multi-hop wireless sensor networks are expected to be untethered and of large scale, they demand automatic management [5]. Therefore, self-configuration is required in these networks, and it needs to be scalable to large network sizes. Moreover, self-healing is required

in wireless sensor networks, because such complex perturbations as node join, node leave, node movement, node crash, and state corruption are expected to occur in these networks. Since even node crash and message loss perturbation can drive a network protocol into arbitrary states [13], self-healing of a network from arbitrary states is desirable. Furthermore, given that wireless sensor networks are of large scale, self-healing that is local is essential for the stability, availability, and scalability of a network [2]. By local self-healing, perturbations are dealt with and their impact contained in the locality near where the perturbations have occurred.

The demand of geography-aware clustering and local self-healing are not readily achievable in general large-scale systems. However, wireless sensor networks offer some distinguishing properties such as node distribution is dense, location information of nodes is effectively available, the degree of node mobility is low, and there exist some gateways between a sensor network and external networks such as Internet [21, 22]. These properties offer opportunities to solve the problem in efficient ways, and we exploit them in the paper.

Contributions of the paper In this paper, we design a distributed algorithm (GS^3) for configuring a wireless planar network into clusters (which we henceforth call *cells* due to their geographic nature). More specifically, the network nodes configure themselves into a *cellular hexagonal structure*, in which the network nodes are partitioned into hexagonal cells each with a radius that is tightly bounded with respect to a given value R (an ideal cluster radius) and zero overlap between neighboring cells. One node in each cell is distinguished, as the *head* of the cell, to represent this cell in the network. All heads in a network form a directed graph, called *head graph*, which is rooted at a "big node" that is the interface between the wireless sensor network and external networks such as Internet.

Our algorithm yields a local self-healing system. The head graph and cellular hexagonal structure are self-healing in the presence of various perturbations, such as one or more node joins, leaves, deaths, movements, and state corruptions. The self-healing capability and the modular design of GS^3 enable different modules to be integrated so as to cater to different network models, in static as well as dynamic networks, in immobile as well as mobile networks, and in networks with one big node as well as multiple big nodes. Moreover, the self-healing is local such that the head graph and the cellular hexagonal structure remain stable upon perturbations in the following ways: 1) unanticipated node leaves within a cell are masked by the cell; 2) in case multiple cells experience node deaths at about the same time (due to energy exhaustion), independent shift of each cell enables the head graph as well as the cellular hexagonal structure to slide as a whole yet maintain consistent relative location among cells and heads; 3) in case the root of the head graph moves d away from its previous location, only the part of the head graph that is within $\sqrt{3}d/2$ radius from the root needs to change accordingly. Thus, a stable communication infrastructure for other services, such as routing, is configured in a dynamic or mobile network.

Our algorithm achieves scalability in three respects: 1) *local knowledge* enables each node to maintain the identities of only a constant number of nearby nodes; 2) *local self-healing*

guarantees that all perturbations are dealt within and their impact is confined to a tightly bounded region around the perturbed area; the cellular hexagonal structure self-stabilizes within the time to diffuse an one-way message across the perturbed area; 3) only *local coordination* is needed in both the self-configuration and self-healing processes. (The complexity and convergence properties of GS^3 are summarized in Appendix 1.)

The rest of the paper is organized as follows. In Section 2, we present the system model and problem statement. We then develop algorithm for static networks, dynamic networks, and mobile dynamic networks in Section 3, 4, and 5 respectively. We discuss related work in Section 6. Section 7 concludes the paper and makes further comments on system model. For reasons of simplicity, we relegate the detailed description of algorithm modules, and proofs for theorems to Appendix.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1 System Model

The system model consists of two parts: models for system nodes and perturbations.

System nodes A system consists of a set of nodes in a 2D plane, each having a certain wireless transmission range.

> ***Node distribution*** There exists R_t (called *radius tolerance*) such that, with high probability, there are multiple nodes in each circular area of radius R_t in the plane.[1]

There are two kinds of nodes: big and small. Intuitively, the big node acts as the initiator as well as the access point for small nodes[2]. That is, the big node initiates operations (such as clustering) at small nodes, and acts as the interface between small nodes and external systems such as Internet. For convenience, we assume that the system has one big node, and all the other nodes are small (in Section 7, we discuss the case of multiple big nodes).

> ***Wireless transmission*** Nodes can adjust transmission range, and detect relative location with respect to other nodes. Destination-aware message transmission is reliable, but destination-unaware message transmission (such as broadcast) may be unreliable. [3]

Perturbations We consider two types of perturbations: dynamic and mobile. The former consists of node joins, leaves, deaths, and state corruptions, and the latter consists of node movements.

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[1] More specifically, nodes are distributed uniformly in the plane and the number of nodes in a circular area of certain radius is a Poisson random variable. We discuss this in detail in Section 4.3.4.

[2] Many wireless sensor networks have some central points that control system wide operations. For example, in a field of disaster recovery, there is usually a commander for a group of rescue workers that is the central point. Sensor networks are also used to sample environment for sensory information (e.g. temperature) and propagate it to some central points [8].

[3] A network node can detect the strength of a received signal, and calculate its distance from the communicating peer [19]. Thus nodes can calculate relative location among themselves just by local information exchange in a dense network, even without GPS support. Moreover, when a node sends a message to some known node(s), the message transmission can be made reliable through such mechanisms as acknowledgement and retransmission.

Perturbation frequency Node joins, leaves, and state corruptions are unanticipated and thus rare. Node death is predictable (e.g. as a function of its rate of energy dissipation). The probability for a node to move distance d decreases as d increases.

For pedagogical reasons, we classify networks into three: in a *static network*, there are neither dynamic nor mobile nodes; in a *dynamic network*, there can be dynamic nodes, but no mobile nodes; in a *mobile dynamic network*, both mobile nodes and dynamic nodes can exist.

2.2 Problem of Self-configuration and Self-healing

Informally, the self-healing configuration problem is to partition a system such that the maximum distance between nodes within a partition is bounded, each partition, called *cell*, has a unique distinguished node, called *head*, and the heads are organized into a *head graph* that is self-healing under perturbations. Nodes other than the head in a cell are called *associates*, and they communicate with nodes beyond their cell only through the cell head.

We define:

- *Head graph*: a tree that is rooted at the big node and consists of all cell heads.
- *Cell radius*: the maximum geographic distance between the head of a cell and its associates.

Formally, the problem is to design an algorithm that given R (*ideal cell radius*) where $R \geq R_t$, constructs and maintains a set of cells and head graph that meet the following requirements:

a) Each cell is of radius $R \pm c$, where c is a small value with respect to R, and is a function of node distribution density characterized by R_t. [4]
b) Each node is in at most one cell. [5]
c) A node is in a cell if and only if the node is connected to the big node (i.e. there is a path between the node and the big node, and every two neighboring nodes in the path are within transmission range of each other). [6]
d) The number of children for each node in the head graph is bounded. [7]
e) The set of cells and the head graph are self-healing in the presence of dynamic as well as mobile nodes. By self-healing, a system can recover from a perturbed state to its stable state by itself. [8]

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[4] The primary goal of geography aware self-configuration is to organize nodes into cells with certain ideal radius R that depends on application scenarios (e.g. data aggregation ratio and node distribution). In practice, a system may not be able to organize itself into cells of exactly the ideal radius R due to discrete node distribution, but the deviation of the actual radius from R still need to be small enough, and be a function of node distribution density. Since the density of node distribution is characterized by R_t, the deviation should be a function of R_t.

[5] By guaranteeing that each node belongs to only one cell, energy can be saved, and the number of cells as well as control complexity is reduced.

[6] If a node is able (unable) to communicate with the big node before configuration, it should still be able (unable) to do so after configuration.

[7] Since network traffic flows from children to parents along the head graph until reaching the big node, in order to guarantee load balancing and uniform rate of energy dissipation, the number of children for each node in the head graph should be bounded.

3. STATIC NETWORK

3.1 Concepts

Recall that in static networks, nodes are neither dynamic nor mobile. So we solve the problem without considering perturbations (i.e., requirement *e)* is ignored). Moreover, we assume there is no R_t-*gap* in static networks, where an R_t-gap is a circular area of radius R_t with no node inside. R_t-gaps are dealt with as a rare perturbation in dynamic networks in Section 4.

Let us first consider an ideal case of the problem: given a plane with a continuous distribution of nodes, we may divide it into cells of equal radius R with minimum overlap between neighboring cells to obtain a cellular hexagonal structure as shown in Figure 1. In this structure, each cell is a hexagon with the maximum distance between its geometric center and any point in it being R. Let the geometric center of a cell be the "head" of all points in the cell.[9] Then the distance between the heads of any two neighboring cells is $\sqrt{3}R$. And each cell that is not on the boundary of the plane is surrounded by 6 neighboring cells.

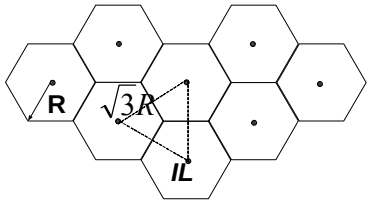

Figure 1: Cellular hexagonal structure

Of course, node distribution is not continuous in reality. Thus there may be no node at the geometric center of some cell and it may be impossible to divide the network into exact hexagons as in Figure 1. But in scenarios where there are multiple nodes in any circular area of radius R_t, we can still approximate this structure by letting some node within R_t distance from the geometric center of a cell be a head, as is allowed in cellular networks [16].

Our solution is achieved in three steps. First, we cover a system with a hexagonal virtual structure as in Figure 1 such that the big node is located at the geometric center of some cell. Second, for each cell C in the virtual structure, we choose a node k closest to the geometric center of C as a head, and the geometric center of C is called the Ideal Location (simply denoted as *IL* hereafter) of k, *IL(k)*; Third, every non-head small node j covered by a cell C becomes an associate and chooses the best (e.g. the closest in a clockwise sense) head as its head, *H(j)*. A head together with its associates form a cell, and the *IL* of the head is also called the *IL* of the cell.

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[8] In large-scale wireless sensor networks, complex perturbations can drive a network protocol into arbitrary state, and the network cannot be managed manually. Therefore, self-healing of a network from arbitrary states is demanded. This goal is achieved by the technique of self-stabilization [1].

[9] The advantage of a cell head being at the center of the cell is that communication as well as energy efficiency is improved, since most communication within a cell is between the cell head and non-head nodes and the distance between the cell head and non-head nodes is minimized if the cell head is at the center of the cell.

We designate the cell where the big node is as the central cell, and each set of cells of equal minimum distance from the central cell in terms of the number of cells in between as a cell band. If cells in a band are of *d*-cell distance from the central cell, this band is called a *d*-band, and the central cell alone forms the 0-band.

Next, we discuss a scalable distributed algorithm that implements the above concepts.

3.2 Algorithm

Overview Generally speaking, there are two kinds of clustering methods: bottom-up and top-down. In the wireless sensor network literature, only bottom-up clustering has been considered. But it cannot guarantee the exact placement and tight radius of clusters [10, 12], therefore it does not solve the self-configuration problem as described in this paper. In contrast, the existence of big nodes in wireless sensor networks enables the top-down clustering approach, and our algorithm explores this approach with distributed control.

The self-configuration algorithm consists of a one-way diffusing computation across the network. The big node H_0 initiates the computation by acting as the head for the 0-band cell (i.e. the cell whose *IL* is at H_0), and selecting the heads of its neighboring cells in its *search region*. Then each newly selected head selects the heads of its neighboring cells in its search region, and so on until no new head is selected. Every node that has participated in the computation but not been selected as head becomes an associate and chooses the best head in the system as its head.

In the diffusing computation, the actual location of selected cell heads may deviate from the *IL* of the cell due to discrete node distribution. In order to prevent the accumulation of such deviation as the diffusing computation propagates far away from the big node, and to guarantee the exact placement as well as tight radius of cells, a unique Global Reference ($\overrightarrow{GR}$) direction[10] is diffused across the network along with the diffusing computation. Moreover, when a head selects its neighboring cell heads, it uses the *IL* of its cell instead of the actual location of itself (see Figure 3 for detail).

If head *i* is selected by head *j*, we say that *j* is the *parent* of *i*, *P(i),* and *i* is a *child* of *j*, *CH(j)*. $P(H_0)$ is H_0. Then the search region of a head *i* is defined as the area that is within $\sqrt{3}R + 2R_t$ distance from *IL(i)* and between the two directions: *L* direction (*LD*) and *R* direction (*RD*) with respect to direction $\overrightarrow{IL(P(i)),IL(i)}$ (see Figure 3). In order to guarantee that every node connected to H_0 is covered by the diffusing computation, <*LD, RD*> is chosen as $<0^o, 360^o>$ and $<-60^o-\alpha, 60^o+\alpha>$ for H_0 and the other heads respectively, where $\alpha = \sin^{-1}(R_t/\sqrt{3}R)$.

In most cases, a *(d+1)*-band cell head is selected by a *d*-band head ($d \geq 0$). But in the case where the speed of the diffusing computation differs at different directions with respect to H_0, it is also possible that a *(d+1)*-band head is selected by a *(d+2)*-band head ($d \geq 1$). But this does

[10] The global reference direction $\overrightarrow{GR}$ can be any one, even though it needs to be consistent across the network.

not affect the correctness of GS^3-S, and it is dealt implicitly in the algorithm in Section 4. For simplicity, we do not discuss this any further here.

Algorithm modules The algorithm (GS^3-S) consists of two programs[11] (described in Figure 2): *Big_node* for the big node and *Small_node* for small nodes. Underlying these two programs are modules used for head organization: HEAD_ORG, used to organize heads, and HEAD_ORG_RESP as well as ASSOCIATE_ORG_RESP, used to respond to HEAD_ORG.

Program *Big_node*

var q: {*bootup, work*}; //node status

q = *bootup* → HEAD_ORG(0^o, 360^o, R, R_t); //Big node organizes the 1-band cells;
//Then transits to status *work*

Program *Small_node*

var q: {*bootup, head, associate, work*}; //node status

q = *bootup* → ASSOCIATE_ORG_RESP; //Small nodes listen to nearby HEAD_ORG;
//Then transits to status *head* or *associate*

[]

q = *head* → HEAD_ORG(-60^o-α, 60^o+α, R, R_t); //Heads organize neighboring heads in their search regions;
//Then transits to status *work*

[]

q = *work* → HEAD_ORG_RESP;

[]

q = *associate* → ASSOCIATE_ORG_RESP; //Associates respond to HEAD_ORG;
//Remain status *associate*

Figure 2: Self-configuration algorithm for static networks (GS^3-S)

In HEAD_ORG, a head i (including the big node) organizes neighboring heads in its search region. It first gets the state (e.g. geographic location) of all the nodes in its search region by local information exchange; then it selects the neighboring heads using the low-level module HEAD_SELECT; last, it broadcasts the selected set of heads to nodes within $\sqrt{3}R + 2R_t$ distance.

In HEAD_SELECT (described in Figure 3), head i first calculates the *IL*s for the neighboring cells in its search region; then for each *IL* that is not the *IL* of an existing head, i selects the best node less than R_t away from the *IL* as a head.

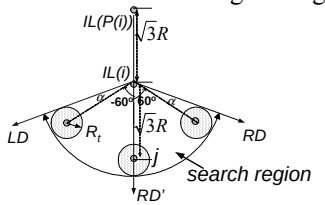
In HEAD_ORG_RESP, a head sends its state in response to HEAD_ORG at another head at most $\sqrt{3}R + 2R_t$ away. In ASSOCIATE_ORG_RESP, which is executed by a small node i in response to HEAD_ORG at a head j at most $\sqrt{3}R + 2R_t$ away, if i already has a head, i sets j as its head only when j is better than its current head; if i does not have a head, it sends its state to j, and waits for j's decision of whether i is selected as a head, and sets its status accordingly.

[11] We use the notation of Guarded Command [9] to write algorithms.

Module *HEAD_SELECT* (SmallNodes, ExistingHeads, LD, RD, R, R_t)

Step 1: Calculate *IL*s of neighboring heads, *NH*, in the search region of *i*.

Use $\overrightarrow{IL(P(i)),IL(i)}$ as reference direction (*RD'*) (if *P(i) = i*, *RD'* can be any direction), *IL(i)* as origin, and $\sqrt{3}R$ as radius, go both clockwise and counterclockwise, the points on the arc that are $j\times60^o$ ($\lfloor LD/60 \rfloor \le j \le \lfloor RD/60 \rfloor$) degree from *RD'* are the *IL*s of neighboring heads.

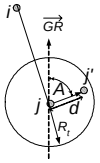


Step 2: Remove the set of *IL*s that are the *IL*s of some existing heads from *NH*. I.e. $NH \leftarrow (NH - EH)$, where $EH = \{j : j \in NH \wedge (\exists\, k \in \text{ExistingHeads} : (\, dist(j, k) \le R_t\,))\}$.

Step 3: For each *IL j* in *NH*, let $CA(j) = \{k : k \in \text{SmallNodes} \wedge dist(k, j) \le R_t\}$. *CA(j)* is the set of small nodes within R_t distance from *j*.

Step 4: For each *IL j* in *NH*, since *CA(j)* is non-empty, select the *highest ranked* node *j'* in *CA(j)* as the cell head corresponding to *j*, and set *CH(i)* as (*CH(i)* $\cup$ *{j'}*).

Every node *k* in *CA(j)* is lexicographically ordered by <*d*, |*A*|, *A*>, where *d* is the distance between *j* and *k*, *A* stands for the angle ($-180^o \le A \le 180^o$) formed by $\overrightarrow{GR}$ and $\overrightarrow{j,k}$ (*A* is negative if $\overrightarrow{j,k}$ goes clockwise with respect to $\overrightarrow{GR}$ and positive if counter-clockwise), and *d* has the highest significance.



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Time complexity: θ (|SmallNodes|)

Figure 3: Module HEAD_SELECT used in HEAD_ORG

A more detailed description of the individual modules of GS^3-S is given in Appendix 2.

3.3 Analysis

In this subsection, we discuss the invariant, fixpoint, self-stabilization, and other properties of algorithm GS^3-S (proofs are given in Appendix 4).

Notation

Physical network: $G_p = (V_p, E_p)$, where $V_p = \{ j: j$ is a node in the system$\}$ and $E_p = \{(i, j): i \in V_p \wedge j \in V_p \wedge (i$ and j are within transmission range of each other$)\}$

Head Graph: $G_h = (V_h, E_h)$, where $V_h = \{i: i \in V_p \wedge i$ is a cell head$\}$ and $E_h = \{(i, j): i \in V_h, j \in CH(i)\}$

Head level structure: the set of heads in a system and the geographic relation (such as relative direction and distance) among them

Geographic coverage: the geographic coverage of a node is the circular area on a plane that is centered at the node and has a radius equal to the current transmission range of the node. The geographic coverage of a system is the union of the geographic coverage of all the nodes in a system

Boundary cell: a cell that is on the boundary of the geographic coverage of a system or is a neighbor of an R_t-gap perturbed cell (Section 4)

Inner cell: a cell that is not a boundary cell

3.3.1 Invariant

We show the correctness of algorithm GS^3-S using an invariant, i.e. a state predicate that is always true in every system computation. Note that an invariant depends on the granularity of actions. Here we consider each algorithm module (e.g. HEAD_ORG) as an atomic action. The invariant SI for GS^3-S is $I_1 \wedge I_2 \wedge I_3$, where I_j (j = 1, 2, 3) is individually closed under algorithm actions. The predicates are as follows.

I_1 (Connectivity) = $I_{1.1} \wedge I_{1.2}$, where

- $I_{1.1}$: Every pair of heads that is connected in the head graph G_h is connected in the physical network G_p, and vice versa.
- $I_{1.2}$: The head graph G_h is a tree rooted at the big node H_0.

I_2 (Hexagonal Structure) = $I_{2.1} \wedge I_{2.2} \wedge I_{2.3} \wedge I_{2.4}$, where

- $I_{2.1}$: Each inner cell head i has exactly 6 neighboring heads that form a cellular hexagon centered at i and of edge length $\sqrt{3}R$, with vertices′ location deviation at most R_t. That is, the distance between neighboring heads is bounded by $[\sqrt{3}R - 2R_t, \sqrt{3}R + 2R_t]$.
- $I_{2.2}$: Each boundary cell head has less than 6 neighboring heads, and the distance between neighboring heads is bounded by $[\sqrt{3}R - 2R_t, \sqrt{3}R + 2R_t]$.
- $I_{2.3}$: Each head, except for the big node H_0, has at most 3 children heads. H_0 has 6 children heads if it is an inner cell head and at most 5 children heads otherwise.
- $I_{2.4}$: Each cell is of radius (R+R_{random}), where R_{random} is bounded by $[-2R_t/\sqrt{3}, 2R_t/\sqrt{3}]$. Each associate is no more than (R+R_{random}) away from its head.

I_3 (Inner Cell Optimality): Each associate in an inner cell belongs to only one cell and chooses the best (e.g. closest) head as its head.

Theorem 1: SI is an invariant of algorithm GS^3-S, where SI = $I_1 \wedge I_2 \wedge I_3$.

Theorem 1 and I_2 imply

Corollary 1: The distance between neighboring cell heads is bounded by $[\sqrt{3}R - 2R_t, \sqrt{3}R + 2R_t]$.

Corollary 2: The heads and their cells form a cellular hexagonal structure (shown in Figure 4) with bounded head location deviation R_t.

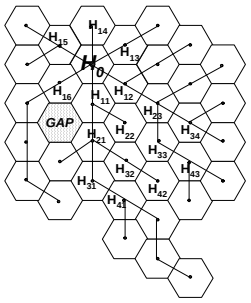


Figure 4: Self-configured cellular hexagonal structure

3.3.2 Fixpoint

A fixpoint is a set of system states where either no action is enabled or any enabled action does not change any system state we are interested in (e.g. G_h). It therefore characterizes the result of the self-configuration process. The fixpoint SF for GS^3-S is $F_1 \wedge F_2 \wedge F_3 \wedge F_4$ as follows.

F_1 (Connectivity) and **F_2** (Hexagonal Structure) are the same as I_1 and I_2 respectively.

F_3 (Cell Optimality): Each associate belongs to only one cell and chooses the best (e.g. closest) head as its head.

F_4 (Coverage): The set of heads and cells covers all the nodes that are connected with the big node in the physical network G_p.

Theorem 2: SF is a fixpoint of algorithm GS^3-S, where SF = $F_1 \wedge F_2 \wedge F_3 \wedge F_4$.

Requirements *a), b),* and *d)* in the problem statement are satisfied by Theorem 1 and 2.

Theorem 2, F_1 and F_4 imply

Corollary 3: At a state in SF, a node is in a cell if and only if it is connected to the big node in the physical network G_p, and vice versa.

Requirement *c)* in the problem statement is satisfied by Corollary 3.

3.3.3 Self-stabilization

Theorem 3: Starting at an arbitrary state, every computation of GS^3-S reaches a state in SI within a constant amount of time.

Theorem 4: Starting at an arbitrary state in SI, every computation of GS^3-S reaches a state in SF within time $\theta(D_b)$, where D_b is the maximum Cartesian distance between the big node and any small node in the system.

Theorem 3 and 4 imply

Corollary 4: Starting at an arbitrary state, every computation of GS^3-S reaches a state in SF within time $\theta(D_b)$.

Termination of the diffusing computation follows from Corollary 4.

3.3.4 Scalability

The self-configuration algorithm GS^3-S is scalable in that it only requires *local coordination* among nodes within $\sqrt{3}R + 2R_t$ distance from one another, and each node maintains the identities (e.g. MAC address) of only a *constant* number of nodes, 1 for associates and at most 6 for heads, irrespective of network size.

4. DYNAMIC NETWORK

4.1 Concepts

Recall that, in dynamic networks, nodes can join, leave (e.g. fail-stop), die, and node state can be corrupted. Excluding node death, which is predictable, the other perturbations are unanticipated and therefore rare. There may also be R_t-gaps in node distribution. In this section, we extend GS^3-S to GS^3-D to deal with these perturbations.

We propose three mechanisms to deal with node leave and death: head shift, cell shift, and cell abandonment. Self-stabilization easily handles the remaining perturbations, i.e. node joins and state corruptions.

Head shift In dynamic networks, the associates in a cell are divided into two categories: *candidate* and *non-candidate*. Associates within R_t distance from the *IL* of the cell are head candidates, with the rest being non-candidates. In the case where only unanticipated head leaves occur, a new head can be found with high probability from the set of candidates, due to the low probability of all candidates in a cell leaving at the same time. Moreover, the extreme case where all candidates leave can be dealt with by *cell shift*.

Cell shift In case node death occurs, it is possible that the set of candidates of a cell becomes empty due to energy exhaustion after long enough system operation. In this case, the *IL* of the cell is changed to another point *IL´* within the geographic coverage of the cell such that the corresponding candidate set is non-empty, which is enabled by the fact that energy usually dissipates faster at a head than at an associate. In many envisioned large-scale wireless sensor networks, the traffic load across a network is statistically uniform due to in-network processing such as data aggregation [24], which means statistically uniform energy dissipation across the network. Given the fact that statistically there are multiple nodes in any circular area of radius R_t at the beginning of the self-configuration, the lifetime of any two sets of candidates at different cells is statistically the same with low deviation, especially for cells close by. Therefore, if the *IL*s at different cells change independently but in the same deterministic manner in terms of the relative position between *IL* and *IL´*, the head graph as well as head level structure will *slide* as a whole but maintain consistent relative location among cells and heads.

Cell abandonment It is possible albeit rare that a cell is so heavily perturbed that nodes in an area of radius larger than R_t die at the same time. Even though cell shift may be able to change the *IL* of the cell to *IL´*, the distance between *IL´* and the *IL*s of all neighboring cells may deviate beyond the range $[\sqrt{3}R - 2R_t, \sqrt{3}R + 2R_t]$. In this case, we let the cell to be abandoned in the sense that every node in it becomes an associate of one of the neighboring cells. (Note that, because of the sliding of the head level structure resulted from cell shift, a new head can be selected within the abandoned cell later.)

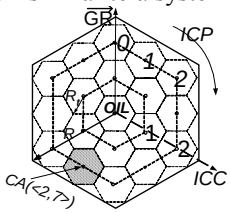
4.2 Algorithm

Overview In GS^3-D, when a head *i* tries to select the heads for its neighboring cells in its search region, it is possible that there is an R_t-gap at the *IL* of a neighboring cell *C* (in this case, *C* is called an R_t-gap perturbed cell). Given the low probability of this case, *i* does not select head for cell *C*, and every node in *C* becomes an associate of a neighboring cell of *C* (this is similar to cell abandonment). Due to node join and the sliding of head level structure, new nodes may show up in the area of *C* or the *IL* for *C* is changed such that there is a node within R_t distance to the *IL* of *C* later. By periodically checking this, head *i* will select the head for *C* whenever it shows up later.

When a node *j* joins an existing system, it tries to find the best existing head as its head if there is any within $\sqrt{3}R + 2R_t$ distance. Otherwise, *j* tries to find the best associate as its *surrogate head* if there is any associate within its radio transmission range. If both trials fail, *j* gives up and retries the above process after a certain amount of time. In the above process, if a head *k* within $\sqrt{3}R + 2R_t$ distance is executing HEAD_ORG, *j* responds with ASSOCIATE_ORG_RESP and becomes either a child head or an associate of *k*.

Node leave or death is dealt with by intra-cell and inter-cell maintenance. In *intra-cell maintenance*, *head shift* enables the highest ranked candidate to become the new head of a cell when the head of the cell fails or proactively becomes an associate when it is resource scarce or a candidate better serves as head; when the candidate set is weak (e.g. empty), *cell shift* enables the cell head to strengthen the candidate set by selecting a better *IL* for this cell if any such *IL* exists (described in Figure 5); *cell abandonment* enables nodes within a heavily perturbed cell to become an associate in one of its neighboring cells. In *inter-cell maintenance*, a parent head and its children heads monitor one another. If a head *h* leaves and the intra-cell maintenance in its cell fails, the parent of *h*, *P(h)*, tries to recover it first. If *P(h)* fails too, each child of *h* tries

We call a cell *C* formed in the initial phase of self-configuration an *original cell*, and the *IL* of *C* an *original ideal location (OIL)*. To maximize the lifetime of the hexagonal structure, for any original cell *C*, the union of its candidate sets of all the *IL*s should cover all nodes in *C*. Let $CA(IL_k)$ be the R_t-radius circular area centered at an ideal location IL_k. Then a cell can be divided into a set of such *CA*s as shown in the following figure, which is self-similar to a system being divided into a set of cells:



Analogous to "bands", we call each set of *CA*s of equal minimum distance to its *OIL* (in terms of *CA*s in between) an *Intra Cell Cycle (ICC)*. The set of *CA*s on the same *ICC* is numbered, called *Intra Cycle Position* (*ICP*), in an increasing order clockwise with respect to $\overrightarrow{GR}$ (for a certain *ICC*, the range for *ICP* is [0, 6×*ICP*-1]). Then the *IL*s in a cell can be lexicographically ordered by tuple <*ICC, ICP*>, and act as the "current *IL*" of a cell in an increasing order.

Figure 5: Method to change the *IL* of a cell

to find a new parent by themselves; also, a head chooses the neighboring head closest to the big node as its parent; an optional action is for a cell to synchronize its *IL* with that of its neighboring cells, which affects the tightness of cell radius with respect to *R* locally within its one-hop neighborhood.

Node state corruption is dealt with by "sanity checking". Periodically (with low frequency) each head *h* checks the hexagonal relation with its neighboring heads, according to the system invariant. If the invariant is violated, *h* asks its neighboring heads to check their state. If all its neighboring heads are valid, the state of *h* must be corrupted, and *h* becomes an associate; if some of its neighboring heads are invalid, *h* cannot decide whether it is valid at this moment, and will check this next time.

Algorithm modules Compared with GS^3-S, GS^3-D (described in Figure 6) has modified modules for head organization, new modules for node-join, intra-cell maintenance, inter-cell maintenance, and sanity checking.

Modified modules for head organization are as follows. In HEAD_ORG, executed by a head *i*, *i* maintains not only its children heads set, but also its neighboring heads set and candidates set. In HEAD_SELECT executed by a head *i*, *i* does not select head for a cell in its search region if there is an R_t-gap at the *IL* of the cell. In HEAD_ORG_RESP, executed by a head *i* in response to the HEAD_ORG at a head *j*, *i* sets *j* as its parent if *j* is better (e.g. closer to the big node) than its current parent.

Node-join consists of three modules: SMALL_NODE_BOOT_UP used by a bootup node trying to find a nearby head or associate; HEAD_JOIN_RESP and ASSOCIATE_JOIN_RESP used by a head or an associate respectively in response to the SMALL_NODE_BOOT_UP at a nearby bootup node, where it sends its state to the bootup node and listens to its decision to join or not.

Intra-cell maintenance consists of four modules: HEAD_INTRA_CELL, CANDIDATE_INTRA_CELL, ASSOCIATE_INTRA_CELL, and BIG_SLIDE.

In HEAD_INTRA_CELL, executed by a head *i*, it exchanges heartbeats with associates in its cell.[12] Head *i* becomes an associate when it is resource scarce, a candidate better serves as head, or the big node is in its cell and resumes its role as head. When the candidate set is weak, *i* strengthens it using the low-level module STRENGTHEN_CELL that implements the concept of *cell shift*. If its cell is heavily perturbed such that the hexagonal property within its neighborhood has deviated too much, *i* abandons its cell and transits to status *bootup*.

In CANDIDATE_INTRA_CELL, executed by a candidate *i*, *i* exchanges heartbeats with its head. When its head fails or becomes an associate, *i* coordinates with other candidates in its cell to elect a new head. When its head transits to status *bootup*, *i* transits to status *bootup* too. When a head *j* that is better than its current head shows up, *i* sets *j* as its new head.

[12] The frequency of heartbeat exchanges can be tuned to minimize the control overhead and to adapt to such network states as traffic and degree of dynamics in a network.

Program *Big_node*

GS³-S with modified HEAD_ORG;

[]

q = *work* → HEAD_JOIN_RESP; //Deals with node join; remains status *work*

[]

q = *work* → [HEAD_INTRA_CELL | HEAD_INTER_CELL]; //Deals with node leave:
//Remains status *work* or transits to status
// *big_slide*

[]

q = *big_slide* → BIG_SLIDE; //The big node does not act as head; remains status *big_slide* or transits
//to *work*

Program *Small_node*

GS³-S with modified HEAD_ORG and HEAD_ORG_RESP;

[]

q = *bootup* → SMALL_NODE_BOOT_UP; //Remains status *bootup* or transits to *associate* or *surrogate*
// *associate*

[]

/* *Head node* */

q = *work* → HEAD_JOIN_RESP; //Deals with node join; remains status *work*

[]

q = *work* → [HEAD_INTRA_CELL | HEAD_INTER_CELL]; //Deal with node leave;
//Remain status *work* or transits to
// *associate*

[]

q = *work* $\xrightarrow{Ts}$ SANITY_CHECK; //Sanity checking; remains status *work* or transits to
//*associate*

[]

/* *Associate node* */

(q = *associate* ∨ q = *candidate*) → ASSOCIATE_JOIN_RESP; //Deals with node join;
//Remains status *associate* or
//*candidate*

[]

q = *candidate* → CANDIDATE_INTRA_CELL; //Deal with node leave;
//Remain status *candidate,* or transits to *head*
//or *bootup*

[]

q = *associate* → ASSOCIATE_INTRA_CELL; //Deal with node leave;
//Remain status *associate,* or transits to *head*
//or *bootup*

Figure 6: Self-configuration algorithm for dynamic networks (GS³-D)

ASSOCIATE_INTRA_CELL executed by a non-candidate i is almost the same as CANDIDATE_INTRA_CELL except that i transits to status *bootup* when its head fails.

In BIG_SLIDE executed by the big node H_0, H_0 keeps the head in the coverage of its original cell as head, and resumes head role when the *OIL* of its cell becomes the current *IL*.

Inter-cell maintenance is implemented by the module HEAD_INTER_CELL. In HEAD_INTER_CELL, executed by a head i, i exchanges heartbeats with its neighboring cell heads. If a neighboring head j is closer to H_0 than its current parent, i sets j as its new parent. If a child j fails and the intra-cell maintenance at its cell fails too, i tries to deal with it using HEAD_ORG in the direction of j. If the parent of i (i.e. $P(i)$) fails, and the failure is not recovered by the intra-cell maintenance at $P(i)$'s cell or by $P(i)$'s parent, i tries to find a new parent using low-level module PARENT_SEEK. If i is a boundary cell head, it periodically checks, using HEAD_ORG, whether new nodes show up in the direction where it does not have a child. When a neighboring head, a child, or its parent changes *IL*, i optionally synchronizes its *IL* using low-level module SYN_CELL.

Sanity checking is implemented by the module SANITY_CHECK whose time complexity is $\theta(D_c)$, where D_c is the diameter of a contiguous state-corrupted area.

A more detailed description of the individual modules of GS^3-D is given in Appendix 2.

4.3 Analysis

New notation

Head Neighboring Graph: $G_{hn} = (V_{hn}, E_{hn})$, where $V_{hn} = V_h$ of the head graph G_h, and $E_{hn} = \{(i, j)$: i and j are neighboring heads$\}$.

4.3.1 Invariant

The invariant of GS^3-D is the same as that of GS^3-S except for the following (formal descriptions are given in Appendix 3):

- In $I_{2.1}$ and $I_{2.2}$, if the <*ICC, ICP*> value (see figure 5) of a head i is different from that of a neighboring head j, the distance between them is bounded by $[d - 2R_t, d + 2R_t]$, where d is the distance between *IL(i)* and *IL(j)*, and is bounded by $(0, 2\sqrt{3}R)$.
- In $I_{2.3}$, the number of children heads of a head other than the big node is at most 5.
- In $I_{2.4}$, the radius of an inner cell is bounded by $(0, 2R + R_t]$ if its <*ICC, ICP*> value is different from that of any of its neighboring cell; and $|R_{random}|$ is at most $(\sqrt{3} - 1)R + 2R_t + d_p$ for boundary cells, with d_p being the diameter of the R_t-gap perturbed area adjoining the boundary cell (d_p is 0 if there is no R_t-gap perturbed area).

Theorem 5: Let DI be SI (invariant of GS^3-S) with I_2 relaxed as above, then DI is an invariant of algorithm GS^3-D.

4.3.2 Fixpoint

The fixpoint of GS^3-D is the same as that of GS^3-S except for the following:

- $F_{1.2}$ is strengthened as: the head graph G_h is a minimum-distance (with respect to the big node H_0) spanning tree of the head neighboring graph G_{hn} rooted at H_0, i.e. the path between H_0 and a head i in G_h is a shortest path between H_0 and i in G_{hn}.
- $F_{2.4}$ is relaxed as: ($F_{2.4}$ of GS^3-S) $\wedge$ ($|R_{random}|$ is at most $2R_t/\sqrt{3}+d_p$ for boundary cells).

Theorem 6: Let DF be SF (fixpoint of GS^3-S) with $F_{1.2}$ and $F_{2.4}$ updated as above, then DF is a fixpoint of algorithm GS^3-D.

F_1, F_2, F_3, and F_4 imply

Corollary 5: At a state in DF, Corollary 1, 2, and 3 hold in dynamic networks.

4.3.3 Self-stabilization

Theorem 7: Starting at an arbitrary state, every computation of GS^3-D reaches a state in DI within time O(D_c), where D_c is the diameter of a contiguous state-corrupted area.

Theorem 8: Starting at an arbitrary state in DI, every computation of GS^3-D reaches a state in DF within time $O(\max\{D_d/c_1, T_d\})$, where D_d is the geographic diameter of the network, c_1 is the average speed of message diffusing, and T_d is the maximum difference between the lifetime of the candidate sets of two neighboring cells.

Theorem 7 and 8 imply

Corollary 6: Starting at an arbitrary state, every computation of GS^3-D reaches a state in DF within time $O(\max\{D_d/c_1, T_d\})$.

Requirement *e)* in the problem statement is satisfied by Theorem 7 and 8.

4.3.4 Statistically low deviation from ideal hexagonal structure

Of course, R_t-gaps may always exist in networks, and this implies the potential existence of non-ideal cells that are not hexagonal. If the *IL* of a cell *C* in the ideal virtual structure (as shown in Figure 1) lies in an R_t-gap, then every node in the geographic coverage of *C* joins some neighboring cell *C'* of *C* in the self-configured cell structure, which makes *C'* assume a shape other than the "ideal" hexagon. Moreover, due to the existence of R_t-gaps, the radius of such non-ideal cells as *C'* depends on the diameter of the R_t-gap perturbed region (i.e. the set of contiguous R_t-gap perturbed cells such as *C*) adjoining it. However, as shown below, the number of non-ideal cells and the diameter of R_t-gap perturbed regions are small due to dense node distribution in wireless sensor networks.

We assume that nodes are uniformly distributed such that the average number of nodes within any circular area of radius 1 is λ. Let m_0 be the number of nodes within any circular area of radius 1, then m_0 is a Poisson random variable with probability distribution function

$$P_{m_0}(k,\lambda)=e^{-\lambda}\frac{\lambda^k}{k!},$$

where $P_{m_0}(k,\lambda)$ is the probability that $m_0 = k$. Then the number of nodes m_t in any circular area of radius R_t is a Poisson random variable with probability distribution function

$$P_{m_t}(k,\lambda,R_t)=e^{-R_t^2\lambda}\frac{\left(R_t^2\lambda\right)^k}{k!},$$

where $P_{m_t}(k,\lambda,R_t)$ is the probability that $m_t = k$. Thus, the probability α that there exists no node in an area of radius R_t is $e^{-R_t^2\lambda}$.

Therefore, if there are n cells in the ideal virtual structure, the expected number of non-ideal cells G_e after configuration is

$$\sum_{k=0}^{n} k\cdot\binom{n}{k}\cdot(\alpha)^k\cdot(1-\alpha)^{n-k}=n\alpha$$

and the expected ratio of non-ideal cells in the system is G_e/n which is α. Moreover, the expected diameter of an R_t-gap perturbed region is

$$2R\sum_{k=0}^{\infty}k\cdot\alpha^k=\frac{2\alpha}{(1-\alpha)^2}R$$

For example, in a system of radius 1000, if R=100 and λ = 10, the expected ratio of non-ideal cells as a function of R_t/R is shown in Figure 7. The ratio is small and converges to 0 quickly as R_t/R increases. The expected diameter of an R_t-gap perturbed region is shown in Figure 8. The expected diameter is also small and converges to 0 quickly as R_t/R increases. From Figure 7 and 8, we see that both the expected ratio of non-ideal cells and the expected diameter of an R_t-gap perturbed region are proximately 0 once R_t/R is greater than or equal to 0.02.

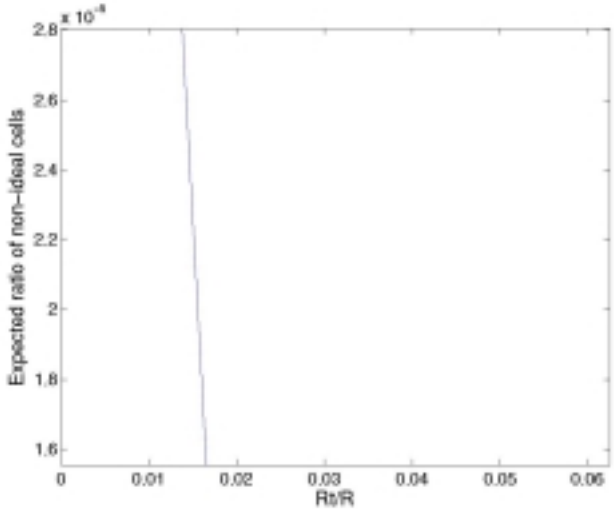


Figure 7: The expected ratio of non-ideal cells, when λ = 10

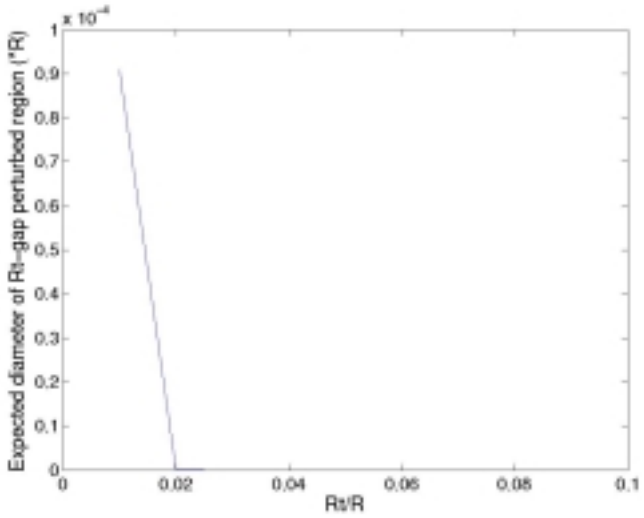


Figure 8: The expected diameter of an R_t-gap perturbed region, when $\lambda = 10$

4.3.5 Stability and scalability

4.3.5.1 Stable cell structure

In the presence of dynamic nodes, the cell structure is stable in the following senses: 1) In case of *node join*, the cell structure remains unchanged except for the possibility that the head of some cell is replaced by a new node if the new node better serves as head; 2) *Node leave* within a cell is masked within the cell by head shift such that the rest of the structure remains unchanged; 3) In case of *node death* such that candidate sets of many cells die, independent cell shift at each cell enables the head level structure to slide as a whole but maintain consistent relative location among cells and heads, which lengthens the lifetime of the structure by a factor of $\Omega(n_c)$, where n_c is the number of nodes in a cell; 4) In case *intra-cell maintenance fail*s, inter-cell maintenance enables a system to stabilize to its stable state within a one-way message diffusing time across the perturbed area; 5) In case of *state corruption*, sanity checking ensures that the erroneous state is corrected by checking the hexagonal properties among heads.

4.3.5.2 Scalable self-healing

The self-healing of the head graph and hexagonal structure is scalable in three senses: 1) *local self-healing* enables the system to stabilize from a perturbed state to its stable state in a one-way message diffusing time across the perturbed area through local coordination among nodes within $\sqrt{3}R + 2R_t$ distance from one another; 2) *local knowledge* enables each node to maintain the identities of only a constant number of nodes within $\sqrt{3}R + 2R_t$ distance, irrespective of network size; 3) the head graph and hexagonal structure can *tolerate multiple simultaneous perturbations* due to the locality property of GS^3-D.

5. MOBILE DYNAMIC NETWORK

5.1 Concepts

Recall that, in mobile dynamic networks, not only can nodes be dynamic, they can also move. The probability of movement is inversely related to the distance of movement. In this section, we extend GS^3-D to GS^3-M to deal with node mobility.

Conceptually, node mobility is modeled as a correlated node join (at the new location) and leave (from the old location). GS^3-D is easily adapted to deal with the mobility of small nodes (more detailed description is given in Appendix 2). Thus, we focus on how to deal with big node movements.

In mobile dynamic networks, the head graph needs to be maintained such that, in spite of the movement of the big node H_0, it is connected and the path between H_0 and every head is of minimum distance. To achieve this, the closest head to H_0 in the network acts as the *proxy* for H_0 during the time when H_0 is not a head, and the distance from the proxy to H_0 is set as 0. Then, just by algorithm GS^3-D, the head graph can be maintained as a minimum distance tree to the proxy, and thus every head is of minimum hops to H_0. Moreover, the impact of the movement of H_0 on the head graph is contained within a local range of radius $\sqrt{3}d/2$, where d is the distance that H_0 moves.

5.2 Algorithm

Overview In mobile dynamic networks, if the big node H_0 moves more than R_t away from the *IL* of its cell, it retreats from the head role, and transits to status *big_move* where it moves around and maintains a proxy-relationship to its proxy. Whenever H_0 moves within R_t distance to the *IL* of a cell later, it replaces the existing head of the cell to act as head.

Algorithm modules Compared with GS^3-D, GS^3-M (described in Figure 9) has a new module BIG_MOVE, modified modules for big node, intra-cell maintenance, and inter-cell maintenance. (A more detailed description is given in Appendix 2.)

Program *Big_node*
GS^3-D with removed BIG_SLIDE, modified intra-cell as well as inter-cell maintenance modules;
[]
q=*big_move* → BIG_MOVE; //The big node moves; remains status *big_move* or transits to *head*

Program *Small_node*
GS^3-D with modified intra-cell as well inter-cell maintenance modules;

Figure 9: Self-configuration algorithm for dynamic mobile networks (GS^3-M)

5.3 Analysis

5.3.1 Invariant and Fixpoint

The invariant as well as fixpoint of GS^3-D is preserved in GS^3-M, except for one more fixpoint predicate F_5 for GS^3-M as follows.

F_5 (Proxy optimality): The big node chooses the closest neighboring head as its proxy.

Theorem 9: Let MI be DI (invariant of GS^3-D), then MI is an invariant of algorithm GS^3-M.

Theorem 10: Let MF be DF (fixpoint of GS^3-D) $\wedge$ F_5, then MF is a fixpoint of algorithm GS^3-M.

5.3.2 Self-stabilization

Theorem 11: When the big node moves from point A to B on a plane, its impact on the head graph G_h is contained within a circular area centered at point C and of radius $\sqrt{3}d/2$, where C is the midpoint of segment $\overline{AB}$ and d is the cartesian distance between A and B.

Theorem 12: Starting at an arbitrary state, every computation of GS^3-M reaches a state in MI within time $O(D_c)$, where D_c is the diameter of a contiguous state-corrupted area.

Theorem 13: Starting at an arbitrary state in MI, every computation of GS^3-M reaches a state in MF within time $O(\max\{D_d/c_1, T_d\})$, where D_d is the diameter of the network, c_1 is the average speed of message diffusing, and T_d is the maximum difference between the lifetime of the candidate sets of two neighboring cells.

Theorem 12 and 13 imply

Corollary 7: Starting at an arbitrary state, every computation of GS^3-M reaches a state in MF within time $O(\max\{D_d/c_1, T_d\})$.

5.3.3 System stability

In mobile dynamic networks, node mobility is dealt as a special kind of node dynamics. So the stability property of the head level structure and head graph in dynamic networks is preserved in mobile dynamic networks. The invariant and fixpoint of GS^3-M only depend on local coordination, which enables them to tolerate high degree of node mobility because local coordination converges fast.

6. RELATED WORK

In [10], a distributed algorithm LEACH is proposed for clustering in wireless sensor networks. But, as mentioned by its authors, LEACH guarantees neither the placement nor the number of clusters in a system, and perturbations are dealt with by globally repeating the clustering operation, which is not scalable. In [3], another algorithm for clustering is designed, but it only considers logical radius of clusters instead of their geographic radius, which can reduce wireless transmission efficiency because of large geographical overlap between clusters [8]. The radius difference among clusters can be large too. Moreover, its healing procedure is not local, because the healing there depends on multiple rounds of message diffusing across the whole system, instead of a one-way diffusing just within perturbed areas as in our algorithm. And, given a certain density of node distribution, the geographic radius ensured by our algorithm implicitly guarantees a bound on the logical radius of clusters, but not vice versa. In

[12], an access-based clustering algorithm is presented that focuses on the stability of clusters, but the algorithm does not consider the size of clusters and it requires GPS at every node.

In [16], a cellular hexagonal structure is described for cellular networks, but it is pre-configured and there is no ability of self-healing. In [6, 18], different algorithms for topology control in networks are developed, but they are either centralized or semi-centralized, and thus are not scalable.

In [15, 17, 20], algorithms for topology control in wireless sensor networks for energy saving are developed. In [22], adaptive fidelity control and routing algorithms are developed for wireless sensor networks. Our self-configuration algorithm provides a stable network infrastructure for tasks such as routing or power control, and thus is orthogonal to these works.

In [14], self-stabilizing algorithms are proposed that mend faults locally in time, but they are not local in space. [1] proposes self-stabilizing algorithms for tree maintenance that is local in space but not local in time. The self-stabilization in GS^3 is local both in time and in space.

7. CONCLUSION

In this paper, we have presented an algorithm (GS^3) for self-configuring a network into cells of tightly bounded geographic radius and low overlap between cells. GS^3 enables network nodes to organize themselves into a cellular hexagonal structure with a set of proved properties. The structure configured by GS^3 is self-healing, thus GS^3 is applicable to both static networks and networks with dynamic as well as mobile nodes. Moreover, the self-healing is local, which makes GS^3 applicable to networks with high degree of dynamics and mobility. GS^3 is also scalable due to its properties of local knowledge, local self-healing, and local coordination. GS^3 yields a stable structure even in the presence of dynamic and mobile nodes, which enables a more stable as well as available infrastructure for other network services such as routing, power control, and QoS.

GS^3 is readily extended to the following cases: 1) in a mobile dynamic network where there are multiple big nodes, GS^3 enables each small node to choose the best (e.g. closest) big node to communicate, by letting each small node maintain the current big node it chooses. 2) Due to its locality property, GS^3 is also applicable to the case where nodes are not deployed on an exact 2D plane, but nodes within each neighborhood (e.g. a circular area of radius R) are locally planar. 3) GS^3 is also applicable to the case where the ideal cell radius R is larger than the maximum transmission range of small nodes, because R does not affect the correctness of the algorithm.

In the paper, we have discussed local self-healing in GS^3, but we have not studied in detail how to deal with different degrees of node dynamics and mobility. This is a subject of future work. Moreover, the tightness of the bound on cluster radius in GS^3 reflects the density of node distribution in a network, and we plan to study how to incorporate other properties such as network traffic characteristics in the bound.

GS^3 takes advantage of such model properties of wireless sensor networks as dense node distribution, relative location information among nodes, and the existence of big nodes to solve the problem of scalable self-configuration and self-healing. We believe these model properties can be exploited in a richer class of problems in wireless sensor networks and deserve further exploration.

8. REFERENCES

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APPENDIX

In the appendix, we present the complexity and convergence properties of GS^3, detailed description of modules in GS^3-S, GS^3-D and GS^3-M, the invariant as well as fixpoint of GS^3-D, and proofs for theorems in the paper.

Appendix 1: Complexity and convergence properties of GS^3

Information maintained at each node	$\theta(log\ n)$
Factor of *lengthened lifetime* of head level structure by intra-cell & inter-cell maintenance	$\Omega(n_c)$
Convergence time under *perturbations*	$O(D_p)$
Convergence time to the stable state in *static networks*	$\theta(D_b)$
Convergence time from an arbitrary state to the stable state in *dynamic/mobile networks*	$O(D_d)$

n: the number of nodes in a system;
n_c: the number of nodes in a cell;
D_p: the diameter of a contiguous perturbed area;
D_b: $\max\{dist(H_0, i)$: i is a small node, and $dist(H_0, i)$ is the cartesian distance between the big node H_0 and $i\}$;
D_d: the diameter of the system, i.e. $\max\{dist(i, j)$: i and j are small nodes, and $dist(i, j)$ is the cartesian distance between i and $j\}$.

Appendix 2: Description of modules in GS^3-S, GS^3-D and GS^3-M

In this subsection, we give more detailed description of some algorithm modules in GS^3-S, GS^3-D and GS^3-M as follows. The complete program is presented in [23].

1) Algorithm GS^3-S

a) HEAD_ORG (LD, RD, R, R_t)

There are four arguments to HEAD_ORG: 1) L direction (LD) and R direction (RD) with respect to direction $\overrightarrow{P(i),i}$ (see Figure 3). LD and RD determine the search region of a head in the process of organizing its neighboring cell heads. 2) ideal radius R and radius tolerance R_t.

The function of HEAD_ORG executed by a head *i* is for head *i* to organize the neighboring cell heads in its search region. HEAD_ORG executed by head *i* works as follows: first, head i reserves wireless channel and broadcasts message *org* within $\sqrt{3}R + 2R_t$ distance; second, head i listens to replies (message *org_reply* or *head_org_reply*) from nodes no more than $\sqrt{3}R + 2R_t$ away and within (LD, RD) search region for certain amount of time and calculates the set of small nodes and head nodes (*SmallNodes* and *ExistingHeads* respectively) in the search region; Third, using the low level module HEAD_SELECT (see Figure 3), head i selects neighboring cell heads *HeadSet*; fourth, head i broadcasts message ⟨*HeadSet*⟩ to nodes within $\sqrt{3}R + 2R_t$ distance, revokes channel reservation, and transits to status *work*.

In HEAD_SELECT executed by head *i*, head i needs to select neighboring cell heads in its search region. It achieves this in two steps: first, it calculates the ideal locations for those possible neighboring cell heads; second, for each possible neighboring cell, if there is any small node that is in the R_t-radius circular area centered by the ideal location of the cell, select the highest ranked such node as the cell head. The algorithm is described in Figure 3 and its time complexity is $\theta(|SmallNodes|)$.

b) HEAD_ORG_RESP

When a head node i (at status *head* or *work*, and not including the big node) receives a message *org* from a head *j*, it replies with a message *head_org_reply*, and waits until head j's HEAD_ORG process finishes (by overhearing its message ⟨*HeadSet*⟩). No status transition in this module.

c) ASSOCIATE_ORG_RESP

When a small node i is at status *bootup* or *associate*, it will execute ASSOCIATE_ORG_RESP process upon receiving a message *org* from a head j. If node i is at status *bootup* or status *associate* but head *j* is better (such as closer, with higher remaining energy) than its current head H(i), node i replies a message *org_reply* to head j. Then waits for head j's message ⟨*HeadSet*⟩. If node i is selected as a cell head, it sets head *j* as its parent head, and transits to status *head*; otherwise, node i sets head *j* as its head, and transits to status *associate.* On the other hand, if node i fails to hear the message ⟨*HeadSet*⟩ from head j after a certain amount of time, it transits back to its status at the beginning of the process (i.e. *bootup* or *associate*).

2) Algorithm GS^3-D

Intra-cell maintenance

a) HEAD_INTRA_CELL

In HEAD_INTRA_CELL executed by a head *i*, head *i* executes the following actions:

i. It periodically broadcasts message *head_intra_alive* within its cell, and updates its candidate as well as associate set according to replies from the associates in its cell.

ii. If head i receives a message *associate_alive* or *associate_retreat* from an associate, it needs to update candidate as well as associate set properly.
iii. If i is resource scarce or a candidate better serves as head, i broadcast a message *head_retreat* within its cell and retreats back to be an associate.
iv. If i receives message replacing_head from the big node H_0 or a head candidate j, it retreats to be an associate, and sets H_0 or j as its head.
v. If the candidate set of its cell is weak, i calls STRENGTHEN_CELL to strengthen it.
vi. If the distance IL of its cell that of all its neighboring cells deviates too much from $\sqrt{3}R$, exceeding certain threshold T_d, it abandons the cell by broadcasting a message *cell_abandoned* within its cell and transiting to status *bootup*.

In STRENGTHEN_CELL, head *i* first finds the next ideal location (IL) of its cell whose corresponding candidate set is not empty, according to the cell's current <ICC, ICP> value and the ordering of all ILs in its cell (see Figure 5). Then it calculates the new candidate set with respect to the new IL. Last, it broadcasts two messages (*head_intra_alive* containing the new candidate set, and *head_retreat*) within its cell, and retreats to be an associate. Time complexity is $O(n_c)$, where n_c is the number of nodes in a cell.

b) CANDIDATE_INTRA_CELL

In CANDIDATE_INTRA_CELL executed by a candidate *i*, i executes the following actions:

i. Upon receiving a message *head_intra_alive* from a head j: if j is its head, i checks whether it is still in j's candidate set, and transits to status *associate* if not; otherwise, replies a *head_intra_ack* message. If j is not its head and is better than its current head, i sends an *associate_retreat* message to its current head and *associate_alive* message to head j.
ii. If i receives a message *head_retreat* from or detects the failure of its current head, it coordinates with other candidates in this cell to elect the highest ranked candidate as the new head. The head candidates in a cell are ranked in the same way as that in HEAD_SELECT (see Section 3).
iii. If i receives a message *cell_abandoned*, *head_retreat_corrupted*, *head_disconnected,* or *syn_cell* from its head, it transits back to boot up status.

Inter-cell maintenance

a) HEAD_INTER_CELL

In HEAD_INTER_CELL executed by a head *i*, head i executes the following actions:

i. Periodically broadcasts message *head_inter_alive* as heartbeat to its parent as well children heads.
ii. Upon receiving a message *head_inter_alive* from head j, update children set, and neighboring head set properly. If j is not i's parent head but is better (closer to the big node, for example) than its current parent head, i sets j as parent head, and sends a message *new_child_head* to j.

iii. If i receives a message *new_child_head* from j, update children heads set as well neighboring heads set accordingly.

iv. If a neighboring cell C_n (including child as well as parent cell) has a new head due to intra-cell maintenance, i updates neighboring head set, children head set, or parent head accordingly. If C_n has a newer <ICC, ICP> value, head i synchronizes its cell to the new <ICC, ICP> by calling SYN_CELL process (this is optional).

v. If i receives a *syn_cell* message from a neighboring cell's head j, it updates (remove j) neighboring head and child head sets accordingly. If j is i's parent head, i executes PARENT_SEEK to find a new parent head. If *syn_cell* message carries a newer <ICC, ICP> value, i executes SYN_CELL.

vi. If i is a boundary head and there is no head at certain neighboring cell area in its search region, it periodically executes HEAD_ORG to check whether new nodes have shown up in this direction.

vii. If a child head j fails, i executes HEAD_ORG in j's direction, trying to organize a new head.

viii. If i's parent head P(i) fails, and P(i)'s failure has not been recovered by P(i)'s parent head, i executes to PARENT_SEEK. If i receives a message *parent_seek* from a head j and they don't have the same parent head, it replies a *parent_seek_ack* message.

ix. If i receives a message *sanity_check_req* from a neighboring head j, it checks its own status. If its status is valid, i replies a message sanity_check_valid message to j; otherwise, i executes SANITY_CHECK.

x. If i receives a *head_retreat_corrupted* message from a neighboring cell's head j, it updates (remove j) its neighboring head set and children head sets accordingly. If j is i's parent head, i executes PARENT_SEEK.

In SYN_CELL, head i first calculates the new IL with respect to the new <ICC, ICP> value. Then it calculates the candidate set corresponding to this IL. If the candidate set is not empty, i broadcasts a message *head_retreat* within its cell; otherwise, it broadcasts a message *syn_cell* to its neighboring heads that includes the current <ICC, ICP> value. Last, i transits to status *big_slide* if it is the big node or status *associate* otherwise. Time complexity is O(C), where C is a constant.

In PARENT_SEEK, let ST denote the sub-tree of G_h rooted at head i. Head i ranks its neighboring heads in almost the same way as that in HEAD_SELECT, except that $\overrightarrow{i,P(i)}$ instead of $\overrightarrow{GR}$ is used as reference direction. Then i tries to find a neighboring head as parent head in an increasing order. If it succeeds in finding such a head j, i sets j as its parent; otherwise i lets its children heads on the boundary of ST's geographic coverage try to find a new parent head in the same way. If any of its child head j succeeds, i sets j as its parent; otherwise i broadcasts a message *head_disconnected* within its cell, and transits back to boot up status. Its time complexity is O(|FNH|), where FNH denotes the set of head in (G_h-ST) that has a neighboring head in ST.

b) ASSOCIATE_INTER_CELL

If an associate (including both candidate and non-candidate) receives a message *org*, it calls ASSOCIATE_ORG_RESP.

Sanity checking

In order to deal with status corruption, every head periodically executes SANITY_CHECK. In SANITY_CHECK executed by head *i*, it first checks if its <ICC, ICP> value is equal to that of all its neighboring cells. If yes, it checks whether its status satisfies the hexagonal relationship of the system invariant. If no, it broadcasts a message *sanity_check_req*, and waits for replies from its neighboring cells' heads. If all its neighboring cells' heads reply a message *sanity_check_valid*, head i broadcasts a message *head_retreat_corrupted* within its cell. If it has not got the message *sanity_check_valid* from any of its neighboring cells after certain amount of time, head i exit this module without changing its status. Time complexity is $\theta(A)$, where A denotes the size of the contiguously affected area.

3) Algorithm GS^3-M

BIG_MOVE

In BIG_MOVE, the big node keeps listening to heartbeats (*head_intra_alive* message) from all nearby heads, and always chooses the best (closest, for example) head as its proxy. When its proxy is replaced by a candidate h_n in the proxy's cell, the big node reset its proxy as h_n. When the big node moves into the R_t-radius circular area of a cell, it replaces the existing head as head, and transits back from status *big_move* to status *work*.

Modified intra-cell and inter-cell maintenance

The modification to the intra-cell as well as inter-cell maintenance is to maintain the cell head, candidate set, and big node's proxy relationship in the presence of mobile nodes. As for big node, if it retreats from the head role because of the IL change of any of its neighboring cells, it transits to status *big_move* instead of *big_slide* in dynamic mobile networks.

Appendix 3: Invariant and fixpoint of GS^3-D in dynamic networks

Notation

Visible node: a node that is connected to the big node H_0 in V_p

Neighboring_heads(i): { j: j is a head $\wedge$ (head i and j's geographic coverage adjoins}

Dist(i, j): cartesian distance between nodes i and j

H(i): the head of the cell that the associate node i is in

1) Invariant

The invariant of GS^3-D differs from that of GS^3-S at I_2 when a cell and its neighboring cells have different <ICC, ICP> values.

- **I_1** (connectivity)

Same as in static networks.

- **I_2** (Hexagonal structure)
  - $I_{2.1}$: (for inner heads)

$I_{2.1}$ *for static networks* $\wedge$
($\forall$ inner_head i: $\forall$j $\in$ neighboring_heads(i):
<ICC(i),ICP(i)> $\neq$ <ICC(j),ICP(j)> $\Rightarrow$ ((dist(IL(i),IL(j))-2Rt $\leq$ dist(i, j) $\leq$ dist(IL(i),IL(j))+2Rt) $\wedge$ (0 < dist(IL(i), IL(j)) $\leq 2\sqrt{3}$ R))
)

  - $I_{2.2}$: (for boundary heads)

$I_{2.2}$ *for static networks* $\wedge$
($\forall$ boundary_head i: $\forall$j $\in$ neighboring_heads(i):
<ICC(i),ICP(i)> $\neq$ <ICC(j),ICP(j)> $\Rightarrow$ ((dist(IL(i),IL(j))-2Rt $\leq$ dist(i, j) $\leq$ dist(IL(i),IL(j))+2Rt) $\wedge$ (0 < dist(IL(i), IL(j)) $\leq 2\sqrt{3}$ R))
)

  - $I_{2.3}$: modify $I_{2.3}$ *for static networks* by changing

($\forall$ head i: | CH(i) | $\leq$ 3) to *($\forall$ head i: | CH(i) | $\leq$ 5)*

  - $I_{2.4}$: (cell radius)

$I_{2.2}$ *for static networks* $\wedge$
($\forall$ inner cell C: ($\exists$j$\in$ neighboring_heads(i):<ICC(i),ICP(i)> $\neq$ <ICC(j),ICP(j)>)$\Rightarrow$ ($\forall$ associate i $\in$ C : dist(i, H(i)) < 2R+Rt)) $\wedge$
($\forall$ boundary cell *C'*: $\forall$associate i $\in$ *C'*: dist(i, H(i)) $\leq \sqrt{3}$ R+2R_t+d_p)

- **I_3** (Inner cell optimality)

Same as in static networks.

2) Fix Point

The fixpoint of GS^3-D differs from that of GS^3-S at $F_{1.2}$ that is strengthened in GS^3-D.

- **F_1** (connectivity)
  - $F_{1.1}$: Same as in static networks
  - $F_{1.2}$: G_h is a minimum-distance (with respect to the big node H_0) spanning tree of G_{hn}, and G_h is rooted at H_0.

$F_{1.2}$ *for static networks* $\wedge$ ($\forall$ $v_i \in (V_h - \{H_0\})$: hops(H0,v_i) = MIN(H_0, v_i)),
where MIN(v_1, v_2) is the length (by hops) of the shortest path between v_1 and v_2 in G_{hn}.

- **F_2** (hexagonal structure)

$F_{2.1}$, $F_{2.2}$, and $F_{2.3}$ are the same as in static networks.

$F_{2.4}$ is relaxed as: ($F_{2.4}$ of GS^3-S) $\wedge$ ($|R_{random}|$ is at most (($\sqrt{3}$ -1)R+2R_t+d_p) for boundary cells)

- **F_3** (cell optimality): Same as in static networks.
- **F_4** (coverage): Same as in static networks.

Appendix 4: Proofs for theorems

For simplicity, we only present proofs for some theorems in static networks, and the complete set of proofs for all the theorems in the paper is relegated to [23].

1) Theorem 1

SI is an invariant of algorithm GS^3_S, where $SI = I_1 \wedge I_2 \wedge I_3$.

- **I_1**: Connectivity (safety property of head level graph)
  - *$I_{1.1}$:* Any pair of heads that are connected in G_h are also connected in G_p, and vice versa.
    ($\forall$ v_{h1}, $v_{h2} \in V_h$: there is a path between v_{h1} and v_{h2} in $G_h \Leftrightarrow$ there is a path between v_{h1} and v_{h2} in G_p)

Proof:

G_p only depends on the nodes in the system and their communication capability, thus has nothing to do with the program actions. G_h only depends on the set of head nodes in the system and the parent-child relationship among them. Thus the set of actions that are related to G_h are those of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP. At the same time, HEAD_ORG_RESP and ASSOCIATE_ORG_RESP operate under the control of HEAD_ORG, so the critical module is HEAD_ORG.

In order to prove this invariant, we only need to prove it is closed under a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP. The proof is as follows:

1) Suppose $G_h'(V_h', E_h')$ is the G_h before a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP. G_h' and G_p satisfy $I_{1.1}$;
2) After a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, G_h becomes $G_{h2}(V_{h2}, E_{h2})$.

   $\Rightarrow$:

   Case one: V_h' is empty

   If V_h' is empty, G_{h2} would be such that V_{h2} is the set composed of the big node H_0 and its children heads generated in HEAD_ORG process and E_{h2} is the set of edges that goes from the big node to its children heads. By the process HEAD_ORG, the big node and its children heads are within transmission range of one another and they are at most ($\sqrt{3}R+2R_t$) away from each other. Thus the big node and its children heads must be directly connected in G_p.

   So we only need to prove that any two different heads h1 and h2 are connected in G_p. And this is obvious because both h1 and h2 are connected to H_0 in G_p.

   So the claim holds in this case.

   Case two: V_h' is not empty

If V_h' is not empty, there must be a head h1 in V_h' such that $V_{h2} = V_h' \cup CH(h1)$ and $E_{h2} = E_h' \cup \{(h1, j): j \in CH(h1)\}$. By the proof of case one, we can easily know that the claim holds for any two nodes that are in the set of $\{h1\} \cup CH(h1)$.

So we only need to prove the claim between a node $h2 \in (V_h' - \{h1\})$ and a node $h3 \in CH(h1)$. If the set $(V_h' - \{h1\})$ is empty, the claim trivially holds. If the set $(V_h' - \{h1\})$ is not empty, then there must be a path p1 between h2 and h1 in G_p and a path p2 (actually just one hope edge) between h3 and h1. So there must be a path p3 between h2 and h3 and p3 is the concatenation of p1 and p2 by head node h1.

So the claim holds in this case.

⇐:

By $I_{1.2}$, G_h is a tree, thus any two heads h1 and h2 are connected in G_h and there would always be a path between then in G_h. So this claim trivially holds.

Thus, after a round of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, G_h and G_p still satisfy $I_{1.1.}$

❑

- *$I_{1.2}$:* G_h is a tree rooted at the big node H_0. That is,

  $(hops(H_0) = 0) \wedge (P(H_0) = H_0) \wedge$

  $(\forall\ v_i \in (V_h - \{H_0\})$: (there is a path between v_i and H_0) $\Rightarrow$ $(hops(H_0, v_i) = hops(H_0, P(v_i))+1) \wedge$

  $(\forall\ v_i, v_j \in V_h$: there is a path between v_i and v_j in $G_h) \wedge$

  $(\forall\ v_i, v_j \in V_h$: there is a path of length no fewer than 2 between v_i and $v_j \Rightarrow (P(v_i) \neq v_j \wedge P(v_j) \neq v_i))$,

  where $hops(v_1, v_2)$ denotes the length of the path from v_1 to v_2 in G_h.

Proof:

Same as the analysis in the proof of $I_{1.1}$, the modules that can affect this invariant are HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP.

1) Suppose $G_h'(V_h', E_h')$ is the G_h before a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, and G_h' satisfies $I_{1.2}$;
2) After a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, G_h becomes $G_{h2}(V_{h2}, E_{h2})$.

   Case one: V_h' is empty

   If V_h' is empty, G_{h2} would be such that V_{h2} is the set composed of the big node H_0 and its children heads ($CH(H_0)$) generated in HEAD_ORG process and E_{h2} is the set of edges that goes from H_0 to nodes in $CH(H_0)$. By the way HEAD_ORG works, for any node $h \in CH(H_0)$, hops(h) would be 1. For any two different heads $h1, h2 \in CH(H_0)$, $(P(h1) \neq h2 \wedge P(h2) \neq h1))$ must hold.

   So the claim holds in this case.

   Case two: V_h' is not empty

If V_h' is not empty, there must be a head h1 in V_h' such that $V_{h2} = V_h' \cup CH(h1)$ and $E_{h2} = E_h' \cup \{(h1, j): j \in CH(h1)\}$. By the proof of case one, we could easily know that the claim holds for the set of heads of $\{h1\} \cup CH(h1)$. So we only need to prove the claim between a node $h2 \in (V_h' - \{h1\})$ and a node $h3 \in CH(h1)$.

If the set $(V_h' - \{h1\})$ is empty, the claim trivially holds.

If the set $(V_h' - \{h1\})$ is not empty, then there must be a path p1 between h2 and h1 in G_h and a path p2 (actually just one hope edge) between h3 and h1. So there must be a path p3 between h2 and h3 in G_h and p3 is the concatenation of p1 and p2 by head h1. At the same time, h2 must have a parent head $P(h2) \in V_h'$, P(h3) is h1 that is different from h2, and $h3 \notin V_h'$. So $(P(h3) \neq h2 \wedge P(h2) \neq h3))$ must hold.

So the claim holds in this case.

Thus, after a round of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, G_h and G_p still satisfy $I_{1.2.}$

❑

- **I_2**: Hexagonal map of heads and inner cells
- *$I_{2.1}$:* Each inner cell head *i* has exactly 6 neighboring heads that form a cellular hexagon centered at *i* and of edge length $\sqrt{3}R$, with vertices′ location deviation at most R_t. That is, the distance between neighboring heads is bounded by $\left[\sqrt{3}R - 2R_t, \sqrt{3}R + 2R_t\right]$.

($\forall$ inner cell head i:

(| neighboring_heads(i) | = 6) $\wedge$ ($\forall$j$\in$ neighboring_heads(i): $\sqrt{3}$ R-2R_t $\leq$ dist(i, j) $\leq$ $\sqrt{3}$ R+2R_t

)

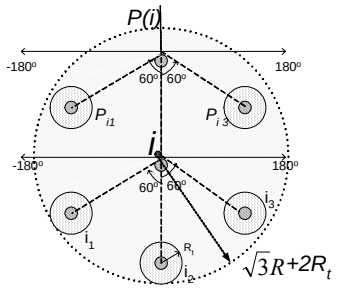
)

Proof:

Same as the analysis in the proof of I_1, the modules that can affect $I_{2.1}$ are HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP.

Suppose $I_{2.1}$ holds before a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP execution. We just need to prove that after the execution of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, initiated by a head i that executes HEAD_ORG, $I_{2.1}$ still holds. Because this round of head organization will only affect head i and its children heads i_1, i_2, i_3, we only need to prove that the $I_{2.1}$ holds for head *i*, i_1, i_2, and i_3. Let's first consider head i.

1) head i is an inner_head $\Rightarrow$ | neighboring_heads(i) | = 6
   a) if head i is the big node, this claim holds obviously just by the way the process HEAD_ORG works;
   b) if head i is a small head node, we can get the picture below from the design of HEAD_ORG and the program of Big Node and Small Node.



From the picture above, we can see that head i has 3 next-band heads (i_1, i_2, i_3), 1 parent head (P(i)), and 2 neighboring heads (p_{i1}, p_{i3}) at the same band that are under the care of the same parent head as head *i*, even though they might not be generated by P(i). Thus node i has 6 neighboring heads around within ($\sqrt{3}$ R+2R_t) radius.

Also it is easy to see that head almost centers the hexagon formed by its 6 neighboring heads, with possible deviation at most R_t.

2) head i is an inner_head $\Rightarrow$ ($\forall$j $\in$ neighboring_heads(i): $\sqrt{3}$ R-2R_t) $\le$ dist(i, j) $\le$ $\sqrt{3}$ R+2 R_t)

From HEAD_ORG and the picture above, we can see that: for all neighboring head j of node *i*, dist(IL(i), IL(j)) = $\sqrt{3}$ R. At the same time "dist(k, IL(k)) $\le R_t$" holds for any head k, thus "$\sqrt{3}$ R-2 $R_t \le$ dist(i, j) $\le$ $\sqrt{3}$ R+2 R_t" holds too.

As for head i_1, i_2, and i_3, we can prove, in the same way as above for head *i*, that $I_{2.1}$ also holds for them.

Thus $I_{2.1}$ still holds after a round of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP execution.

❑

- $I_{2.2}$: Each boundary cell head has less than 6 neighboring heads, and the distance between neighboring heads is bounded by $\left\lfloor \sqrt{3}R - 2R_t, \sqrt{3}R + 2R_t \right\rfloor$. That is,

($\forall$ boundary_head i: | neighboring_heads(i) | < 6) $\wedge$

($\forall$ boundary_head i: ($\forall$j $\in$ neighboring_heads(i): $\sqrt{3}$ R-2 $R_t \le$ dist(i, j) $\le$ $\sqrt{3}$ R+2 R_t)

Proof:

Since we have proved that $I_{2.1}$ is an invariant, we just need to prove that $I_{2.1} \Rightarrow I_{2.2}$ in proving that $I_{2.2}$ is an invariant. The proof of $I_{2.1} \Rightarrow I_{2.2}$ is as follows:

Boundary heads are generated in the same way as inner heads. The only difference is that their cells are on the boundary of the system's geographic coverage such that there is no neighboring head in certain $(60+2\alpha)^o$ region around itself, where α denotes the angular deviation corresponding to the R_t head's location deviation. Since each inner head has exactly 6 neighboring heads in its ($\sqrt{3}$ R+2R_t) radius, each boundary head should have less than 6 neighboring heads in its ($\sqrt{3}$ R+2R_t) radius. And the distance between boundary head i and its neighboring heads is bounded in the same way as inner head does.

❑

- $I_{2.3}$: Each head, except for the big node H_0, has at most 3 children heads. H_0 has 6 children heads if it is an inner cell head and at most 5 children heads otherwise. That is,

($\forall$ head i: | CH(i) | $\leq$ 3) $\wedge$

(H_0 is not on the boundary of system coverage$\Rightarrow$(|CH(H0)| = 6)) $\wedge$

(H_0 is on the boundary of system coverage but not disconnected $\Rightarrow$ (1 $\leq$ | CH(H_0) | $\leq$ 5))

Proof:

The modules that can affect $I_{2.3}$ are HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP.

Suppose $I_{2.3}$ holds before a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP execution. We just need to prove that after the execution of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP, initiated by a head i that executes HEAD_ORG, $I_{2.3}$ still holds. Because this round of head organization will only affect head i and its children heads i_1, i_2, i_3, and we are only considering a head's children heads, we only need to prove that the $I_{2.3}$ holds for head i.

If head i is not the big node, from the design of HEAD_ORG, its search region is only 180^o. Ideally there would be only one head in 60^o, thus at most there would be no more than three next-band heads (children heads) initiated by head i. Also, if the default value of R_t is R/4, the way HEAD_ORG works also guarantees that no more than three next-band heads initiated by i.

If head i is the big node, its search region is 360^o, thus it would have 6 children heads if the big node is not at the boundary of the system's geographic coverage. If it is at the boundary of the system but not disconnected, the big node H_0 would have 1~5 children heads because there is no neighboring head in certain $(60+2\alpha)^o$ region around the big node, where α denotes the angular deviation corresponding to the R_t head's location deviation.

Thus $I_{2.3}$ still holds after a round of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP execution.

❑

- $I_{2.4}$: Each cell is of radius (R+R_{random}), where R_{random} is bounded by $\left[-2R_t/\sqrt{3}, 2R_t/\sqrt{3}\right]$. Each associate is no more than (R+R_{random}) away from its head.

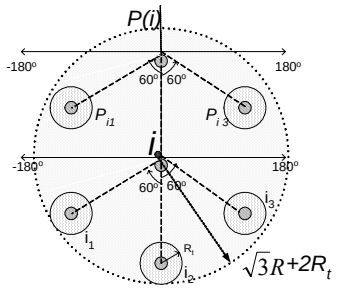
  ($\forall$ cell C: $\forall$associate i $\in$ C : R–($2R_t/\sqrt{3}$) $\leq$ dist(i, H(i)) $\leq$ R+($2R_t/\sqrt{3}$))

Proof:

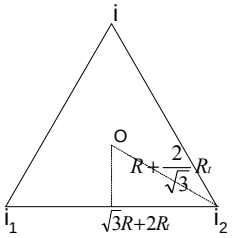
The modules that can affect $I_{2.4}$ are HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP.

Suppose $I_{2.4}$ holds before a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP execution. We just need to prove that after this round of execution of these modules, initiated by a head i that executes HEAD_ORG, $I_{2.4}$ still holds. This round of head organization can affect head *i*, its possible children heads i_1, i_2, i_3, its parent head P(i), and the two neighboring heads (p_{i1}, p_{i3}) at the same band that are under the care of the same parent head as head *i*, and their covered cells, as shown in the picture below. But we only need to

prove that the $I_{2.3}$ holds for the cell C_i covered by head i without loss of generality, because we can prove that $I_{2.3}$ holds for all other related cells in the same way as for cell C_i.



If head i is an inner cell head and thus C_i is an inner cell, then head i is surrounded by six neighboring heads as shown above. Then C_i is also surrounded by six neighboring cells. So, any point in C_i will lie in the triangle formed by head i and two of its immediately neighboring heads (i_1 and i_2, for example), as shown in the following figure. According to the way ASSOCIATE_ORG_RESP works, any point in this triangle chooses the closest head to join. Thus, the maximum distance between a point in head i's cell and head i is ($R+2R_t / \sqrt{3}$), as shown in the figure. Thus, the radius for any inner cell is at most ($R+2R_t / \sqrt{3}$).



Thus $I_{2.4}$ still holds after a round of a round of HEAD_ORG, HEAD_ORG_RESP, and ASSOCIATE_ORG_RESP execution.

❑

- **I_3**: <u>Inner Cell Optimality</u> (for associate nodes)
  - Each associate node in an inner cell chooses the best (closest, most remaining energy, etc.) neighboring head to join. That is,

(∀ associate i in any inner cell: ∀ head j ≠ H(i): (H(i) <u>*better than*</u> j))

Proof:

The modules that can affect I_3 are HEAD_ORG and ASSOCIATE_ORG_RESP.

Suppose I_3 holds before a round of HEAD_ORG and ASSOCIATE_ORG_RESP. We just need to prove that after this round of execution of these modules, initiated by a head i that executes HEAD_ORG, I_3 still holds. Because HEAD_ORG only happens at boundary heads at any moment, any round of HEAD_ORG execution could only, from the perspective of I_3, affect those associates that was in a boundary cell before the execution but is in an inner cell after the execution. For any such associate node j, it must have chosen the best head around it as its head because the way ASSOCIATE_ORG_RESP works. So I3 holds for every such associate j.

Thus I3 holds after any round of HEAD_ORG and ASSOCIATE_ORG_RESP execution.

❑

2) Theorem 3

Starting at an arbitrary state, every computation of GS^3-S reaches a state in SI within a constant amount of time. That is,

TRUE *leads to* Invariant ($I_1 \wedge I_2 \wedge I_3$)

Proof:

In order to prove "TRUE *leads to* Invariant", we just need to prove "$\overline{In\,\mathrm{var}\,iant}$ *leads to* Invariant" because "Invariant *leads to* Invariant" is obvious.

Because Invariant ($I_1 \wedge I_2$) is closed under all the program actions and there is no state corruption in static networks (according to the definition), the system will not be able to reach any state where $\overline{In\,\mathrm{var}\,iant}$ would hold. Thus $\overline{In\,\mathrm{var}\,iant}$ is FALSE all the time. So "$\overline{In\,\mathrm{var}\,iant}$ *leads to* Invariant" is equal to "FALSE leads to Invariant" that is trivially true. Therefore, "$\overline{In\,\mathrm{var}\,iant}$ *leads to* Invariant" is true.

According to the analysis above, "TRUE *leads to* Invariant" hold.

❑

3) Theorem 4

Starting at an arbitrary state in SI, algorithm GS^3-S reaches a state in SF (SF = $F_1 \wedge F_2 \wedge F_3 \wedge F4$) within time $\theta(D_b)$, where $D_b = \max\{\mathrm{dist}(H_0, i)$: i is a small node and $\mathrm{dist}(H_0, i)$ is the cartesian distance between H_0 and $i\}$. That is,

Invariant ($I_1 \wedge I_2 \wedge I_3$) *leads to* fix point ($F_1 \wedge F_2 \wedge F_3 \wedge F_4$)

Proof:

a) Invariant ($I_1 \wedge I_2 \wedge I_3$) *leads to* F_3

We only need to prove that $I_1 \wedge I_2 \wedge I_3 \wedge \overline{F_3}$ leads to F_3, since F_3 naturally leads to F_3, and I_1 as well as I_2 is invariant.

For any associate node i, the scenario where $\overline{F_3}$ could hold is when some better neighboring head j around it is still at state q_{head} and has not carried out the process HEAD_ORG yet. Because HEAD_ORG and HEAD_ORG_RESP guarantee that two neighboring heads within ($\sqrt{3}$ R+2R_t) range cannot initiate HEAD_ORG in parallel, associate i is able to hear the ORG messages from its entire neighboring heads, including head j. The way ASSOCIATE_ORG_RESP works guarantees that associate i will choose the best (such as closest, highest remaining energy, etc.) head to associate with after all such better heads js finish their HEAD_ORG process. So $\overline{F_3}$ will be false and F_3 will be true after all the better neighboring heads around associate i finish their HEAD_ORG processes.

Suppose the number of better heads around associate i is N_{bh} when $\overline{F_3}$ is true. Then N_{bh} is no less than 0. When $\overline{F_3}$ is true, at least one HEAD_ORG process is enabled, and whenever a HEAD_ORG process finishes, the value of N_{bh} will decrease by 1. Thus, it only takes N_{bh} rounds of HEAD_ORG process for associate i to transfer from state $\overline{F_3}$ to state F_3, which is a finite procedure. Thus "$I_1 \wedge I_2 \wedge \overline{F_3}$ *leads to* F_3" holds.

Since it only takes finite time C_{head_org} for a HEAD_ORG process to finish, the state transition from $\overline{F_3}$ to F3 would only take $N_{bh} \times (C_{head\_org} + C_{gap})$ (i.e. $\theta(N_{bh})$) amount of time, where C_{gap} denotes the maximum interval between two neighboring heads' HEAD_ORG process.

b) Invariant $(I_1 \wedge I_2 \wedge I_3)$ *leads to* $F_1 \wedge F_2$

If we could prove that $I_1 \wedge I_2 \wedge I_3 \Rightarrow F_{2.4}$, then "$I_1 \wedge I_2 \wedge I_3$ *leads to* $F_{2.4}$" holds, which also means "Invariant $(I_1 \wedge I_2 \wedge I_3)$ *leads to* $F_1 \wedge F_2$" since I_1 is the same as F_1 and F_2 is equal to $I_2 \wedge F_{2.4}$.

Now let's prove $I_1 \wedge I_2 \wedge I_3 \Rightarrow F_{2.4}$. Because $I_{2.4} \equiv F_{2.4} \wedge$ (There is no R_t -radius gap in the system $\Rightarrow R' \leq R+\frac{2}{\sqrt{3}} R_t$), we only need to prove that $I_1 \wedge I_2 \wedge I_3 \Rightarrow$ (There is no R_t -radius gap in the system $\Rightarrow R' \leq R+\frac{2}{\sqrt{3}} R_t$). According to the way HEAD_ORG works, the boundary cell would be no bigger than the inner cell, if there is no R_t -radius gap. Otherwise, the HEAD_ORG process will be continuously initiated. Thus the boundary cell's radius is still no more than $(R+\frac{2}{\sqrt{3}} R_t)$ according to $I_{2.4}$ that says any inner cell's radius is no more than $(R+\frac{2}{\sqrt{3}} R_t)$.

c) Invariant $(I_1 \wedge I_2 \wedge I_3)$ *leads to* F_4

We only need to prove that $I_1 \wedge I_2 \wedge I_3 \wedge \neg F_4$ leads to F_4, since it is obvious that $I_1 \wedge I_2 \wedge I_3 \wedge F_4$ leads to F_4.

Since the way HEAD_ORG works guarantees that all the visible areas of the system can be covered by the HEAD_ORG process in the end (see the proof of this claim later), there will be a HEAD_ORG process waiting to take place whenever $\neg F_4$ holds. Because the system's coverage is finite and every HEAD_ORG process is able to cover another $(\sqrt{3} R+2R_t)$-radius circular area, the number of possible HEAD_ORG process occurrence is finite. Therefore, "$I_1 \wedge I_2 \wedge I_3 \wedge \neg F_4$ leads to F_4" holds.

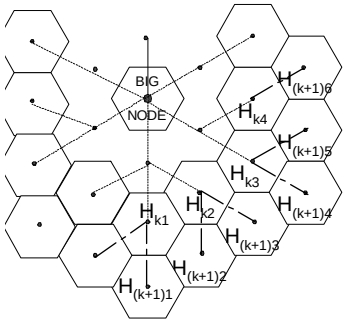
Now, let us prove that all the visible areas of the system can be covered by the HEAD_ORG process in the end. We prove it by induction on the area encircled by heads of

i-band away from the big node, denoted Round Area *RA(i)* (i.e. area of radius ($\sqrt{3}$R × i + R_t + R) and i is the number of hexagons away from the big node).

Base: when i = 0, 1, clearly holds

Hypothesis: the claim holds when i = k

Induction: when i = (k+1),



As we can see from the picture, any point that is in RA(k+1) but not in RA(k) will be covered by some (k+1)-band head. And each (k+1)-band head can be taken care of by some k-band head, either directly or indirectly, even though some of them might not be generated directly by a k-band head due to different progress speeds of the self-configuration process at different directions spreading from the big node. Thus the claim holds when i is (k+1).

❑