Lab 2

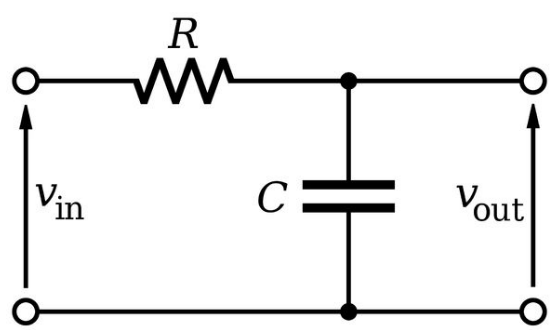
EE 324: Signals and Systems II

In this Lab we will derive mathematical models of several simple dynamical systems and simulate their response.

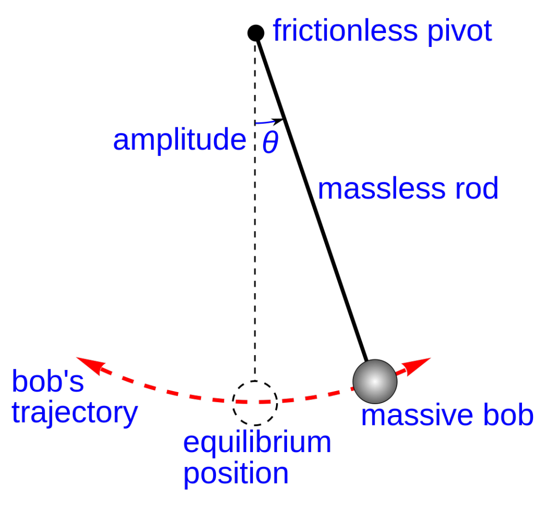
1 Prelab assignment

1. Get familiar with the differential equation solver ode45 `https://www.mathworks.com/help/matlab/ref/ode45.html`
2. Derive the differential equations associated with the following systems

i) Input $x = v_{in}$, output $y = v_{out}$



ii) Output $y = \theta$. Note there is no input here. Denote the length of the rod and the mass as ℓ and m respectively. The standard gravity is $g = 9.8m/s^2$.



2 Lab assignment

1. Simulate the response of system (i) with parameters $R = 100k\Omega, C = 1\mu F$, and input $x = 1V$. Plot your results.

2. Change the parameters to $R = 1000k\Omega, C = 1\mu F$ (with the same input). Plot your results and describe your observation.

3. Simulate the response of system (ii) with parameters $\ell = 0.5m, m = 1kg$ and initial condition $y(0) = 0.2rad, \frac{dy}{dt}(0) = 0$. Plot your results.

4. Change the parameters to $\ell = 1m, m = 10kg$ (with the same initial condition). Plot your results and describe your observation

5. van der Pol equation

$$\frac{d^2y}{dt^2} - \mu(1 - y^2)\frac{dy}{dt} + y = 0.$$

describes the dynamics of a limit cycle system. Simulate the system with $\mu = 1$ and initial condition $y(0) = 2, \frac{dy}{dt}(0) = 0$. Plot the result in the phase space. More specifically, plot the path of $y(t)$ with $y(t)$ as the x-axis and $\frac{dy}{dt}$ as the y-axis.

(feel free to skip 6 and 7)

6. Euler scheme is a discretization method. It approximates $\frac{dy}{dt}(t)$ with $(y(t+h) - y(t))/h$ where h is the step size. Write down the new discretized version of

$$\frac{dy}{dt} = -y + x.$$

7. Numerically solve the above difference equation with input $x = 1$ and initial condition $y(0) = 0$. Try several step size $h = 0.01, 0.1, 1$. Plot the results and describe your observation.