

Lecture 6 M week 3 01/22

1. review LTI systems

Continuous time

$$y(t) = h * x(t) = \int_{-\infty}^t h(t-z) x(z) dz$$

discrete time

$$y[k] = h * x[k] = \sum_{m=-\infty}^{\infty} h[k-m] x[m]$$

add causality

$$y(t) = \int_0^t h(t-z) x(z) dz$$

$$y[k] = \sum_{m=0}^k h[k-m] x[m]$$

equivalent to

continuous ordinary differential equation e.g. $\dot{y} + y - 2y = x$

discrete ordinary difference equation e.g. $y[k] - y[k-1] = x[k]$

$$y[k] = \frac{1}{2}x[k] + \frac{3}{2}x[k-1]$$

2. difference equations

$$x[0] \uparrow \quad \quad x[1] \uparrow \quad \quad \dots$$

investment in k th month $x[k]$

interest: r

$$y[k] = (1+r)y[k-1] + x[k]$$

Let $\rho = (1+r)$, compute output

with input $x[k] = \begin{cases} 1 & 0 \leq k \leq 24 \\ 0 & k > 24 \end{cases}$

$$y[k] = \sum_{m=0}^k h[k-m] x[m]$$

$$h[0] = 1$$

$$h[1] - \rho h[0] = 0 \Rightarrow h[1] = \rho$$

$$h[2] - \rho h[1] = 0 \Rightarrow h[2] = \rho^2$$

$$\dots \Rightarrow h[k] = \rho^k$$

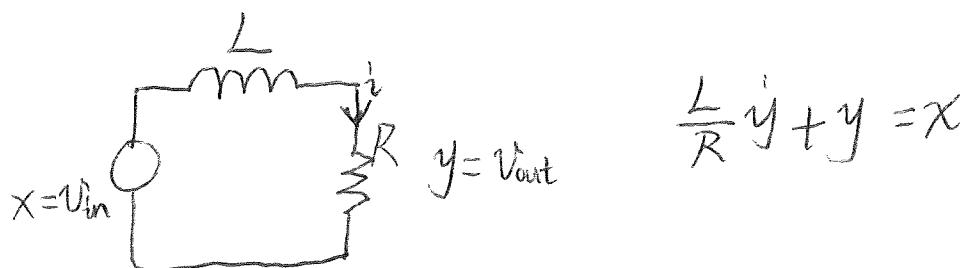
$$y[k] = \sum_{m=0}^k h[k-m]x[m]$$

$$= \begin{cases} \cancel{1 + \rho + \dots + \rho^k} & \text{if } k \leq 24 \\ \rho^{k-24}(1 + \rho + \dots + \rho^{24}) & \text{if } k > 24 \end{cases}$$

$$\cancel{h[k]} = \begin{cases} \frac{1 - \rho^{k+1}}{1 - \rho} & \text{if } k \leq 24 \\ \frac{\rho^{k-24}(1 - \rho^{25})}{1 - \rho} & \text{if } k > 24 \end{cases}$$

$\rho > 1$ unstable, $\rho < 1$ stable

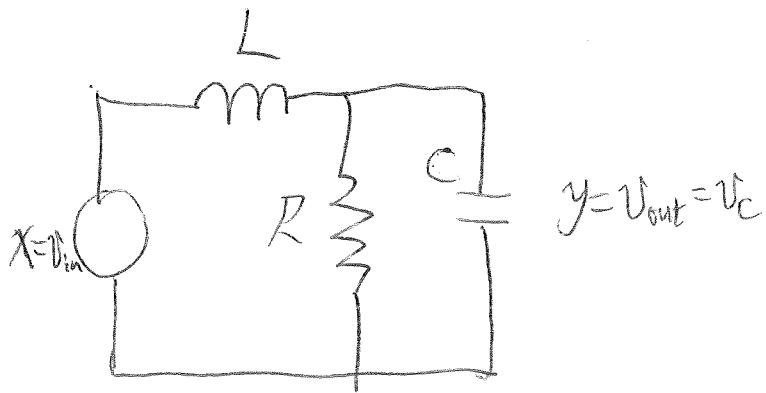
3. differential equation



$$\frac{L}{R} \dot{y} + y = x$$

$$\frac{L}{R} \dot{y} + y = 0 \quad y = b e^{at}$$

$$\frac{L}{R} b a e^{at} + b e^{at} = 0 \Rightarrow (\frac{L}{R} a + 1) e^{at} = 0 \Rightarrow a = -\frac{R}{L}$$



$$LC\ddot{y} + \frac{1}{R}\dot{y} + y = x$$

impulse response

$$h(t) = b e^{at} \sin(\omega t + \phi)$$

$$\text{or } b_1 e^{a_1 t} + b_2 e^{a_2 t}$$

more complicate for high order systems.