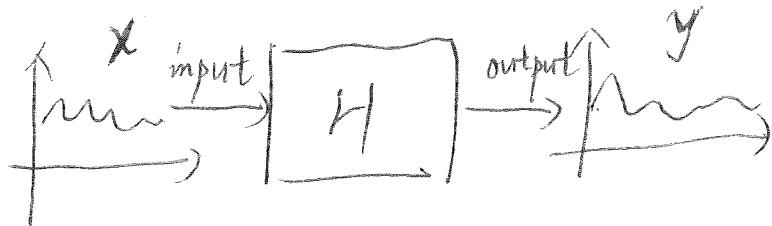


Lecture 4 W 01/17 week 2

1. signals, Fourier transform recap

2. Systems: definition: maps for input signals to output signals



e.g. Cruise control

measuring devices

communication channels

signal / image processing

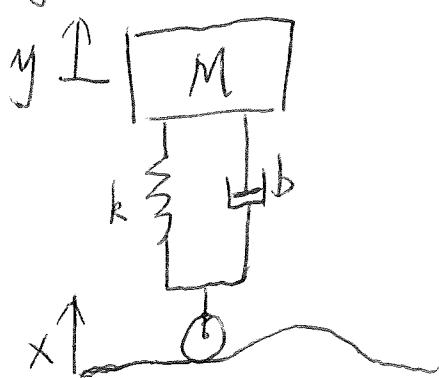
3. mathematical modeling

static: $y = f(x)$ function e.g. $y = 2x$

dynamical: $\dot{y} = -y + x$ differential equation

$y[k] - y[k-1] = x[k] + x[k-1]$ difference equation

e.g. suspension system



$$F = Ma$$

$$k(x+l-y) + b(\dot{x}-\dot{y}) - Mg = M\ddot{y}$$

$$k(x+l-y) \uparrow \quad \uparrow b(\dot{x}-\dot{y})$$

$$\downarrow Mg$$

4. Properties

① Stability

Bounded input bounded output (BLBO)

Def: if $|x(t)| \leq M_x < \infty$ for any t ,
then there exists $M_y > 0$ such that $|y(t)| \leq M_y < \infty$

e.g. stable: $y = 2x$, $\dot{y} = x^2$, $\ddot{y} = -y + x$

unstable: $\dot{y} = x$, $\ddot{y} = y + x$

② static vs dynamic

$$y = 2x \quad \dot{y} = x$$

$$y = x^3 \quad \dot{y} = -y + 2x$$

③ Causality: $y(t)$ depends on $x(s)$, $s \leq t$

causal: $\dot{y} = x$, $\ddot{y} = -y + x$

uncausal: $y(t) = \int_0^{t+1} x(s) ds$, $y[k] = \frac{1}{2}x[k+1] + \frac{1}{2}x[k-1]$

④ Time-invariant



$$y(t) = H(x(t)) \Rightarrow y(t-t_0) = H(x(t-t_0)) \text{ for any constant } t_0$$



⑤ Linearity

if $y_1 = H(x_1)$, $y_2 = H(x_2)$.

$$\begin{aligned}\text{then } H(2x_1 + \beta x_2) &= 2H(x_1) + \beta H(x_2) \\ &= 2y_1 + \beta y_2\end{aligned}$$

e.g. $y = 2x$, $\dot{y} = x$

nonlinear: $y = x^2$, $y = e^x$

5. Linear time-invariant system LTI

Linear + Time-invariant + Causal

e.g. $\dot{y} = x$, $\ddot{y} = -y + x$

$$y[k] - y[k-1] = \cancel{x[k]}$$

convolution $y[k] = y[0] + \sum_{m=0}^k h[k-m]x[m]$

$$y(t) = y(0) + \int_0^t h(t-z)x(z)dz$$

impulse response h