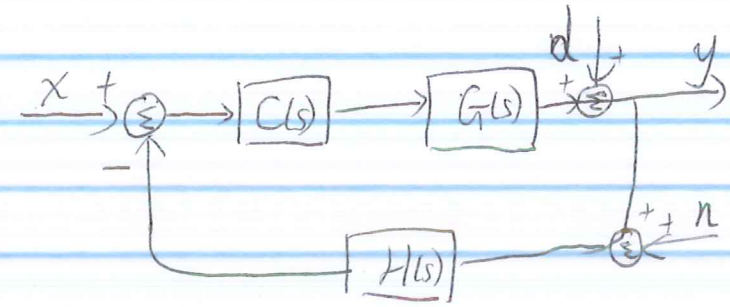
Lecture 37 W Week 13 04/11

Feedback control



G(s): plant car

H(s): sensor eyes, cameras

C(s): controller you

x: reference signal trajectory following

y: output

d: disturbance

n: measurement noise

goal: y follows x as close as possible.

$$x \to y: \quad \frac{C(s)G(s)}{1+H(s)C(s)G(s)} \approx 1$$

$$d \to y: \quad \frac{1}{1+H(s)C(s)G(s)} \approx 0$$

$$n \to y: \quad \frac{H(s)C(s)G(s)}{1+H(s)C(s)G(s)} \approx 0$$

stability:

$$T: \quad \frac{C(s)G(s)}{1+H(s)G(s)} = \frac{L(s)}{1+L(s)} = \frac{L(s)}{F(s)}$$

$$L(s) = \frac{A_1(s)}{B(s)} \qquad F(s) = 1+L(s) = \frac{A_1(s)+B(s)}{B(s)} =: \frac{A(s)}{B(s)}$$

$$T(s) = \frac{L(s)}{F(s)} = \frac{A_1(s)/B(s)}{A(s)/B(s)} = \frac{A_1(s)}{A(s)}$$

poles: $A(s) = 0$.

$$A(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

Row n: $a_n \quad a_{n-2} \quad a_{n-4} \quad \cdots$

Row $n-1$: $a_{n-1} \quad a_{n-3} \quad a_{n-5} \quad \cdots$

Row $n-2$: $\dfrac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \quad \dfrac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$ $\Rightarrow$ Routh array

$$-\frac{1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} \quad -\frac{1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix} \cdots$$

e.g. $a_1 s + a_0 \qquad -\frac{a_0}{a_1} < 0$

$a_2 s^2 + a_1 s + a_0 \qquad s_1 s_2 = \frac{a_0}{a_2} > 0 \qquad a_0 > 0,\ a_2 > 0$

$s_1 + s_2 = -\frac{a_1}{a_2} < 0 \Rightarrow a_1 > 0$

all the roots of $A(s)$ lie in the left half plane if all the entries in the ~~left most~~ first column are nonzero and have the same sign. Otherwise, the number of such changes is the number of roots of $A(s)$ in the right half plane

$s^2 + 3s + 1$

$$\begin{array}{cc} 1 & 1 \\ 3 & 0 \\ 1 & 0 \end{array}$$

no sign change.

$$A(s) = s^4 + 3s^3 + 7s^2 + 3s + 10$$

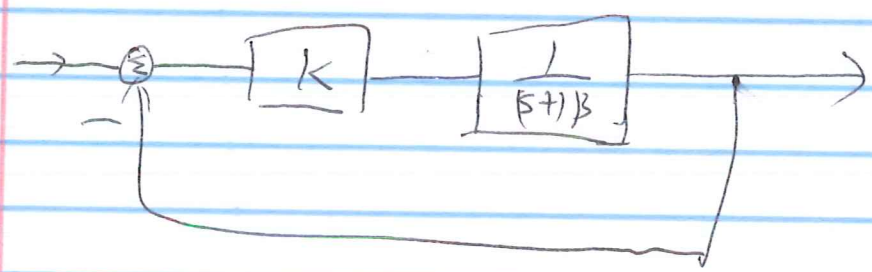
Row 4:	1	7	10
Row 3:	3	3	0
Row 2:	$\frac{3\times7-3}{3}=6$	$\frac{3\times10-0}{3}=10$	0
Row 1:	$\frac{6\times3-3\times10}{6}=-2$	0	0
Row 0:	$\frac{-2\times10}{-2}=10$	0	

$1 \quad 3 \quad 6 \quad \underline{-2} \quad 10$

2 roots in the right half plane

$-1.6597 \pm 1.9426j$

$0.1597 \pm 1.2273j$



$$L(s) = \frac{K}{(s+1)^3}$$

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{K}{(s+1)^3+K} = \frac{K}{s^3+3s^2+3s+1+K}$$

s^3:	1	3
s^2:	3	$K+1$
s^1:	$\frac{9-K-1}{3}$	0
s^0:	$\frac{\frac{(9-K-1)(K+1)}{3}}{\frac{9-K-1}{3}}$	

$$\frac{9-K-1}{3} > 0$$

$$K+1 > 0$$

$$\Rightarrow \quad -1 < K < 8$$