

$$1. y[k] - 0.5 y[k-1] = x[k]$$

discrete-time system, no initial condition under the assumption

$$x[k]=0, y[k]=0 \text{ for all } k < 0$$

2. ROC region of convergence

$$e^{-t} u(t) \sim \frac{1}{s+1} \quad \text{ROC: } \operatorname{Re}(s) > -1$$

$$-e^{-t} u(-t) \sim \frac{1}{s+1} \quad \text{ROC: } \operatorname{Re}(s) < -1$$

two signals may have the same Laplace transform

3. zeros, poles

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{s^n + b_{n-1} s^{n-1} + \dots + b_0} =: \frac{N(s)}{D(s)}$$

assumption $m \leq n$ proper transfer function

zeros: roots $\cdot N(s) = 0$

poles: roots $D(s) = 0$

e.g.

$$\frac{s+1}{s+2}$$

$$\frac{s+3}{s^2+3s+2}$$

$$\frac{s^2+6s+9}{(s+2)(s^2+2s+1)}$$

$$\frac{s^2+4s+4}{(s+2)(s^2+2s+1)}$$

zeros

N/A

-3

-3, -3

-2

poles

-2

-2, -1

-2, -1, -1

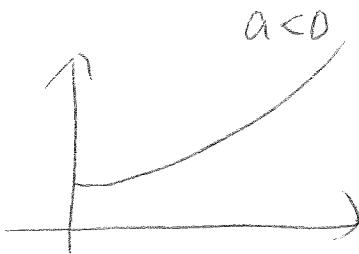
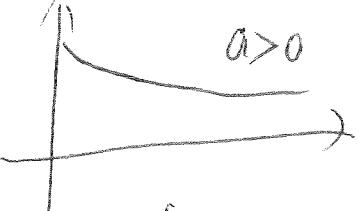
-1, -1

pole, zero cancellation

4. stability

$$H(s) = \frac{1}{s+a}$$

$$h(t) = e^{-at}$$



stable if $a > 0$, unstable if $a < 0$

BIBO (bounded input bounded output) stability

Input $x(t) = u(t)$

$$Y(s) = H(s)X(s) = \frac{1}{s+a} \frac{1}{s} = \frac{1}{as} - \frac{1}{a(s+a)}$$

$$\Rightarrow y(t) = \frac{1}{a}u(t) - \frac{1}{a}e^{-at}$$

converge if $a > 0$, diverge if $a < 0$

A system is stable if and only if all the poles are in the left half plane

e.g.

$$\frac{1}{s^2 + s - 2}$$

$$\frac{s+3}{s^2 + 3s + 2}$$

$$\frac{s-1}{s^2 + 2s + 1}$$

poles
2, -1

-2, -1

-1, -1

unstable

stable

stable

5. state space model

$$H(s) = \frac{1}{s^2 + 3s + 2} \Leftrightarrow Y(s) = H(s)X(s)$$

$$\Rightarrow (s^2 + 3s + 2)Y(s) = X(s)$$

$$\ddot{y} + 3\dot{y} + 2y = x$$

define $z_1 = y$

$$z_2 = \dot{y}$$

then $\dot{z}_1 = \dot{y} = z_2$

In matricial form $\dot{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x$

$$y = (1 \ 0) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + 0x$$

general form

$$\dot{z} = Az + bx$$

$$y = cz + dx$$

z : state variables

x : input

y : output

→ memory

eigenvalues of A $\det(AI - A) = 0$

$$\begin{vmatrix} \lambda & -1 \\ 2 & \lambda + 3 \end{vmatrix} = \lambda(\lambda + 3) + 2 = 0 \Rightarrow \lambda = -2, -1$$

poles of $H(s)$: $-2, -1$

eigenvalues of A = poles of $H(s)$

General form

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{s^n + b_{n-1} s^{n-1} + \dots + b_0}$$

$$\dot{\underline{z}} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -b_0 & -b_1 & -b_2 & \cdots & -b_{n-1} \end{pmatrix} \underline{z} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} x$$

$$y = (a_0, a_1, \dots, a_m, \underbrace{0, \dots, 0}_{n-m-1})$$

$$\dot{\underline{z}} = A\underline{z} + Bx$$

$$y = C\underline{z} + Dx \quad \text{not unique}$$

$$\text{Let } \hat{\underline{z}} = T\underline{z} \quad \text{for invertible } T$$

$$\dot{\hat{\underline{z}}} = T \cdot A \cdot T^{-1} \hat{\underline{z}} + TBx$$

$$y = CT^{-1}\hat{\underline{z}} + Dx$$