Fourier Transform

EE 324: Signals and Systems II

1 Continuous time Fourier transform (FT)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

2 Discrete time Fourier transform (DTFT)

$$X(e^{j\Omega}) = \sum_{n} x[n]e^{-j\Omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$$

3 Fourier series (FS) for periodic functions with period T

$$X[k] = \frac{1}{T}\int_{0}^{T} x(t)e^{-j\frac{2\pi}{T}kt}dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j\frac{2\pi}{T}kt}, \quad 0 \leq t \leq T$$

4 Discrete Fourier transform (DFT, DTFS) for $x[n]$ with $0 \leq n \leq N-1$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}, \quad 0 \leq k \leq N-1$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn}, \quad 0 \leq n \leq N-1$$