Local control

- Def: Supervisor called local if it can control only those events that it can also observe.

**Observation mask:** \( M_{\text{loc}} : \Sigma \rightarrow \Sigma_{\text{loc}} \cup \Sigma_{\text{loc}} \) (projection type).

- Uncontrollable events for local supervisor: \( \Sigma_{u,\text{loc}} = \Sigma u \cup (\Sigma - \Sigma_{\text{loc}}) \)

- Stronger condition needed for existence of supervisors

- **Normality:** \( M(s) = m^{+}, \ s \in pr(k), \ t \in L(G) \Rightarrow t \in pr(k). \) 
  equivalently, \( M^{-1}M (pr(k)) \cap L(G) \leq pr(k) \)

- **Fact:** Normality \( \Rightarrow \) Observability (normality is stronger condition)

- **Theorem:**
  1. \( K \Sigma_{u,\text{loc}} \)-controllable, \( M_{\text{loc}} \)-observable \( \iff \)
  2. \( K \Sigma_{u} \)-controllable, \( M_{\text{loc}} \)-normal \( \iff \)
  3. \( K \Sigma_{u,\text{loc}} \)-controllable, \( M_{\text{loc}} \)-normal.

\( (1 \Rightarrow 2) \): Sufficient to show \( M_{\text{loc}} \)-normality. Suppose for contradiction

\( M^{-1}M (pr(k)) \cap L(G) \neq pr(k) \). Let \( s \) smallest string in \( M^{-1}M (pr(k)) \cap L(G) \). Let \( s \in L(G) \). Then \( s \neq \emptyset \), since \( \emptyset \in pr(k) \). So \( s = \emptyset \sigma \).

Since \( \emptyset \in pr(k) \), \( s = \emptyset \sigma \in L(G) \), and \( K \Sigma_{u,\text{loc}} \)-controllable \( \Rightarrow \emptyset \notin \Sigma_{u,\text{loc}} \)

\( \Rightarrow \emptyset \notin \Sigma_{\text{loc}} \) \( \Rightarrow M(\emptyset) \neq \emptyset \).

Since \( \emptyset \in M^{-1}M (pr(k)) \), there exists \( t = \emptyset \sigma \in pr(k) \), \( M(t) = M(\emptyset) = M(\emptyset) \) \( \sigma \) contradicts normality to observability.

\( (2 \Rightarrow 3) \): Sufficient to show \( \Sigma_{u,\text{loc}} \)-controllability. Suppose for contradiction

\( pr(k) \Sigma_{u,\text{loc}} \cap L(G) \neq pr(k) \). Pick smallest \( s \in pr(k) \Sigma_{u,\text{loc}} \cap L(G) \). \( s = \emptyset \sigma \) with \( \emptyset \in \Sigma_{u,\text{loc}} \Rightarrow \emptyset \in \Sigma_{\text{loc}} \Rightarrow M(\emptyset) = \emptyset \). So \( M(s) = M(\emptyset) \). 

This contradicts normality since \( \emptyset \in pr(k) \), \( s \in L(G) = pr(k) \).

\( (3 \Rightarrow 1) \): obvious.
Test for Normality

- Need to test \( M^* M \{ \text{pr}(k) \} \cup \text{L}(\mathcal{E}) \subseteq \text{pr}(k) \)
- Construct \( SNRM \) by adding transitions in \( S \) st. \( \text{L}(SNRM) = M^* M \{ \text{pr}(k) \} \)
  
- \((y_1, y_2, y_3)\) a transition in \( S \) \(\Rightarrow\) add \((y_1, y_1', y_2)\) where \( M(y') = M(y) \)
  \( M(y) = \varepsilon \) \(\Rightarrow\) add \((y, y, y)\) at every state \( y \) of \( S \)

- Then \( k \) normal iff \( \text{L}(SNRM) \cap \text{L}(S) \cap \text{L}(\overline{S}) = \emptyset \)

  computational complexity: \( O(mn^2) \)

- Normality is preserved under union & intersection (for pre-determinised) exist.

  Acceptor for supremal normal sublanguage:

- start with \( S \Rightarrow L_m(S) = L_e(S) = K \), \( L(S) = \Sigma^* \)
- add transition \( \delta \) as above to obtain \( (S)_{NRM} \Rightarrow L_m((S)_{NRM}) = M^* M \{ \text{pr}(k) \} \)
- determinize \((S)_{NRM}\) to get \( \hat{S} \Rightarrow L_m(\hat{S}) = M^* M \{ \text{pr}(k) \} \), \( L(\hat{S}) = \Sigma^* \)

- Consider \( G \mid \epsilon \| S \Rightarrow L_m(G \mid \epsilon \| S) = K \), \( L(G \mid \epsilon \| S) = L(S) \).

- A typical state looks like \( r = (x, y, \hat{y}) \), where \( x \in X, y \neq y' \) for \( y' \neq y \), \( \hat{y} = \hat{y}_1, \ldots, \hat{y}_r \) \( \in \hat{Y} \)

- \( r_i \) and \( r_2 \) are called matching if \( \hat{y}_i = \hat{y}_2 \)

For each string leading to \( r_1 \), exists indistinguishable string leading to \( r_2 \)

(a) \( \mathcal{E}_0 := \{ y \mid \text{second coordinate is a dump state} \} \)

(b) \( \mathcal{E}' := \mathcal{E}_K \cup \{ r \in \epsilon \mid \exists \text{ matching } r' \in \mathcal{E}_K \} \)

\( \mathcal{E}_K := \mathcal{E}' \cup \{ r \in \epsilon \mid \text{r does not belong to trim component of } \epsilon \} \)

(c) Stop when \( \mathcal{E}_K = \mathcal{E}_{K+1} \); else \( k = k+1 \), goto (b).
Maximally Permissive Supervision

- Consider \( \sup P\left[\tilde{M}^{-1} \tilde{M}\left(pr(H)\right)\right] \land L(G) \leq pr(H) \)
  \[ f(H) \quad g(H) \]

- \( f \) monotone, not disjunctive; \( g \) monotone, not conjunctive

\[ \sup 0(k) := \sup \{H \in K \mid H \text{ observable}\}, \quad \inf \delta(k) := \inf \{H \in K \mid H \text{ observable}\} \]

need not exist.

- Example:

\[ \begin{array}{ccc}
\delta & b & \rho \\
G & M & M(b) = M(b) \neq \emptyset \\
\end{array} \]

\( K_1 = \{b\}, \quad K_2 = \{aa\} \Rightarrow \text{both } K_1, K_2 \text{ observable.} \)

\( K = K_1 \cup K_2 = \{b, aa\} \text{ not observable since } b, a \in \text{pr}(k); \quad M(b) = M(b), \quad M(b) = M(b) \)

\( ba \in L(G) \) - pr(K).

\( K_1 = \{b\}, \quad K_2 = \{aa, ba, aa, aa\} \quad K_2 = \{b, aa, ba\} \Rightarrow \text{both } K_1, K_2 \text{ observable} \)

\( K = K_1 \land K_2 = \{b, aa\} \text{ not observable.} \)

- Extremal prefix-closed and observable languages:

- Consider \( \sup P\left[\tilde{M}^{-1} \tilde{M}\left(pr(H)\right)\right] \land L(G) \leq pr(H) \) and \( pr(H) \leq H \)

Equivalently, \( \sup P\left[\tilde{M}^{-1} \tilde{M}\left(pr(H)\right)\right] \land L(G) \leq H \)

\( \Rightarrow (2) \) obvious; \( (2) \) implies 1st inequality of \( (1) \); for 2nd inequality of \( (1) \) use:

Also, \( pr(H) \leq \sup P\left[\tilde{M}^{-1} \tilde{M}\left(pr(0)\right)\right] \land L(G) \leq H \)

- \( f \) monotone, not disjunctive; \( g \) conjunctive

\( \sup \partial 0(k) \text{ need not exist}; \quad \inf \partial 0(k) \text{ exists} \)

Since \( f \) is idempotent,

\[ \inf \partial 0(k) = K \cup (f(k) \land L(G)) = \sup P\left[\tilde{M}^{-1} \tilde{M}\left(pr(0)\right)\right] \land L(G). \]

- Since \( \sup 0(k), \sup \partial 0(k), \sup \partial 0(k) \) do not exist, maximally permissive control under partial observation does not exist.
• A unique maximally permissive supervisor under partial obs. does not exist.

  "Sub-optimal" solution: \( \text{sup } \text{PCN}(K) \) or \( \text{sup } \text{RCN}(K) \)

  Alternative: Find an observable sublanguage of \( K \):

  \[
  K_{p0} = K - \left[ \tilde{M}^{-1} \tilde{M} \left( - \left( L \left( \text{pc} (K) \right) \cap K_2 \right) \right) \right] \Sigma^* .
  \]

  Then: Suppose \( K \) is prefix-closed, then

  1) \( \text{sup } \text{PCN}(K) \leq K_{p0} \leq K \)

  2) \( K_{p0} \) is prefix-closed and observable

  3) \( K_{p0} \) is controllable, whenever \( K \) is controllable

• If \( K \) is not prefix-closed, the replace \( K \) by \( \text{sup } \text{PC}(K) \).

• Above computation is also useful in design of local supervisors.

  \[
  \text{sup } \text{PC} \left( \bigcup_{u \in M_{in2}} (K) \right) = \left[ \text{sup } \text{PC} \left( \text{rule } 1, 2 \right) \right] \text{po}_{M_{in2}}.
  \]

  I.e., a modular computation is possible.
Consider $M^{-1} M (pr(H)) \cap L(G) \leq pr(H)$

If disjunctive, $g$ monotone but not conjunctive; $f^{-1}(H) = [M^{-1} M(H)] \Sigma^*$

So $\sup N(K) = \{ H \in K | H \text{ normal} \}$ exists; $\inf N(K) = \{ H \in K | H \text{ normal} \}$ may not exist

Iterative computation of $\sup N(K)$:

$k_0 = K$; $k_{i+1} = k_i - f^{-1}(L(k_i) - g(k_i)) = k_i - [M^{-1} M(L(G) - pr(k_i))] \Sigma^*$.

Extremal prefix-closed and normal languages:

consider $(M^{-1} M (pr(H)) \cap L(G) \leq pr(H)) \land [pr(H) \leq H] \iff [M^{-1} M (pr(H)) \land L(G) \leq H]$.

If disjunctive, $g$ conjunctive $\Rightarrow \sup PN(K)$ and $\inf PN(K)$ exist.

If idempotent, so

$\sup PN(K) = k - f^{-1}(L(k) - k) = k - [M^{-1} M (L(G) - k)] \Sigma^*$

$\inf PN(K) = k U [f(k) \land L(G)] = M^{-1} M (pr(k) \land L(G))$.

Extremal prefix-closed / relative-closed, controllable and normal languages:

$\sup PCN(K)$ and $\inf PCN(K)$ can be computed iteratively.

A modular computation is possible when any pair of controllable and uncontrollable events whenever indistinguishable are both observable, i.e.,

$\sigma_1 \in I_u$, $\sigma_2 \in I - I_u$, $M(\sigma_1) = M(\sigma_2) \Rightarrow M(\sigma_1) = M(\sigma_2) = E$, equivalently,

$M^{-1} [M(\sigma_1) - \Sigma^*] \leq \Sigma_u$

(A projection mask satisfies this condition)

Under this condition: $\sup PCN(K) = \sup N (\sup PC(K))$

$\sup RCN(K) = \sup N (\sup RC(K))$

($\sup N$ and $\inf N$ computations preserve prefix closure and relative closure)