Behavior of DES

* $X$: set of states; $x \in X$ typical element/state
* $\Sigma$: set of events; $\sigma \in \Sigma$ typical element/event

* Behavior of DES may be described by sequences of triples:
  $(x_0, \sigma_1, t_1) (x_1, \sigma_2, t_2) \ldots$

  $x_0 \in X$: initial state
  $x_i \in X$: $i$th state; $\sigma \in \Sigma$: $i$th event
  $t_i \in R$: instance of $i$th state transition

* Such behavioral model is called timed model (contains timing information)

* Timed model used for achieving quantitative goals:
  minimization of average delay in communication network.

* Untimed models ignore timing information; contain information about orderly occurrence of states and events.

* Used for achieving qualitative goals:
  buffer in mfg. system must never overflow
  message sequence be received in the order it was sent

  Such properties do not depend on when events occurred; rather in what order they occurred.

* We will only deal with qualitative or logical behaviors.
Languages

- At qualitative or logical level behavior described by:
  \[(x_0, \sigma_1) (x_1, \sigma_2) \ldots\]

- DES deterministic if given a state and event occurring in that state, next state is uniquely known.

- For deterministic systems behavior may be described by:
  \[\sigma_1 \sigma_2 \ldots\]
  and initial state \(x_0\).

- Sequence of events called trace/ string; collection of strings: language

- \(\Sigma^*\): set of all finite length traces, including zero length trace "\(\epsilon\)".

- language: subset of \(\Sigma^*\); \(H, K\) symbols used

- trace: member of \(\Sigma^*\); \(s, t\) symbols used

- \(|s|\): length of \(s\) ; \(s \leq t \Rightarrow s\) a prefix of \(t\)
  \(s < t \Rightarrow s\) a proper prefix of \(t\)

- Example: Buffer with capacity one.
  states: empty and full; events: arrival and departure.
  state transition from empty to full on arrival
  state transition from full to empty on departure
  initial state: empty

Language of buffer: all sequences of the type:
  arrival, departure, arrival, departure …
Language Models

- \( \mathcal{K} \subseteq \Sigma^* \); \( K \neq \emptyset \) be all traces that occur in a DES; called generated lang.

For a trace to occur all prefixes must occur first \( \Rightarrow \mathcal{K} = \text{pr}(K) \)

- \( \mathcal{K}_m \subseteq \mathcal{K} \): traces whose execution imply completion of task; called marked language

- Language model: \( (\mathcal{K}_m, \mathcal{K}) \) with \( \mathcal{K}_m \subseteq \mathcal{K} = \text{pr}(K) \neq \emptyset \)

- Example: Buffer with capacity one; \( a \): arrival event; \( d \): departure

  generated language = \( \text{pr}(\langle a,d \rangle^*) \)

  Suppose \( s \in \text{pr}(\langle a,d \rangle^*) \) implies completion of task if buffer is empty.

  marked language = \( (a,d)^* \)

Elevator moves between floors 1 & 2. Events = {up, down}.

\[ \mathcal{K} = \text{pr}(K) = \{ t \in \Sigma^* \mid t \leq s, \text{ where } s \in \mathcal{K} \} \]

\[ t \leq s \iff t \in \overline{\mathcal{K}} \]

HW: Design machine & key model of plant & spec. for traffic control.
State Machines

- Alternative way of describing a language model

- SM is a 5-tuple: \( G: = (X, \Sigma, \delta, x_0, \{m\}) \)
  
  \( X \) = set of states 
  
  \( \Sigma \) = finite set of events 

\[ \alpha(x_t, \sigma_t) = \{x_{t+1}, x_{t+2}\} \]

\( \alpha: X \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^X \)  

\( x_0 \in X \) initial state 

\( x_m \in X \) marked states 

- SM is, in general, nondeterministic state machine with \( \epsilon \)-move

  \( \epsilon \)-NSM: \( \alpha: X \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^X \)

  NSM: \( \alpha: X \times \Sigma \rightarrow 2^X \) (no \( \epsilon \)-moves)

  DSM: \( \alpha: X \times \Sigma \rightarrow X \) (deterministic SM)

Example: Buffer of capacity one with language model \( (Pr(ad^*), ad) \)

\[
\begin{align*}
&\xrightarrow{a} &\text{DSM for buffer} \\
&\text{empty} &\xrightarrow{d} &\text{full}
\end{align*}
\]

\( X = \{\text{empty, full}\} \); \( \Sigma = \{a, d\} \); \( x_0 = \text{empty} \); \( x_m = \{\text{empty}\} \)

\( \alpha(\text{empty}, a) = \text{full} \); \( \alpha(\text{full}, d) = \text{empty} \).

- State transition function is a partial map
  
  (defined on a subset of \( X \times (\Sigma \cup \{\epsilon\}) \).
Epsilon-closure of $x$: $E^*_G(x) = \text{set of states reachable from } x \text{ on zero or more } \varepsilon \text{ moves}$

- Extension $\alpha^*$ from events to traces:
  
  $\alpha^*(x, \varepsilon) = E^*_G(x) = \text{set of states reached on zero length string}$
  
  $\alpha^*(x, \varepsilon) = E^*_G(\alpha(\alpha(x, \varepsilon), \varepsilon)) = \text{set of states reached on } \varepsilon \varepsilon$.

Example:

$$
\begin{array}{c}
1 & \varepsilon & 2 & \varepsilon & 3 \\
\end{array}
$$

- $E^*_G(1) = \{2, 2, 3\}$
- $E^*_G(2) = \{1, 2, 3\}$
- $E^*_G(3) = \{1, 2\}$

HW: Compute $E^*_G$ and $\alpha^*(1, \varepsilon)$ for state machine in problem 2, Chapter 1.

Let $L(G) = \{x \in \Sigma^* | \alpha^*(x, \varepsilon) \neq \emptyset \}$

In the example above, $\alpha^*(1, \varepsilon) = \emptyset \Rightarrow 6 \varepsilon (\neq L(G))$.

Let $L_m(G) = \{x \in \Sigma^* | \alpha^*(x, \varepsilon) \wedge x_m \neq \emptyset \}$ generated lang.

Then $(L_m(G), L(G)) \text{ is a language model.}$

- $(L_m(G), L(G)) \text{ language model for any } G$

Conversely, given a language model $(K_m, K)$, exists DSM $G$ s.t.

$(L_m(G), L(G)) = (K_m, K)$

$X := \{ \delta \in K\}$; $X_0 := \emptyset$; $X_m := \{ \delta \in K \}$

For $x \in X$, $\delta \in \Sigma$: $\alpha(x, \delta) := $

- undefined otherwise

Any deterministic DES can be represented as a DSM/lang. model.