Supervisor for discrete event plant

- Deterministic discrete event plant with model \((K_m, K)\)
  or equivalently, \(G := (X, \Sigma, X_0, X_M)\)
  
  Example: \(G = M, \|M \parallel TU\) (in previous example)

- For control purposes: \(\Sigma = \Sigma_u \cup (\Sigma - \Sigma_u)\)
  
  \[\downarrow\] \[\downarrow\]
  uncontrollable controllable

  \[
  \text{supervisor: } f : K \rightarrow 2^{\Sigma - \Sigma_u}
  \]

  \(\forall s \in K : f(s) \subseteq \Sigma - \Sigma_u\) is set of controllable events disabled
  followed by execution of trace \(s\)

  \[
  \text{disabled events}
  \]

  \[
  f : K \rightarrow 2^{\Sigma - \Sigma_u}
  \]

- Controlled language model = \((K^f_m, K^f)\)
  
  \[\exists \in K^f : [\exists \in K^f, \exists \in K, \sigma \notin f(\exists) \Leftrightarrow [\exists \in K^f]\]

  \[K^f_m := K^f \cap K_m\] (marked lang. that survives undisturb)

- \((K^f_m, K^f)\) is a language model \(\Leftrightarrow K^f_m \subseteq K^f = pr(K^f) \neq \emptyset\)
  
  \(\Rightarrow pr(K^f_m) \subseteq K^f\) (may exist generated trace \(\Delta \in K^f\) which is not a prefix of marked trace \(\Rightarrow pr(K^f_m) \Rightarrow\) system may "block"

- \(f\) called nonblocking if \(pr(K^f_m) = K^f \Leftrightarrow K^f \subseteq pr(K^f_m)\)
  
  (each generated trace is a prefix of some marked trace).
Supervisor restricts behavior of plant
This can also be achieved by synchronous composition:

- Let \( S := (Y, \Sigma, \beta, y_0, Y_m) \) be supervisor state machine
  \[ L(G11S) = L(G) \land L(S) \quad ; \quad L_m(G11S) = L_m(G) \land L_m(S). \]

- \( S \) restricts the behavior of \( G \). Additional conditions:
  1. \( S \) must not disable any uncontrollable event, i.e.,
     \[ \forall e \in L(G11S), \sigma \in \Sigma, \sigma e \in L(G) \Rightarrow \sigma e \in L(G11S), \text{ i.e., } L(G11S) \Sigma_u \land L(G) \subseteq L(G11S). \]
  2. \( S \) must also satisfy:
     \[ L_m(G11S) = L(G11S) \land L_m(G). \]

- \( S \) called \( \Sigma_u \)-enabling if \( L(G11S) \Sigma_u \land L(G) \subseteq L(G11S) \)
- non-marking if \( L_m(G11S) = L(G11S) \land L_m(G) = L(G) \land L_m(G) \)
- non-blocking if \( pr(L_m(G11S)) = L(G11S). \)

Note: \( S \) non-marking if \( L_m(S) = L(S) \), i.e., each state in \( S \) is marked.

Example:

\[ \Sigma_u = \{ a, b \}. \]

\[ f(\varepsilon) = f(b) = f(ab) = \emptyset \]
\[ f(a) = f(aba) = f(ba) = \{ a \} \]

\[ L(G11S) \Sigma_u \land L(G) \subseteq L(G11S) \Rightarrow S \ \Sigma_u \text{-enabling.} \]
\[ a \in L(S) \land L_m(G) - L_m(G11S) \Rightarrow S \text{ not non-marking.} \]
\[ pr(L_m(G11S)) \subseteq L(G11S) \Rightarrow S \text{ not non-blocking.} \]