Nodal prices – II

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There are two regions: region 1 and region 2. Each of the two regions has generators and consumers.

The Grid
The loads

In bus 1, the willingness to pay is 70 for the first 200 units, and 50 for the next additional 100 units. There is no demand above 300 units.
The loads

In bus 2, the willingness to pay is 80 for the first 200 units, and 60 for the next additional 200 units. There is no demand above 400 units.
The Generators’ bids

In bus 1, the generation of the first 200 units has a constant marginal cost of 5. The marginal cost jumps to 10 for each of the next 200 units. There is no capacity for more than 400 units.
The Generators’ bids

In bus 2, the generation of the first 200 units has a constant marginal cost of 8. The marginal cost jumps to 20 for each of the next 400 units. There is no capacity for more than 600 units.
The ISO’s problem

\[ \max_{(x,q) \geq 0} \left( 70x_1^1 + 50x_1^2 + 80x_2^1 + 60x_2^2 \right) - \left( 5q_1^1 + 10q_1^2 + 8q_2^1 + 20q_2^2 \right) \]

\[ s.t. \]

\[
\begin{align*}
x_1^1 & \leq 200 \\
x_2^1 & \leq 100 \\
x_1^2 & \leq 200 \\
x_2^2 & \leq 200 \\
q_1^1 & \leq 200 \\
q_2^2 & \leq 200 \\
q_1^1 & \leq 200 \\
q_2^2 & \leq 400 \\
x_1^1 + x_2^1 + x_1^2 + x_2^2 & = q_1^1 + q_1^2 + q_2^1 + q_2^2
\end{align*}
\]
Letting

- \( z = (x_1^1, x_2^1, x_1^2, x_2^2, q_1^1, q_2^1, q_1^2, q_2^2) \)
- \( c = (70, 50, 80, 60, -5, -10, -8, -20) \)
- \( b = (200, 100, 200, 200, 200, 200, 200, 400, 0) \)

\[
M = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1
\end{pmatrix}
\]
In matrix notation

The ISO problem is

\[
\begin{align*}
\text{max} & \quad c \cdot z \\
\text{s.t.} & \quad M \cdot z \leq b
\end{align*}
\]

The solution to this problem is

- Loads: \((x_1^1, x_1^2, x_2^1, x_2^2) = (200, 100, 200, 200)\)
- Generation: \((q_1^1, q_1^2, q_2^1, q_2^2) = (200, 200, 200, 100)\)

This is called the economic dispatch.
The price

Not only does the ISO determine the economic dispatch, but also the price of power. The price of power will be the value of the Lagrangian multiplier of the balance constraint. In order to find it, we can solve the dual of the optimal dispatch problem:

$$\begin{align*}
\min_{y \geq 0} & \quad y \cdot b \\
\text{s.t.} & \quad y \cdot M \geq c
\end{align*}$$

where $b$, $c$, and $M$ have already been defined and $y = (\alpha^1_1, \alpha^2_1, \alpha^1_2, \alpha^2_2, \beta^1_1, \beta^2_1, \beta^1_2, \beta^2_2, \lambda)$ is the vector of dual variables associated to each of the nine constraints. We are especially interested in the value of $\lambda$, which turns out to be equal to 20.
The Dispatch

The outcome is the following:

- Loads: \((x_1, x_2) = (300, 400)\)
- Generation: \((q_1, q_2) = (400, 300)\)
- Price: 20
- Transmission: 100
- Transmission rent: 0
Economic dispatch

The socially optimal dispatch

\[ C_1 \rightarrow \text{1} \rightarrow \text{2} \rightarrow C_2 \]

\[ G_1 \rightarrow \text{1} \rightarrow \text{2} \rightarrow G_2 \]
Economic dispatch

The socially optimal dispatch

\[ p_1 = 20 \]

\[ p_2 = 20 \]
Constrained dispatch

Assume now that the transmission line is constrained to carry no more than 60 units. Using bus 2 as the reference this constraint can be stated as

\[ |x_1^1 + x_1^2 - q_1^1 - q_2^2| \leq 60. \]

Given that in the unconstrained problem power flowed from bus 1 to bus 2, we can guess that in our constrained problem the flow will have the same direction. Hence the relevant constraint can be written as

\[-x_1^1 - x_1^2 + q_1^1 + q_2^2 \leq 60.\]
The ISO’s problem

\[
\max_{(x,q) \geq 0} (70x_1^1 + 50x_2^2) + (80x_2^1 + 60x_2^2) - (5q_1^1 + 10q_1^2) - (8q_2^1 + 20q_2^2)
\]

s.t.

\[
\begin{align*}
x_1^1 & \leq 200 \\
x_1^2 & \leq 100 \\
x_2^1 & \leq 200 \\
x_2^2 & \leq 200 \\
q_1^1 & \leq 200 \\
q_2^1 & \leq 200 \\
q_1^2 & \leq 200 \\
q_2^2 & \leq 400 \\
x_1^1 + x_1^2 + x_2^1 + x_2^2 & = q_1^1 + q_1^2 + q_2^1 + q_2^2 \\
-x_1^1 - x_1^2 + q_1^1 + q_1^2 & \leq 60
\end{align*}
\]
Letting

\[ \tilde{b} = (200, 100, 200, 200, 200, 200, 200, 400, 0, 60) \]

\[ \tilde{M} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0
\end{pmatrix} \]
In matrix notation

The ISO problem is

\[
\begin{align*}
\max_{z \geq 0} & \quad c \cdot z \\
\text{s.t.} & \quad \tilde{M} \cdot z \leq \tilde{b}
\end{align*}
\]

The solution to this problem is

- Loads: \((x_1^1, x_2^1, x_1^2, x_2^2) = (200, 100, 200, 200)\)
- Generation: \((q_1^1, q_1^2, q_2^1, q_2^2) = (200, 160, 200, 140)\)

This is the economic dispatch.
The price

Not only does the ISO determine the economic dispatch, but also the price of power at each bus and the price of transmission.

- The price of power at the reference bus will be the value of the Lagrangian multiplier of the balance constraint.

- The price of transmission will be the Lagrangian multiplier of the transmission constraint.

- The price of power at bus 1 will be the difference between the price at bus 2 and the transmission price.
In order to find these prices, we can solve the dual of the optimal dispatch problem:

\[
\min_{\tilde{y} \geq 0} \quad \tilde{y} \cdot \tilde{b} \\
\text{s.t.} \quad \tilde{y} \cdot \tilde{M} \geq c
\]

where \( b, c, \) and \( M \) have already been defined and \( \tilde{y} = (\alpha_1^1, \alpha_1^2, \alpha_2^1, \alpha_2^2, \beta_1^1, \beta_1^2, \beta_2^1, \beta_2^2, \lambda, \mu) \) is the vector of dual variables associated to each of the nine constraints. We are especially interested in the values of \( \lambda \) and \( \mu \), which turn out to be equal to 20 and 10, respectively.
The Dispatch

The outcome is the following:

- Loads: \((x_1, x_2) = (300, 400)\)
- Generation: \((q_1, q_2) = (360, 340)\)
- Price at bus 1: 10
- Price at bus 2: 20
- Price of transmission: 10
- Transmission flow: 60
- Transmission rent: 600
Economic dispatch

The socially optimal dispatch

\[ C_1 \rightarrow 300 \rightarrow 360 \rightarrow G_1 \]
\[ C_2 \rightarrow 400 \rightarrow 340 \rightarrow G_2 \]

1 – 2
60
Economic dispatch

The socially optimal dispatch

\[ p_1 = 10 \]
\[ p_2 = 20 \]

\[ C_1 \]
\[ C_2 \]

\[ G_1 \]
\[ G_2 \]

\[ \mu = 10 \]

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