Linear programming

- Graphical method
  Draw the feasible region (convex polyhedron)
  evaluate the extreme points

- Simplex method (Write the problem in Std. form, when introduce slack/excess variables to have all constraints in <= form)
  Simpler case: when all constraints have a slack variable.
  Then BV for each constraint equals slack variable for that constraint.

General case: Some constraints have excess variables, or no slack/excess variables.
  Then we need to set up an auxiliary LP by introducing aux. variable in each such constraint.
  Solve aux. LP to minimize sum of all aux. variables.
  For each constraint the BV is either a slack variable or an aux. variable.
  Solution of aux. LP = 0 \iff \exists solution for original LP.
When aux. LP has solution = 0,
Original LP can be solved by using the BVs to
be same as BVs of aux. LP.

1) Table is “ready” if in "RO" coefficients of BUs
   are all zero.

2) Table optimal? No -ve coefficient in "RO" → Optimal.
   Otherwise identify entering BV as the variable with
   most -ve coefficient in RO. (This defines pivot column)
   Identify leaving BV (i.e., identify pivot row): Pick the
   row that is most constraining by doing the ratio test
   (ignore rows whose pivot column entries are "0" or "ve")
   If all rows are “non-constraining” (Since all pivot column
   entries are "< 0") → D No leaving BV =⇒ Optimal is unbounded
   STOP
3) If entering & leaving BVs are found, then modify table with respect to new BVs. This is done by making pivot number = 1 and all other entries in pivot column = 0. Also, the pivot row will have a new BV = Enterig BV.

Repeat 1 - 3.

Aux. LP. min \( z = - \sum a_i \)

\( \Rightarrow \max \left[ -z = \sum a_i \right] \)

\( \Rightarrow -z + a_1 + a_2 + \ldots + a_K = 0 \) "Row"

\( \begin{array}{cccccccc} R_0 & x_1 & \ldots & x_n & s_1 & \ldots & s_m & e_1 & \ldots & e_p \ a_1 \ldots & a_K \end{array} \)

\( \begin{array}{cccccccc} & 0 & -2 & 0 & \ldots & 0 & 1 & 1 & 1 & 0 \end{array} \)
B&B for MIP

1. STOP
   if LP has no optimal solution
2. LP has an optimal solution
   solution is feasible for MIP
   STOP
   LP version of given MIP
   ignore integer constraints
3. solution is not feasible for MIP (25 = 5.7)

   P2: a + b > 5

   P3: P1 + a ≤ 5

   STOP

   Set a new upper bound (max. problem)

   lower bound (min. problem)

   Branch

   obt. 13 inferior
   STOP