Capacitance & Inductance

- Battery (and other sources) supply energy, whereas resistors consume energy.
- Capacitors and inductors store electrical energy and supply that to circuit in future (since they cannot generate any energy of their own, they are considered passive).
- Capacitors: Consists of two separate sheet of conductors, separated by an insulating material, called dielectric, such as air, mica, polyester.

When capacitor is connected to battery, electrons from the plate connected to +ve end start to migrate to the end of battery, and electrons from the -ve end of battery migrate to the plate connected to -ve end. This sets up an electric field, opposing the flow of electrons, which eventually stops. The two plates acquire a net +ve/-ve charge of equal amount.

Acquired charge ∝ applied voltage

\[ q = CV \]

Constant of proportionality called capacitance (capacity to acquire charge when subject to an electric field)

- Behavior of a circuit element described in terms of its voltage-current (v-i) relationship. For capacitor,

\[ \frac{\Delta Q}{\Delta t} = C \frac{\Delta V}{\Delta t} \]
Capacitance (cont.)

Voltage: \( i = c \frac{dv}{dt} = \int_0^t i dt \)

\( \Rightarrow v(t) - v(t_0) = \frac{1}{c} \int_{t_0}^{t} i dt \)

Also, \( v(t) = \frac{1}{c} [q(t) + \int_{t_0}^{t} i dt] \)

Energy: \( w(t) = \int_0^t P(t) dt = \int_0^t v(t) i(t) dt = \int_0^t v(t) c \frac{dv}{dt} dt \)

\( = \int_0^t c v(t) dv(t) = \frac{1}{2} c v(t)^2 \bigg|_0^t = \frac{1}{2} c v(t)^2 \)

Also, \( w(t) = \frac{1}{2} c q^2(t) \)

Example: \( i(t) = \begin{cases} 0 & t < 0 \\ Ae^{-Bt} & t \geq 0 \end{cases} \)

\( \Rightarrow q(t) = \int_{t_0}^{t} i(t) dt \)

\( = \begin{cases} 0 & t < 0 \\ \frac{Ae^{-Bt}}{B} & t \geq 0 \end{cases} \)

\( \Rightarrow v(t) = \frac{2q(t)}{c} = \begin{cases} 0 & t < 0 \\ \frac{Ae^{-Bt}}{2B} [1 - e^{-Bt}] & t \geq 0 \end{cases} \)

\( w(t) = \int_0^t v(t)^2 dt = \begin{cases} 0 & t < 0 \\ \frac{A^2}{2B^2} [1 - e^{-Bt}]^2 & t \geq 0 \end{cases} \)