Equalization

- Equalization needed to compensate for distortion introduced by transmission medium.
- Distortion
  - Amplitude distortion: amp. response not constant fn. freq.
  - Phase distortion: phase response not linear fn. freq.

(Recall for a distortionless system, amp. response = constant, and phase response = linear)

- A transmission medium that introduces distortion is called dispersive. If it is LTI system.

\[
\begin{align*}
\text{transmitted signal} & \rightarrow \text{dispersive medium} & \text{received signal} \\
H_c(j\omega) & \rightarrow \text{compensated signal} & H_e(j\omega)
\end{align*}
\]

- To distortionless behavior, \( H_c(j\omega) H_e(j\omega) = e^{-j\omega t_0} \) for some \( t_0 \), dispersive equalizer

Note: \( t_0 \) is a design parameter (along with \( H_e(j\omega) \)).

- Then \( H_e(j\omega) = \frac{e^{-j\omega t_0}}{H_c(j\omega)} \)

For a FIR filter based equalizer, \( t_0 \) is chosen to be delay corresponding to half window length, i.e.,

\( t_0 = (M/2)T \).

The sampling period \( T \) is chosen based on desired frequency-range for equalization, say \([-w_c, w_c]\).

Then from Nyquist sampling requirement, \( T = 1/(\text{band-width in Hz}) = 1/(2w_c/2\pi) = \pi/w_c \)
Equalizer Example

Consider a channel that behaves like a 1st-order Lowpass Butterworth filter:

\[ H_c(j\omega) = \frac{1}{1 + j\omega/T} \]

\[ H_{eq}(j\omega) = (1 + j\omega/T) e^{-j\omega T} \]

Suppose we need to design an equalizer over freq. range \(|\omega| \leq \pi\) using an order-\(N\)-digital filter:

\[ H_d(j\omega) = \begin{cases} (1 + j\omega/T) e^{-j\omega (M/2)} & |\omega| \leq \pi \\ 0 & \text{otherwise} \end{cases} \]

In this example, \(\omega_c = \pi \Rightarrow T = \frac{\pi}{\omega_c} = 1. \Rightarrow \omega = \omega T = \omega.\)

Thus,

\[ H_d(e^{j\omega}) = \begin{cases} (1 + j\omega/\pi) e^{-j\omega (M/2)} & |\omega| \leq \pi \\ 0 & \text{otherwise} \end{cases} \]

\[ h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1 + j\omega/\pi) e^{j\omega (n-k/2)} d\omega \]

\[ = \frac{1}{2\pi} \left( 2 \sin \left[ \frac{\pi (n-k)}{2} \right] + \frac{\cos \left[ \pi (n-k/2) \right]}{\pi (n-k/2)} - \frac{\sin \left[ \pi (n-k/2) \right]}{\pi^2 (n-k/2)^2} \right) \]

\[ = \begin{cases} 1 & \text{if } n = \frac{k}{2} \\ 0 & \text{otherwise} \end{cases} \]

\[ h[n] = h_d[n] \cdot \text{win} \]

Fig 8.30 shows the plot.
Figure 8.30 (p. 653)
Magnitude response of Butterworth channel of order 1: dashed and dotted (— •— •) curve. Magnitude response of FIR equalizer of order \( M = 12 \): dashed (— —) curve. Magnitude response of equalized channel: continuous curve. The flat region of the overall (equalized) magnitude response is extended up to about \( \omega = 2.5 \).
FIR FILTER (Digital)
  fir1
  fir2
  fir1s
  fir1m
  fircl
  firclm
  c fir pm
  firco

IIR FILTER (Digital)
  butter
  cheby1
  Cheby2
  ellip
  bessel

Analog FILTER
  buttap
  chebap
  cheb2ap
  ellipap
  besselap

Filter Transformations
  lp2bp
  lp2bs
  lp2hp
  lp2lp

Filter Discretization
  bilinear
  impimp

Others
  maxflat
  yulewalk
  filter (num, den, input_seq) (digital)