PF Large Dim State Spaces - 2

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Large dim tracking problems

- **Image Sequences**
  - Boundary contour of a moving/deforming object
  - Optical flow (motion of each pixel)

- **Spectro-temporal receptive fields (STRF)**
  - Neuronal transfer function: varies with time

- **Sensor Networks**
  - Spatially varying physical quantities, e.g. temperature
  - Boundary of a chemical spill or target emissions
Our Problem

- Observation likelihood is multimodal (multiple actual modes or temporary false modes): require a PF
  - e.g. multiple contours, clutter, occlusions, low contrast images

- Large dim state space: PF is expensive & requires impractically large N
  - e.g. contour dim can be as large as image dimension
Small Dim Effective Basis \([\text{PFLD-1}]\)

- Consider \(C_t \in S, \, v_t \in TS_{C_t} \) : “velocity” of \(C_t\)

- “Most of the state change” occurs in a small number of dimensions (effective basis), \(v_{t,s} \in \mathbb{R}^K\)

- State change in rest of the dimensions, \(v_{t,r}\), is small: \(\text{Covar}(v_{t,r}) = \text{Covar}(C_t \mid C_{t-1}, v_{t,s}) = \Delta_K\) small

- Effective basis dim, \(K\), can change over time
State Space Model \([\text{PFLD-1}]\)

- \(v_t = B_s v_{t,s} + B_r v_{t,r}\), AR model on \(v_{t,s}\) & \(v_{t,r}\) i.i.d.
  - state \(X_t = [C_t, v_{t,s}]\)

- **System Model:**
  - \(C_t = C_{t-1} + g(\hat{C}_t, B_r v_{t,r})\), \(v_{t,r} = n_{t,r} \sim N(0, \Delta_K I)\)
  - \(\hat{C}_t = C_{t-1} + g(C_{t-1}, B_s v_{t,s})\)
  - \(v_{t,s} = A v_{t-1,s} + n_{t,s}, n_{t,s} \sim N(0, \Sigma)\)

- **Observation:** \(p(Y_t|X_t) = p(Y_t|C_t) = \exp[-E(C_t)]\)
Assume \[ \text{[PFLD-1]} \]

- **Unimodality of** \( p^{**}=p(C_t|C_{t-1}^i,v_t^i,v_{t,s}^i,Y_t) \): \( \Delta_K \) small enough to ensure this, \( \Delta_K < \min_i \Delta_K^*(C_{t-1}^i,v_{t,s}^i,Y_t) \)

- **Replace IS\( \rightarrow \)MT:** \( \Delta_K \) small enough so that with high probability, there is little error in replacing \( C_t^i \sim N(m_t^i,\Delta_K I) \) by \( m_t^i \)
Proposed PF Algorithm \[PFLD-1\]

- Under the above assumptions, can modify the PF as follows:
  
  - Sample $v_{t,s}^i$ from its state transition prior
  
  - Compute the single mode of $p^{**} = p(C_t|C_{t-1}^i,v_{t,s}^i,Y_t)$
  
  - Deterministically set $C_t^i = \text{mode of } p^{**}$
Current Work: Main Idea

We study

• how to compute the effective basis dim, K, that satisfies these assumptions

• interpret contour deformation as a spatially band-limited periodic signal: way to detect/estimate change in K

• handling errors in estimate of K
Outline

• Recap of assumptions
• Computing $K$ to satisfy the assumptions

• Spatial Frequency Interpretation
• Use to detecting change in $K$, estimating $K_{\text{new}}$

• Handling errors in estimating $K$
• Results & Future Work
Computing K to satisfy unimodality

• If current K, K₀, does not satisfy unimodality, i.e. Δₖ₀ > Δₖ₀ *, then do the following

  • Find a K₁ > K₀ so that Δₖ₁ < ε = Δₖ₀ *
    – K₁ exists if TSₖ has a countable orthogonal basis

  • Δₖ * non-decreasing in K: Δₖ₀ * ≤ Δₖ₁ *

• So Δₖ₁ < Δₖ₁ *: satisfies unimodality
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Contour Deformation “Signal”

- $C_t(p)$: closed contour, $v_t(p)$: contour deformation in normal direction

- $v_t(p)$: periodic function of arc-length of $C_t$ with period = contour length, $L$

- $v_{t,s}$: velocity at $K$ uniformly spaced points (knots) on arc-length (sub-sampled $v_t$)

- $B_s$ = interpolation filter, use B-splines since handles non-uniform spacing of sample points
Spatial Frequency Interpretation

• Small effective basis dim $\Leftrightarrow$ spatial freq of $v_t(p)$ is approx band-limited & $f_{\text{max}} \ll f_s/2$
  – $f_{\text{max}}$ : approx cut-off

• Effective basis dim $K = L \cdot (2f_{\text{max}})$
  – Using a smaller $K \Leftrightarrow$ low pass filtering (estimating a smoothed contour)
  – Using a much larger $K \Leftrightarrow$ estimating noise
Two contours & the deformation (def)
def plotted as a function of contour arc-length, def filtered using a box-car filter with cut-off 0.055 & 0.071 Hz
Fourier transform of def: cut-off freq ~ 0.071Hz (cycles/pixel)
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Need to change $K$

- System model same but $\Delta K^*$ becomes smaller (depends also on the current observation): increase $K$ until unimodality is satisfied

- System model changes, i.e. need larger $K$ to get same value of $\Delta$
  - While tracking, cannot estimate new $\Delta$: not tracking residual state change, use heuristics
Detect Change in Effective Basis

• L changes significantly or distance b/w knots becomes non-uniform
  – Detect: distance b/w consecutive knots much larger or smaller than $1/(2f_{\text{max}})$

• $f_{\text{max}}$ changes significantly
  – Detect: estimate energy in current $f_{\text{max}}$ using past tracked deformation, if much larger than expected: increase K
Computing New Basis

- Compute $K_{\text{new}} = L_{\text{new}} \cdot 2f_{\text{max,new}}$
  - Can estimate $L_{\text{new}}$ (assuming no loss of track)
  - Estimate $f_{\text{max,new}}$: use another method to get $C_t$
  - Or guess: $K_{\text{new}} \in [K, K+A]$ (if increase in $f_{\text{max}}$)

- Allocate $K_{\text{new}}$ knots uniformly on each $C_t^i$ & compute new basis $B_{K,\text{new}}^i$

- Project $v_{t,s}^i$ into new basis
PF with Time Varying Basis

- Importance Sample + Mode Track
- Weight
- Resample

- Detect need to change basis

- If needed, estimate new basis, project $v_{t,s}^i$ in new basis
Implicit Assumptions

1. No error in estimating new basis dim, \( K_{\text{new}} \)

2. No delay in detecting the need to change basis

3. No error in computing \( v_{t,s}^i \) in new basis
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• Handling errors in estimating K
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Error in Estimating $K_{\text{new}}$

• If Assumption 1 not hold, $K_{\text{new}}$ is an unknown static parameter during $[T_j, T_{j+1}]$
  – $\{T_j\}$: basis change times

• Under a weaker set of assumptions, can apply a modified Adaptive PF for static parameters [Papavasiliou’04] during $[T_j, T_{j+1}]$
Weakened Assumption 1

1. \( K_{\text{new}} = K_{\text{old}} + \text{Unif}(0,A) \) : compact prior

2. For any \( K_{\text{new}} \), PF for rest of state space is uniformly convergent

3. Adaptive PF for \( K_{\text{new}} \) converges in finite time, \( T_{\text{conv}} \)

4. Time b/w basis change times: \( T_{j+1} - T_j > T_{\text{conv}} \)
Modified Adaptive PF

• Before $t = T_j$, previous APF converged (Ass. 3,4)
  – Only 1 value of $K$ has survived: $K_{\text{old}}$

• At $t = T_j$, if $A=1$, run PFs for $K_{\text{new}} = K_{\text{old}}, K_{\text{old}} + 1$. If $A=-1$, run PFs for $K_{\text{new}} = K_{\text{old}} - 1, K_{\text{old}}$
  – For each PF, resample $N/2$ times from posterior at $T_j$ to get initial state pdf & then run the PF

• At $t = T_j + T_{\text{conv}}$,
  – Choose MAP value of $K_{\text{new}}$, eliminate other PFs
  – Resample $N$ times from the $N/2$ particles of this PF
Zero delay in detecting change

• If this assumption does not hold, cannot show convergence of PF with N

• Under assumption of bounded delay, can possibly extend existing results to show stability of the PF with $t$, for large $N$
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• Detecting Change in $K$ & Estimating $K_{new}$

• Handling errors in estimating $K$
• Contour Tracking Results & Future Work
Background clutter
(2 OL modes at same affine location) &
Large non-affine deformation per frame:
using K=6 uniformly sampled points to estimate
deformation
Background clutter – light grey object (2 OL modes at same affine location) & Large non-affine deformation per frame: using affine deformation, K=4 (assuming no translation)
Future Work

• Detecting change in $K$
• Practical issues of Modified Adaptive PF
• Non-uniform sampling: spatially varying $f_{\text{max}}$

• Proofs of asymptotic stability of Modified Adaptive PF
• Proof of stability under bounded delay