* Monday $\boxed{2-2:30}$ –

  – Recitation session

Projects → Read a paper & discuss it

& → Tell ~~me~~ us about your research, Questions.

→ Your input. ~~will~~ I'll answer Questions

→ Yu Cao's ~~pro~~ questions : share with class.

→ I can show videos, ppts etc.

Volunteer : make notes of the session & post them

→ MATLAB questions

→ Project related ques.

→ Your Questions : will address in next Wednesday's class.

"Geometric" – ~~evolution eq does not depend on~~ parameter p.

"Arc-length" – geometric Quantity : same value for any parametriz".

Definition ①

"Arc length param" "ds" defined so that $\boxed{\left\|\frac{\partial c}{\partial s}\right\| = 1}$.

Definition ② : "Arc-length" : ~~$ds = \|C_p\| dp$~~ for any param $C(p)$
$p: 0 \to 2$.

$$\frac{\partial c}{\partial s} = \frac{\partial c}{\partial p}\frac{dp}{ds} \qquad \left\|\frac{\partial c}{\partial s}\right\| = 1$$

$$\Rightarrow ds = \left\|\frac{\partial c}{\partial p}\right\| dp$$

Sept 14: [Recap.]

[KWT] ← Kass Witkin Terzopolous

$$E_{int} = \int \left[\alpha(p)\,\|C_p\|^2 + \beta(p)\,\|C_{pp}\|^2\right]$$

$$E_{PANG} = \int_{p=0}^{1} g_{decr}(\nabla I(C))\,dp$$

$+ \; I(x, y)$

$\boxed{\text{MINUS SIGN}}$ — $$\nabla E = \nabla g - \frac{\partial}{\partial p}\left[2\alpha(p)C_p\right] + \frac{\partial^2}{\partial p^2}\left[\beta(p)C_{pp}\right]$$

← Error in Book.

Define $\boxed{\text{"contour"}}$: locus of points traced out by the mapping of unit interval $[0, 1]$ ~~(circle)~~ onto $\mathbb{R}^2$. ~~s.t. $\gamma(0) = (1)$~~

→ Any mapping: $C(p): [0, 1] \to \mathbb{R}^2$.

$C(p) \in \mathbb{R}^2$.

is a "parametrization"

→ $\boxed{\text{Closed}}$ if $C(0) = C(1)$ $\boxed{\text{Simple}}$: $C(p_0) \neq C(p_1)$ $\forall\, p_0 \neq p_1$

→ Many param — can denote the same curve ⇒ NOT UNIQUE.

eg $C(p) = \cos 2\pi p,\ \sin 2\pi p.$ $\quad \cos 2\pi p^2,\ \sin 2\pi p^2$] circle

$\boxed{\text{KWT}}$: ~~Used~~ Defined $\boxed{\text{"gradient flow"}}$ (gradient ~~of~~ over continuous curve is called gradient flow) w.r.t. one particular parametrization.

→ Uniformly sampled the initial curve:

· Arclength param for the original curve but not after deformation

PROBLEMS: (a) change in contour length (scale up or increasing concavities)

(b) change in topology: splitting / merging

(c) Some points come very close to each other in certain region

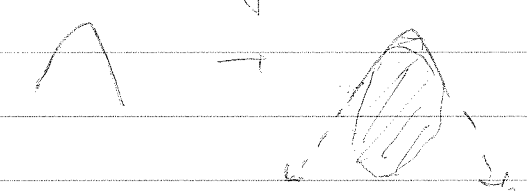
SOLN 1: SNAKE GROWING

- initial guess snake, - split it into smaller pieces → keep low energy pieces ONLY.

If energy not reducing

split snake

→ Each piece: Allow it to "grow" (increase contour length along tangent to edges)



→ Follow up with energy min of this grown snake

SOLN 2: Varying Bspline control points, Topological snakes

Add links / have it on Reserve.

Soln 3 : "Geometric" "Active Contours"

"Geometric" : independent (invariant) param. — can be expressed using arclength param. of contour & derivatives w.r.t. arclength. [Yezzi's class notes].

"Arclength param" — defn: ds. s.t.

$$\left\| \frac{\partial C}{\partial s} \right\| = 1 \quad \forall s$$

→ Let $C(p)$ is any arbitrary param., then

$$\frac{\partial C}{\partial s} = \frac{\partial C}{\partial p} \frac{dp}{ds}.$$

$$\left\| \frac{\partial C}{\partial s} \right\| = 1 \Rightarrow \boxed{ds = \| C_p \| \, dp}$$

$C_p = \frac{\partial C}{\partial p}$

Geom param — value of parameter corresponds to a geometric prop of curve

Other examples: $\boxed{\text{curvature}}$

$$K = \left\| \frac{\partial^2 C}{\partial s^2} \right\|$$

$\frac{\partial}{\partial s}$

$$\boxed{\text{Tangent} \quad T = \frac{\partial C}{\partial s} = \frac{C_p}{\|C_p\|}}$$

$$\boxed{K = \left\| \frac{\partial \vec{T}}{\partial s} \right\|}$$

$$\langle \vec{T}, \vec{T} \rangle = 1$$

$$\langle \vec{T}, \frac{\partial T}{\partial s} \rangle = 0$$

$$\Rightarrow \frac{\partial T}{\partial s} \perp T$$

$$\vec{N} = J\,\vec{T}, \quad J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\boxed{\frac{\partial \vec{T}}{\partial s} = K\,\vec{N}}$$

with sign

$\nabla_c E$: tied to definition of "inner product".

$\nabla_c E$ is defined by

$$\frac{\partial C}{\partial \varepsilon} = \lim_{\varepsilon \to 0} \frac{E(C+\varepsilon) - E(C)}{\|\varepsilon\|} = \nabla_c E \circ \varepsilon$$

Different inner products ~~types~~

(1) For vectors: $\sum_{i=1}^{N} (\nabla_c E)_i\, \varepsilon_i = \nabla_c E \circ \varepsilon$

(2) $C : C(p)$ $\quad \int_0^1 (\nabla_c E)(p)\, \varepsilon(p)\, dp$

(3) $C : C(s)$ $\quad \int_0^L (\nabla_c E)(s)\, \varepsilon(s)\, ds.$

NOTE: p, t indep.

$s\ t$ — not indep.

p: indep of the contour shape $0 - 1$

s: dep. on contour. $s: 0 \to L$.

EDGE BASED GAC: CASELLES et Al
KICHENASSAMY et Al

$$E = \int_0^{L(c)} ds$$ — min arc length

Flow $\nabla_c E = -K\vec{N}$ — $\vec{N}$ INWARD

FLOW — PDE $\frac{\partial c}{\partial t} = +K\vec{N}$ — ~~$\vec{N}$~~ outward normal

CURVE SHORTENING FLOW.

"t" artificial time parameter to facilitate gradient descent

$$C^{n+1} = C^n - \gamma \nabla_c E$$

Mention "s & t" "not indep"

Gamma "s" depends on the contour.

$$E = \int_0^{L(C)} ds(C)$$

$\leftarrow$ der

Limits of integral depend on contour.

$\downarrow$

convert to a form where they do not

$\downarrow$

Assume arbitrary param. $C(p) : 0 \to 1$

$$ds = \|C_p\| \, dp$$

$$E = \int_0^1 \|C_p\| \, dp \quad \Rightarrow \nabla_C E = \frac{d}{dp}\frac{\partial L}{\partial C_p}$$

Now can use CALC OF VARIATIONS formulas.

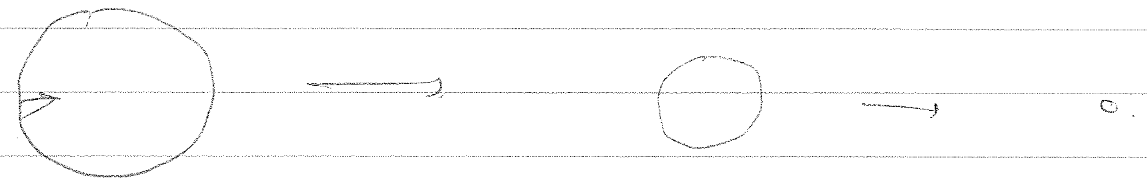
MODIFY ARCLENGTH : weight by $g_{dec}(\|\nabla I\|)$

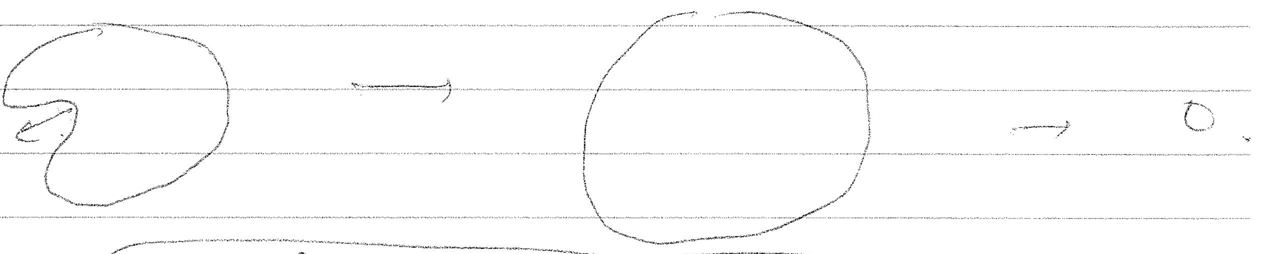
$$E = \int_0^L g_{dec}(\|\nabla I\|) \, ds.$$

FLOW $\nabla_C E = \nabla g - \cancel{g} \, g \kappa \vec{N}$

$g \leftarrow g_{dec}(\|\nabla I(C)\|)$, $\quad \dfrac{\partial C}{\partial t} = g \kappa \vec{N} - \nabla g$

Curve shortening flow : smoothing contour





Concavity moves outwards.

Region Based GAC : Chan & Vese

$E = \cancel{\iint (I - c_1)^2 \, dx\,dy}$

$$\int_{\text{inside}} (I - u_1)^2 \, dx\,dy + \int_{\text{outside}} (I - u_2)^2 \, dx\,dy + \boxed{\int ds}$$

(a) — Using divergence theorem ~~etc~~

& ~~integ by parts~~ — (b)

Convert region integral

to boundary integral

convert w.r.t. to s Boundary integral w.r.t.

(b) Boundary integral → parameter p

Now: limits of integration indep of C

(c) Calc var to integrate by parts

$$\nabla_C E = \left[(I(C) - u_1)^2 - (I(C) - u_2)^2 \right] \vec{N} - \alpha K \vec{N}$$

inward normal

$$\frac{dC}{dt} = \alpha K N - (I(C) - u_1)^2 + (I(C) - u_2)^2 .$$

GENERAL REGION BASED INTEGRAL

$$E = \iint_{\text{inside}} f(x,y)\, dx\, dy$$

$$\nabla_C E = f(C)\, \vec{N}$$

IMPLEMENTATION: LEVEL SET METHOD

PRACTICAL ISSUES : INITIALIZATION

Edge based : Contour Outside object will move inside to decrease arc length - force the gradient

Region based — Half inside, half outside will also work

ALL INSIDE : NOT WORK

↓

then need " $-\nu\vec{N}$ " term.

LEVEL SET METHOD

Today

1

1. ~~Edg~~ Dot product $\oint \int \nabla F(s)\, E(s)\, ds$.
2. Edge based GAC. — $\boxed{2^{nd} \text{ deriv of image}}$
3. Region based :
   - learn u, v.

   $\leftrightarrow$ Alternating maximization.

$$C_t = \cancel{\ldots}\; g \kappa N - (\nabla g \cdot N) \vec{N}$$

geometric

every term is a fn of arclength or its derivatives w.r.t. t

$$C_t = (u - v)\left(I - \frac{u+v}{2} \right) + \alpha \kappa N.$$

4. Interpreting terms.

$\boxed{\kappa N t}$ — smoothes curves, shrink it, removes inflection pts, vertices, (max/min curvature)

discontinuities

$\boxed{g \kappa N}$ — modifies flow to stop where ∇I is large

$-(\nabla g \cdot N) \rightarrow$ pushes evolution along direction where $\|\nabla I\|$ increase (or ∇g decreases)

$(u - v)\left(I - \frac{u+v}{2}\right)$: If $u - v > 0$, move outwards; ~

Any <u>energy min</u>

↳ "clustering" & "contour estimation"

∫

Alternating Min . or as Chan & Vese

<u>Curvature</u> : angular ~~acceler~~ speed when traveling with unit speed. $\frac{\partial T}{\partial s} = \frac{\partial \theta}{\partial s} \vec{N}$

Level set Method.

— Edge : Contour Outside

Region : Anywhere

$$K = \nabla \cdot \left(\frac{\nabla \phi}{\|\nabla \phi\|} \right) \qquad \vec{N} = - \frac{\nabla \phi}{\|\nabla \phi\|}$$