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Coordinated Vehicle Platoon Control: Weighted and Constrained Consensus and Communication Network Topologies

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Abstract— **This paper introduces a new method for enhancing highway safety and efficiency by coordinated control of vehicle platoons. One of our aims is to understand influence of communication network topologies and uncertainties on control performance. Vehicle deployment is formulated as a weighted and constrained consensus control problem. Algorithms are introduced and their convergence properties are established. The main advantages of the methods are demonstrated, including using local control to achieve a global deployment so that communication complexity is reduced; scalability to accommodate dynamic changes of the member vehicles and communication networks; robustness against road conditions and communication uncertainties.**

Keywords. Platoon control, safety, consensus control, networked systems, communications.

I. Introduction

The goal of vehicle platoon control is to ensure that all the vehicles move in the same lane at the same speed with desired inter-vehicle distances. Platoon control adjusts vehicle spatial distribution such that road utilization is maximized while the risk of collision is minimized (within an acceptable bound). In this study, platoon control will be realized in the framework of weighted and constrained consensus control with switching network topologies.

This paper aims to introduce a new framework for vehicle coordination and control, based on the emerging technology of network consensus control. The core target is to achieve suitable coordination of the team vehicles based on road conditions and vehicle types. For instance, wet pavement demands longer inter-vehicle distance than dry surface; departing of a vehicle from the platoon requires re-distribution of space among the remaining vehicles. In this paper, vehicle platoon control is formulated as a weighted and constrained consensus control problem. Consensus control aims to use only local information to coordinate all subsystems such that their formation converges to a desired distribution pattern. In vehicle applications the desired pattern is that the weighted distances between consecutive vehicles are equal. Consensus control is an emerging field in networked control and remains an active research field. At present, most consensus controls are unconstrained and un-weighted. However, inter-vehicle distances are not to be uniform. For example, a heavy truck needs more distance to stop; distances are to be adjusted on hilly roads vs straight highways. Consequently, vehicle platoon control should be weighted. Moreover, a platoon needs to maintain a certain length for its effectiveness in improving safety and road utility. Vehicle coordination needs to be conducted under the constraint of the platoon length. This implies that the related consensus control must be constrained.

Employing and enhancing the averaging consensus control methodologies recently developed by the authors [20], this paper provides a new vehicle control strategy. Algorithms are introduced and their convergence properties are established. We demonstrate the main advantages of this methodology: (1) Global goal: Using neighborhood information and control to achieve a global goal. Although a desired goal is achieved for the entire team, each vehicle only needs to communicate with its neighboring members. As such communication costs and complexity remain minimal. (2) Scalability: Expanding and reduction of the team members does not complicate control strategies. (3) Robustness: Variations in vehicle positions, network topology, and team members can be readily accommodated.

Consensus control has drawn increased attention recently in a variety of application areas, including load balancing in parallel computing [16], [17], sensor networks [13], decentralized filtering, estimation, mobile agents [4], etc. Control methods include deterministic control [4], [15], stochastic approximation algorithms [1], switching network topologies [2], [5], [8], [10], [12], etc. In our recent work [20], a Markov model is used to treat a much larger class of systems, where the network graph is modulated by a discrete-time Markov chain. In addition to the switching topology, nonadditive noise was treated and convergence and rates of convergence for the corresponding recursive algorithms were provided. In this paper, we extend some of the useful features of [20]. One of the new significant developments is to treat weighted and constrained consensus. In addition, the technique of post-iterate averaging is employed in this paper to enhance the vehicle coordination. With the iterate averaging, our algorithms provide the best convergence rate in terms of the best scaling factor and the smallest asymptotic covariance. In fact, they

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achieve asymptotically the well-known Cramér-Rao lower bounds [11], hence are best over all algorithms. This fast convergence feature is highly desirable for achieving fast team formation and robustness against disturbances.

The rest of the paper is organized into the following sections. We start with a description of vehicle platoon control problems in Section II. Section III describes how a typical vehicle deployment problem can be formulated as a weighted and constrained consensus control problem. Algorithms for weighted and constrained consensus control are presented in Section IV. Their convergence properties and convergence rates are established. Section V further enhances the algorithms by post-iterate averaging. It is shown that consensus control for vehicles are subject to noises and their effect can be attenuated by the post-iterate averaging. Optimality of such modified algorithms are established. By using several examples, Section VI demonstrates robustness and scalability of the methods. Finally, Section VII points out directions of further studies.

II. Preliminaries

We concentrate on one-dimensional platoon control. This represents the case of $r+1$ vehicles driving in the same lane, forming a platoon. The leading vehicle serves as a reference, whose position p_0 is used as the origin of the line coordinate (hence, $p_0 \equiv 0$), and its speed v_0 is the reference speed for all remaining vehicles in the platoon to follow. Coordination of vehicle control is to sustain a platoon formation, avoid collision, adjust the formation according to weather and road conditions, converge fast to a new formation after disturbances, reconfigure a formation after vehicle addition and departure. Consequently, inter-vehicle distances are the variables to be controlled.

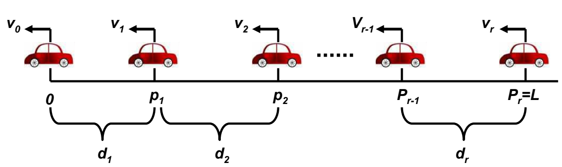

Fig. 1. Platoon coordinates

In a platoon formation (see Figure 1), each vehicle's position is defined by the central point of its length and denoted by $p_j(t)$, $j=1,\ldots,r$, which is the distance of the jth vehicle to the leading vehicle. The vehicle's velocity denoted by $v_j(t) = dp_j(t)/dt \geq 0$, $j=1,\ldots,r$. Let the inter-vehicle distances be defined as $d_j(t) = p_j(t) - p_{j-1}(t)$, $j=1,\ldots,r$. The leading vehicle's speed $v_0(t)$ is the speed target for all the other vehicles in the platoon to follow. Also, a desired distance between consecutive vehicles is a goal that balances efficiency and safety. In principle, the desired distance is a function of weather, road condition, platoon traveling speed, terrain composition (uphill or downhill), and road curvatures, and consequently may change with time.

A team consists of r vehicles to be deployed along a pathway of total length L. In algorithm development, L is treated as a constant. Its changes will be viewed as a disturbance to the consensus control problem. d_i is the distance between vehicle i and vehicle $i-1$. We have the constraint $\sum_{i=1}^{r} d_i(t) = L$. Due to terrain conditions, a desired distance before an vehicle differs at different locations. Each inter-vehicle distance has a terrain factor γ^i. The goal of platoon control is to achieve consensus on weighted distance d_i/γ^i, namely $\frac{d_i(t)}{\gamma^i} \to \beta$, $i=1,\ldots,r$ for some constant β. The convergence is either with probability one (w.p.1.) or in means squares (MS).

The basic scheme of platoon formation employs a sensor-based network topology, in which a vehicle uses sensors to measure its own speed and relative distance to the vehicle ahead of it. As a result, v_{j-1}, v_j, d_j are available to the jth vehicle in its control strategies. On the other hand, inter-vehicle wireless communications allow enhanced information exchange between vehicles. Figure 2 represents a more advanced inter-vehicle communication, in which the jth vehicle receives not only the parameters from the $(j-1)$th vehicle by sensors, but also the information from $(j-2)$th vehicle via wireless communications. Benefits and limitations of communication networks on platoon formation will be studied in this paper.

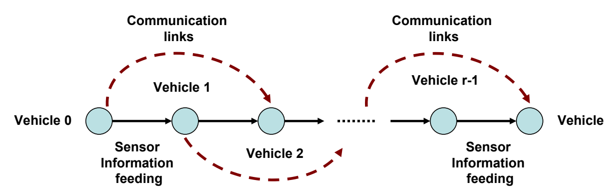

Fig. 2. Information network topologies using inter-vehicle communications

III. Weighted and Constrained Consensus Control for Platoon Coordination

For notational convenience in algorithm development, we use $x^i(t) = P^i(t)$ and denote the state vector $x(t) = [d_1(t),\ldots,d_r(t)]'$. The weighting coefficients are $\gamma = [\gamma^1,\ldots,\gamma^r]'$, and the state scaling matrix $\Psi = \text{diag}[1/\gamma^1,\ldots,1/\gamma^r]$, where v' is the transpose of a vector or a matrix v. Let $\mathbb{1}$ be the column vector of all 1s. Together with the constraint $\sum_{i=1}^{r} d_i(t) = L$, the target of the constrained and weighted consensus control is $\Psi x(t) \to \beta \mathbb{1}$ subject to $\mathbb{1}' x(t) = L$. It follows from $\gamma' \Psi = \mathbb{1}'$ that $\beta = \frac{L}{\gamma' \mathbb{1}} = \frac{L}{\gamma^1 + \cdots + \gamma^r}$.

The vehicles are linked by an information network, represented by a directed graph $\mathcal{G}$ whose element (i,j) (called a directed edge from node i to node j) indicates an observation by vehicle i on the distance d_j. This network defines the information network: $(i,j) \in \mathcal{G}$ indicates estimation of the state d_j by vehicle i via a communication link. Also, the factor γ^j is known. For node i, $(i,j) \in \mathcal{G}$ is a departing edge and $(l,i) \in \mathcal{G}$ is an entering edge. Due to the nature of wireless communication, we assume that if $(i,j) \in \mathcal{G}$ then $(j,i) \in \mathcal{G}$. The total number of communication links in $\mathcal{G}$ is

l_s. From its physical meaning, node i can always observe its own state, which will not be considered as a link in $\mathcal{G}$.

For a selected time interval T,the consensus control is performed at the discrete-time steps $nT, n = 1, 2, \ldots$. At the control step n, the value of x will be denoted by $x_n = [x_n^1, \ldots, x_n^r]'$. Vehicle platoon control updates x_n to x_{n+1} by the amount u_n

$$x_{n+1} = x_n + u_n \tag{1}$$

with $u_n = [u_n^1, \ldots, u_n^r]'$. In platoon control, a distance adjustment a_n^{ij} (called link control) of vehicle i at the nth step based on the weighted separations of vehicles i and j to the vehicles in front of them respectively is the decision variable. The control u_n^i is determined by the link control a_n^{ij} as $u_n^i = -\sum_{(i,j)\in\mathcal{G}} a_n^{ij} + \sum_{(j,i)\in\mathcal{G}} a_n^{ji}$. This implies that for all n, $\sum_{i=1}^r x_n^i = \sum_{i=1}^r x_0^i = L$ that is, the constraint $\sum_{i=1}^r d_i(t) = L$ is always satisfied. Consensus control seeks control algorithms such that $\Psi x_n \to \beta \mathbb{1}$ under the constraint given above.

A link $(i, j) \in \mathcal{G}$ entails an estimate $\widehat{x}_n^{ij}$ of x_n^j by node i with observation noise d_n^{ij}. That is, $\widehat{x}_n^{ij} = x_n^j + d_n^{ij}$. Let $\widetilde{x}_n$ and d_n be the l_s-dimensional vectors that contain all $\widehat{x}_n^{ij}$ and d_n^{ij} in a selected order, respectively. Then, we can get a vector form $\widetilde{x}_n = H_1 x_n + d_n$ where H_1 is an $l_s \times r$ matrix whose rows are elementary vectors such that if the ℓth element of $\widetilde{x}_n$ is $\widehat{x}^{ij}$ then the ℓth row in H_1 is the row vector of all zeros except for a "1" at the jth position. Each link in $\mathcal{G}$ provides information $\delta_n^{ij} = x_n^i/\gamma^i - \widehat{x}_n^{ij}/\gamma^j$, an estimated difference between weighted x_n^i and x_n^j. This information may be represented by a vector δ_n of size l_s containing all δ_n^{ij} in the same order as $\widetilde{x}_n$. δ_n can be written as

$$\delta_n = H_2 \Psi x_n - \widetilde{\Psi}\widetilde{x}_n = H x_n - \widetilde{\Psi} d_n, \tag{2}$$

where the link scaling matrix $\widetilde{\Psi}$ is the $l_s \times l_s$ diagonal matrix whose k-th diagonal element is $1/\gamma^j$ if the k-th element of $\widetilde{x}_n$ is $\widehat{x}_n^{ij}$; H_2 is an $l_s \times r$ matrix whose rows are elementary vectors such that if the ℓth element of $\widetilde{x}(k)$ is $\widehat{x}^{ij}$ then the ℓth row in H_2 is the row vector of all zeros except for a "1" at the ith position, and $H = H_2\Psi - \widetilde{\Psi} H_1$.

Due to network constraints, the information δ_n^{ij} can only be used by nodes i and j. When the platoon control is linear, time invariant, and memoryless, we have $a_n^{ij} = \mu_n g_{ij} \delta_n^{ij}$ where g_{ij} is the link control gain and μ_n is a global time-varying scaling factor which will be used in state updating algorithms as the recursive step size. Let G be the $l_s \times l_s$ diagonal matrix that has g_{ij} as its diagonal element. In this case, the control becomes $u_n = -\mu_n J' G \delta_n$, where $J = H_2 - H_1$. For convergence analysis, we note that μ_n is a global control variable and we may represent u_n equivalently as

$$u_n = -\mu_n J' G (H x_n - \widetilde{\Psi} d_n) = \mu_n (M x_n + W d_n), \tag{3}$$

with $M = -J' G H$ and $W = J' G \widetilde{\Psi}$. This, together with (1), leads to

$$x_{n+1} = x_n + \mu_n (M x_n + W d_n). \tag{4}$$

It can be directly verified that $\widetilde{\Psi} H_1 \Psi^{-1} = H_1$, $H\Psi^{-1} = J$, $J\mathbb{1} = 0$, $\Psi^{-1}\mathbb{1} = \gamma$. These imply that $\mathbb{1}' M = 0$, $\mathbb{1}' W = 0$, $M\Psi^{-1}\mathbb{1} = M\gamma = 0$. Note that for simplicity, we presented the problem using the simplest setup. In the above, the noise $W d_n$ is additive. Much more general noise can be treated as demonstrated in [20]. The following assumption is imposed on the network.

(A0) (1) All link gains are positive, $g_{ij} > 0$.
(2) $\mathcal{G}$ contains a complete tree.

We now use an example to illustrate the above concepts. Since $\mathbb{1}' J' = \mathbb{1}'(H_2 - H_1)' = 0$, we have $\mathbb{1}' M = 0$ and $\mathbb{1}' W = 0$. We can show that under Assumption (A0), M has rank $r - 1$ and is negative semi-definite. The proof uses similar ideas as in [20] and hence is omitted here. Recall that a square matrix $\widetilde{Q} = (\widetilde{q}_{ij})$ is a generator of a continuous-time Markov chain if $\widetilde{q}_{ij} \geq 0$ for all $i \neq j$ and $\sum_j \widetilde{q}_{ij} = 0$ for each i. Note that a generator of the associated continuous-time Markov chain is irreducible if the system of equations

$$\nu \widetilde{Q} = 0, \ \nu \mathbb{1} = 1 \tag{5}$$

for a given constant $C > 0$ has a unique solution, where $\nu = [\nu_1, \ldots, \nu_r] \in \mathbb{R}^{1\times r}$ with $\nu_i/C > 0$ for each $i = 1, \ldots, r$. When $C = 1$, ν is the associated stationary distribution. Consequently, under Assumption (A0), M is a generator of a continuous-time irreducible Markov chain.

IV. Weighted and Constrained Consensus Control Algorithms and Convergence

A. Algorithms

We begin by considering the state updating algorithm (4)

$$x_{n+1} = x_n + \mu_n M x_n + \mu_n W d_n, \tag{6}$$

together with the constraint $\mathbb{1}' x_n = L$, where $\{\mu_n\}$ is a sequence of stepsizes, M is a generator of a continuous-time Markov chain (hence $\mathbb{1}' M = 0$), $\{d_n\}$ is a noise sequence. Since the algorithm (6) is a stochastic approximation procedure, we can use the general framework in Kushner and Yin [7] to analyze the asymptotic properties. Since $\mathbb{1}' M = 0$ and $\mathbb{1}' W = 0$, starting from the initial condition with $\mathbb{1}' x_0 = L$, the constraint $\mathbb{1}' x_n = L$ is always satisfied by the algorithm structure.

(A1) (1) The stepsize satisfies the following conditions: $\mu_n \geq 0$, $\mu_n \to 0$ as $n \to \infty$, and $\sum_n \mu_n = \infty$. (2) The noise $\{d_n\}$ is a stationary ϕ-mixing sequence such that $E d_n = 0$, $E|d_n|^{2+\eta} < \infty$ for some $\eta > 0$, and that the mixing measure $\widetilde{\phi}_n$ satisfies $\sum_{k=0}^{\infty} \widetilde{\phi}_n^{\frac{\Delta}{1+\eta}} < \infty$, where $\widetilde{\phi}_n = \sup_{A\in\mathcal{F}^{n+m}} E^{\frac{1+\eta}{2+\eta}} |P(A|\mathcal{F}_m) - P(A)|^{\frac{2+\eta}{1+\eta}}$, $\mathcal{F}_n = \sigma\{d_k; k < n\}$, $\mathcal{F}^n = \sigma\{d_k; k \geq n\}$.

Under (A0), M has an eigenvalue 0 of multiplicity 1 and all other eigenvalues are in the left complex plan (i.e., the real parts of the eigenvalues are negative). The null space of M is spanned by the vector $\gamma = [\gamma^1, \ldots, \gamma^r]'$. As a consequence of (A1), the ϕ-mixing implies that the noise sequence $\{d_n\}$ is strongly ergodic [6, p. 488] in that for any m $\frac{1}{n}\sum_{j=m}^{m+n-1} d_j \to 0$ w.p.1 as $n \to \infty$. It is noted that typical communication noises are either i.i.d. or having finite memory [3], which are special cases of ϕ-mixing noises.

B. Convergence Properties

To study the convergence of the algorithm (6), we employ the stochastic approximation methods developed in [7]. Due to the page limitation, all proofs are omitted. Instead of working with the discrete-time iterations, we examine sequences defined in an appropriate function space. This enables us to get a limit ordinary differential equation (ODE). We define $t_n = \sum_{j=0}^{n-1} \mu_j$, $m(t) = \max\{n : t_n \le t\}$, the piecewise constant interpolation $x^0(t) = x_n$ for $t \in [t_n, t_{n+1})$, and the shift sequence $x^n(t) = x^0(t + t_n)$. the piecewise constant interpolation $x^0(t) = x(k)$ for $t \in [t_k, t_{k+1})$, and the shift sequence $x^k(t) = x^0(t+t_k)$. Then we can show that $\{x^n(\cdot)\}$ is equicontinuous in the extended sense (see [7, p. 102]) w.p.1. Thus we can extract a convergent subsequence, which will be denoted by $x^{n_\ell}(\cdot)$. Then the Arzela-Ascoli theorem concludes that $x^{n_\ell}(\cdot)$ converges to a function $x(\cdot)$ which is the unique solution (since the recursion is linear in x) of the ordinary differential equation (ODE)

$$\dot{x} = Mx. \tag{7}$$

The significance of the ODE is that the stationary point is exactly the true value of the desired weighted consensus. Then, convergence becomes a stability issue. Furthermore, the algorithm (6) together with $x_n' \mathbb{1} = L$ leads to the desired weighted consensus. The equilibria of the limit ODE (7) and this constraint lead to the following system of equations $Mx = 0$ and $\mathbb{1}' x = L$. The irreducibility of M then implies the above system has a unique solution $x_* = \beta \Psi^{-1} \mathbb{1} = \beta\gamma$, which is precisely the weighted consensus. We record this in the following theorem.

Theorem 1: Under (A0) and (A1), the iterates generated by the stochastic approximation algorithm (6) satisfies $\Psi x_n \to \beta \mathbb{1}$ w.p.1 as $n \to \infty$.

Example 1: A team of four vehicles has an assigned total length L. Vehicle i controls the distance d_i, $i = 1, 2, 3, 4$. Then the condition $d_1 + d_2 + d_3 + d_4 = L$ is imposed as a constraint. The information topology is that in addition to observing their own controlled variables, vehicle 1 observes also d_2, vehicle 2 observes also d_1 and d_3, vehicle 3 observes d_2 and d_4. the controller for d_4 observes d_3 also. The total length $L = 53.9$ m. Terrain factors $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, and $\gamma^4 = 28$. As a result, $\mathcal{G} = \{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3)\}$. $x = [d_1, d_2, d_3, d_4]'$, $\gamma = [12, 15, 20, 28]'$, $\Psi = \mathrm{diag}[1/12, 1/15, 1/20, 1/28]$. Since $L = 53.9$, we have $\beta = \frac{L}{\gamma^1+\gamma^2+\gamma^3+\gamma^4} = 0.7187$ and the weighted consensus is $\Psi x = 0.7187\mathbb{1}$ or $x = 0.7187\Psi^{-1}\mathbb{1} = [8.624, 10.781, 14.374, 20.124]'$.

Suppose that the initial distance distribution from the three vehicles are $d_0^1 = 12$ m; $d_0^2 = 14$ m; $d_0^3 = 10.9$ m; $d_0^4 = 17$ m. Weighted consensus for vehicle control aims to distribute distances according to the terrain conditions defined by $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, $\gamma^4 = 28$, with the total $\mathbb{1}'\gamma = 75$. The target percentage distance distribution over the whole length is $[12/75, 15/75, 20/75, 28/75] = [0.1600, 0.2000, 0.2667, 0.3733]$. From the total length of 53.9 m, the goal of weighted consensus is $d^1 = 8.624$ m; $d^2 = 10.780$ m; $d^3 = 14.373$ m; $d^4 = 20.123$ m.

Suppose that the link observation noises are i.i.d sequences of Gaussian noises with mean zero and variance 1. Figure 3 shows the inter-vehicle distance trajectories. Staring from a large disparity in distance distribution, the top plot shows how distances are gradually distributed according to the terrain conditions. The middle plot illustrates that the weighted distances converge to a constant. The weighted consensus error trajectories are plotted in the bottom figure.

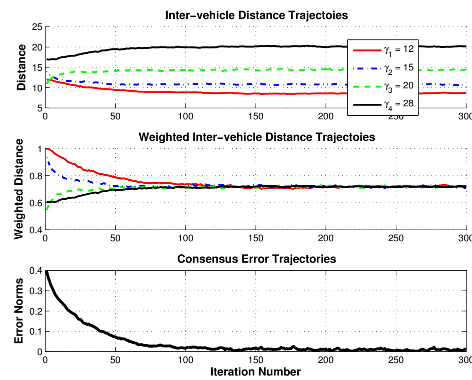

Fig. 3. Vehicle distance control with weighted consensus

The capability of the consensus control in attenuating disturbance's impact on the platoon formation can also be evaluated. Suppose that a sudden braking of the leading vehicle results in a sudden distance change in d_1 by 4 m. Consensus control then restores the desired distance distribution, shown in Figure 4.

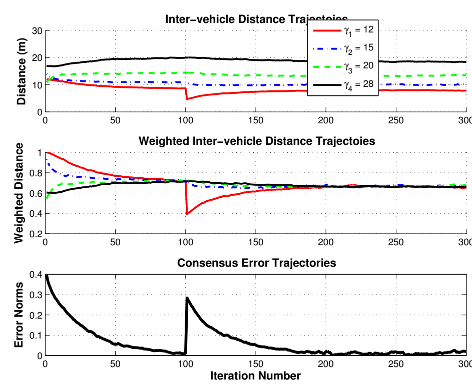

Fig. 4. Disturbance rejection in vehicle distance control

V. Observation Noise and Post-Iterate Averaging

The basic stochastic approximation algorithm (6) demonstrates desirable convergence properties under relatively small observation noises. However, when noises are large, its convergence may not be sufficiently fast and its states show fluctuations. For example, for the same system as in Example 1, if the noise standard deviation is increased from 1 to 20, its state trajectories demonstrate large variations, as shown in Figure 5.

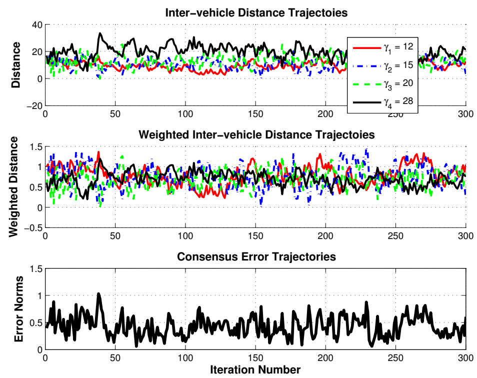

Fig. 5. Vehicle distance control with weighted consensus under large observation noise

To improve the efficiency, we take a post-iterate averaging, resulting in a two-stage stochastic approximation algorithm. Let $\mu_n = 1/n^{\alpha}$ for some $(1/2) < \alpha < 1$ and $c > 0$. The algorithm is modified to

$$\begin{aligned} x_{n+1} &= x_n + \frac{1}{n^{\alpha}} M x_n + \frac{1}{n^{\alpha}} W d_n \\ \overline{x}_{n+1} &= \overline{x}_n - \frac{1}{n+1}\overline{x}_n + \frac{1}{n+1} x_{n+1}. \end{aligned} \tag{8}$$

Since $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$, we have $\mathbb{1}'\overline{x}_n = L$. As a result, the constraint $\sum_{i=1}^{r} d_i(t) = L$ remains satisfied.

Theorem 2: Suppose the conditions of Theorem 1 are satisfied. For iterates generated by algorithm (8) (together with $\mathbb{1}'x_n = L$), $x_n \to \beta\Psi^{-1}\mathbb{1}$ w.p.1 as $n \to \infty$.

We now establish the optimality of the algorithms. Partition the matrix M as $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$, where $M_{11} \in \mathbb{R}^{(n-1)\times(n-1)}$, $M_{12} \in \mathbb{R}^{(n-1)\times 1}$, $M_{21} \in \mathbb{R}^{(n-1)\times 1}$, and $M_{22} \in \mathbb{R}^{1\times 1}$. Accordingly, we also partition $\bar{x}_n$, x_n, and W as $\overline{x}_n = \begin{bmatrix} \widetilde{\overline{x}}_n \\ \overline{x}_n^r \end{bmatrix}$ $x_n = \begin{bmatrix} \widetilde{x}_n \\ x_n^r \end{bmatrix}$, $W = \begin{bmatrix} \widetilde{W} \\ W_1 \end{bmatrix}$, respectively, with compatible dimensions with those of M.

Lemma 1: Under (A0), M_{11} is full rank.

This result indicates that we can concentrate on $r-1$ components of $\overline{x}_n$. We can show that the asymptotic rate of convergence is independent of the choice of the $r-1$ state variables. To study the rates of convergence of $\overline{x}_n$, without loss of generality we need only examine that of $\widetilde{\overline{x}}_n$. It follows from that

$$\left\{ \begin{aligned} \widetilde{x}_{n+1} &= \widetilde{x}_n + \mu_n(\widetilde{M}\widetilde{x}_n + \widetilde{W} d_n), \\ \widetilde{\overline{x}}_{n+1} &= \widetilde{\overline{x}}_n - \frac{1}{n+1}\widetilde{\overline{x}}_n + \frac{1}{n+1}\widetilde{x}_{n+1}, \end{aligned} \right. \tag{9}$$

where $\widetilde{M} = M_{11} - M_{12}\mathbb{1}'_{r-1}$. Note that the noise is now $\widetilde{W} d_n$, which is $r-1$ dimensional but is a function of l_s dimensional link noise d_n. Let $D = I_{r-1} + \mathbb{1}_{r-1}\mathbb{1}'_{r-1}$. It can be shown that under (A0), $\widetilde{M} = M_{11}D$ and is full rank. For convergence speed analysis, let $e_n = \overline{x}_n - \beta\Psi^{-1}\mathbb{1}_n$. Decompose $e_n = [\widetilde{e}_n', e_n^r]'$.

Theorem 3: Suppose that $\{d_n\}$ is a sequence of i.i.d. random variables with mean zero and covariance $Ed_n d_n' = \Sigma$. Under (A0), the weighted consensus errors $\widetilde{e}_n$ satisfies that $\sqrt{n}\widetilde{e}_n$ converges in distribution to a normal random variable with mean 0 and covariance given by $\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})'$.

Note that the above result does not require any distributional information on the noise $\{\varepsilon(k)\}$ other than the zero mean and finite second moments. We now state the optimality of the algorithm when the density function is smooth.

Theorem 4: Suppose that the noise $\{d_n\}$ is a sequence of i.i.d. noise with a density $f(\cdot)$ that is continuously differentiable. Then $\widetilde{\overline{x}}_n$ is asymptotically efficient in the sense of the Cramér-Rao lower bound on $E\widetilde{e}_n'\widetilde{e}_n$ being asymptotically attained, $nE\widetilde{e}_n'\widetilde{e}_n \to \operatorname{tr}(\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})')$.

Corollary 1: Under the conditions of Theorem 4, $\{\overline{x}_n\}$ is asymptotically efficient in the sense of the Cramér-Rao lower bound on $Ee_n'e_n$ being asymptotically attained. The asymptotically optimal convergence speed is $nEe_n'e_n \to \operatorname{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})')$, where $D = I_{r-1} + \mathbb{1}_{r-1}\mathbb{1}'_{r-1}$.

Example 2: We now use the system in Example 1 to illustrate the effectiveness of post-iterate averaging. Suppose that the link observation noises are i.i.d sequences of Gaussian noises of mean zero and standard deviation 20. Now, the consensus control is expanded with post-iterate averaging. Figure 6 shows the distance trajectories. The distance distributions converge to the weighted consensus faster with much less fluctuations.

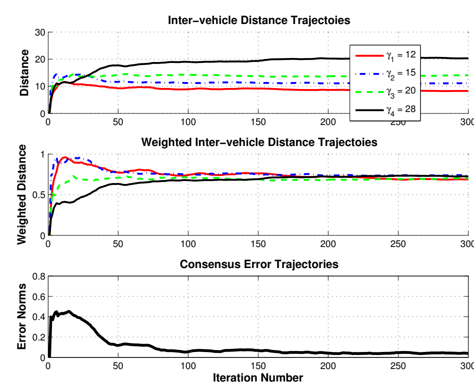

Fig. 6. vehicle distance control with post-iterate averaging on weighted consensus algorithms

VI. Network Topology and Platoon Control Performance

We investigate now the benefits of using communication systems to enhance platoon control. In a sensor-based information network topology, we assume that the vehicles are equipped with front and rear distance sensors, but controls its front distance only. On the other hand, if wireless communications are allowed, inter-vehicle information flows can be further expanded. From the previous analysis, as long as the information topology is connected, convergence of consensus control can be achieved. The main difference is the speed of convergence, which is essential for system robustness, disturbance attenuation, and platoon re-configuration.

Example 3: This example compares the two types of information network topologies. The platoon contains a lead vehicles and 5 other vehicles. In the sensor-based topology, vehicle j measures d_j by the front distance sensor and d_{j+1} by the rear distance sensor. In this case, the last vehicle will control the overall platoon length L (presumably by using a GPS device and communicates with the lead vehicle or the control tower). On the other hand, if inter-vehicle communication is allowed, d_1 is transmitted to vehicle 3, d_2 to vehicle 4, etc., adding new network branches in the information network topology. Under the same consensus control gains and step sizes, Figure 7 illustrates inter-vehicle distance trajectories, starting from the same initial condition. Figure 8 compares the convergence rates under the two topologies. It is clear that the communication network can potentially enhance consensus control by adding new information in the platoon control strategies.

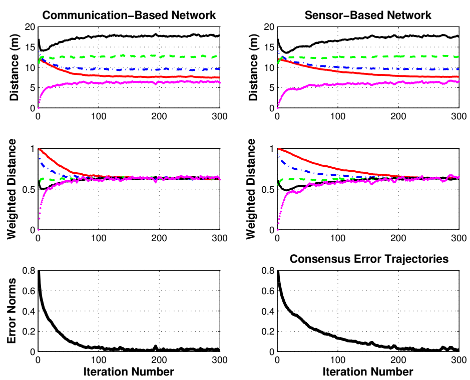

Fig. 7. Consensus control using different information topologies

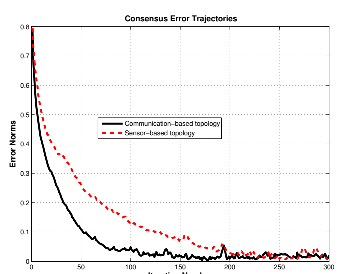

Fig. 8. Comparison of convergence speeds between the sensor-based topology and communication-based topology

VII. Concluding Remarks

This paper covers some typical environments in which vehicle platoons are coordinated. These include communication noises, vehicles' departure from and arrival in the platoon etc. However, there are many other uncertainties in communication systems that are not considered in detail here. Typical scenarios include communication latency, signal fading and interference, packet loss, irregular data arrival rates, etc. These are interesting and important but open issues in weighted and constrained consensus control.

This paper considers only linearly weighted and summation-constrained consensus control. Practical systems often introduce nonlinearity, which needs to be investigated. Finally, this paper deals with only the system-level (or cyber-space) vehicle coordination. Actual vehicle dynamics and control are left to driver-vehicle control. Interaction of the cyber-space with the physical vehicle control is the ultimate goal of this study, but not covered in this paper.

References

[1] J. Cortes and F. Bullo, Coordination and geometric optimization via distributed dynamical systems, *SIAM J. Control Optim.*, no. 5, pp. 1543-1574, May 2005.

[2] M. Huang, S. Dey, G.N. Nair, J.H. Manton, Stochastic consensus over noisy networks with Markovian and arbitrary switches, *Automatica*, **46** (2010), 1571–1583.

[3] S. Haykin, Communication Systems, 5th Ed., Wiley, 2009.

[4] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules, *IEEE Trans. Automat. Contr.*, vol. 48, pp. 988-1000, June 2003.

[5] S. Kar and J.M.F. Moura, Distributed consensus algorithms in sensor networks with imperfect communication: Link failures and channel noise, *IEEE Trans. Signal Processing*, **57** (2009), no.1, 355–369.

[6] S. Karlin and H.M. Taylor, *A First Course in Stochastic Processes*, 2nd ed., Academic Press, New York, NY, 1975.

[7] H.J. Kushner and G. Yin, *Stochastic Approximation Algorithms and Applications*, Springer-Verlag, New York, 2nd Ed., 2003.

[8] T. Li and J. F. Zhang, Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises, IEEE Trans. on Automatic Control, Vol.55, No.9, 2043-2057, 2010.

[9] Tao Li, Minyue Fu, Lihua Xie and Ji-Feng Zhang, Distributed consensus with limited communication data rate, IEEE Trans. on Automatic Control, Vol.56, No.2, 279-292, 2011.

[10] L. Moreau, Stability of multiagent systems with time- dependent communication links, *IEEE Trans. Autom. Control*, vol. 50, no. 2, pp. 169-182, Feb. 2005.

[11] M.B. Nelson and R.Z. Khasminskii, *Stochastic Approximation and Recursive Estimation*, Amer. Math. Soc., Providence, RI, 1976.

[12] R. Olfati-Saber, J.A. Fax, and R.M. Murray. Consensus and cooperation in networked multi-agent systems, *IEEE Proc.*, vol. 95, no. 1, pp. 215-233, Jan. 2007.

[13] P. Ogren, E. Fiorelli, and N.E. Leonard, Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment, *IEEE Trans. Autom. Control*, vol. 49, no. 8, pp. 1292-1302, Apr. 2005.

[14] B. T. Polyak, New method of stochastic approximation type, *Automation Remote Control* **7** (1991), 937–946.

[15] W. Ren and R.W. Beard. Consensus seeking in multiagent systems under dynamically changing interaction topologies, *IEEE Trans. Automat. Control*, vol. 50, no. 5, pp. 655-661, 2005.

[16] J.N. Tsitsiklis, D.P. Bertsekas, and M. Athans, Distributed asynchronous deterministic and stochastic gradient optimization algorithms, *IEEE Trans. Automat. Control*, **31**, no. 9, pp. 803-812, 1986.

[17] L. Xiao, S. Boyd, and S. J. Kim, Distributed average consensus with least-mean-square deviation, *Journal of Parallel and Distributed Computing*, **67**, pp. 33-46, 2007.

[18] L.Y. Wang, G. Yin, J.-F. Zhang, Y. Zhao, *System Identification with Quantized Observations*, Birkhäuser, Boston, 2010.

[19] G. Yin, On extensions of Polyak's averaging approach to stochastic approximation, *Stochastics Stochastics Rep.*, **36** (1991), 245-264.

[20] G. Yin, Y. Sun, and L.Y. Wang, Asymptotic properties of consensus-type algorithms for networked systems with regime-switching topologies, *Automatica*, **47** (2011) 1366–1378.

[21] G. Yin, L.Y. Wang, Y. Sun, D. Casbeer, R. Holsapple, and D. Kingston, Asymptotic optimality for consensus-type stochastic approximation algorithms using iterate averaging, to appear in *J. Control Theory Appl.*