P1. (10 points) Bob needs to use a 3-bit up-counter. However, he only has a 4-bit synchronous down-counter and several NOT gates. He is NOT allowed to modify the internal structure of the down-counter. How can he construct the 3-bit up-counter using only the devices he has?

P2. (30 points) Design a modulo-6 counter, which counts in the sequence 0, 1, 2, 3, 4, 5, 0, 1,... The counter counts the clock pulses if its enable input w is equal to 1, otherwise it does not increment its count.
   a) Use D flip-flops in your circuit and include the state diagram, state-assigned table, next-state expressions and output expressions all clearly labeled. Let y₂, y₁, and y₀ be the current state values.
   b) Repeat Problem 1 using JK flip-flops. Work through the entire design process rather than simply converting the D flip-flops to JK flip-flops. Use the same state diagram.
   c) Repeat Problem 1 using T flip-flops. Work through the entire design process rather than simply converting the D flip-flops to T flip-flops. Use the same state diagram.

P3. (10 points) Derive the circuits that implement the state tables shown below. Compare the costs of these circuits.

P4. (10 points) Consider the state machine specified by the following state transition table.
(a) Draw the state transition diagram of the machine.
(b) Write two next-state expressions for X and Y that will implement the transitions of the state machine. Make your expressions as simple as possible, and use XOR gates and NOT gates only.
(c) Implement the state machine using D flip-flops, XOR gates, and NOT gates.
(d) Suppose the machine is initially in 00 (i.e., X=0 and Y=0). Indicate for each input sequence below, the state the machine is in after the last digit has been read in. Assume the digits are read in from left to right.
   a. 11111
   b. 100111000101100
   c. 436 1s followed by 325 0s

P5. (10 points) Deduce the purpose of a state machine with the next state logic and output logic given below. Let X, Y, and Z be the values of the three D flip-flops, \(X_{next}\), \(Y_{next}\), and \(Z_{next}\) be the next state expressions, and ABC be the three-bit output of the circuit.
(a) Construct the state assigned table from the expression.
(b) Draw the state diagram from the table.
(c) Explain in one sentence the functionality of the circuit.

\[
\begin{align*}
X_{next} &= X'YZ + XY' + XZ' \\
Y_{next} &= Y'Z + YZ' \\
Z_{next} &= Z' \\
A &= X' \\
B &= Y \\
C &= Y + Z 
\end{align*}
\]

P6. (10 points) A state machine has one input P in addition to the clock input and one output Q. The value of Q is 1 if the total number of 1’s in the sequence of input P is either a multiple of 2 or a multiple of 3. Otherwise, the output value Q is 0. Draw a state transition diagram for the state machine using as few states as possible.

P7. (10 points) Derive a minimal state table for an FSM that acts as a three-bit parity generator. For every three bits that are observed on the input w during three consecutive clock cycles, the FSM generates the parity bit \(p = 1\) if and only if the number of 1s in the three-bit sequence is odd.

P8. (10 points) A state machine has one input P in addition to the clock input and one output Q. The value of Q is 1 if the total number of 1’s and the total number of 0’s in the sequence of input P are both even. Otherwise, the output value Q is 0. Draw a state transition diagram for the state machine using as few states as possible.