P1. (10 points) Use Venn diagrams to prove the following two equations:

\[(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x_3}) = x_1 + x_2\]

\[x + y = \overline{x} \cdot \overline{y}\]

P2. (10 points) Use Boolean algebra to prove the first equation in P1.

P3. (10 points) Using a truth table to check the validity of the following:

\[(x_1 + x_3)(\overline{x_1} + \overline{x_2} + \overline{x_3})(\overline{x_1} + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x_1} + \overline{x_3})\]

P4. (10 points) Draw logic circuits for the following three expressions:

\[
\overline{x_1}x_3 + x_1x_2\overline{x_3} + \overline{x_1}x_2 + x_1\overline{x_2} \\
x_1\overline{x_3} + x_2x_3 + \overline{x_2}\overline{x_3} \\
(x_1 + x_3)(\overline{x_1} + \overline{x_2} + \overline{x_3})(\overline{x_1} + x_2)
\]

P5. (20 points) Consider the logic function:

\[f(a,b,c) = a'b'c' + ab'c + a'bc + a'b'c + ab'c',\]

where \(a'\) is an alternative notation for \(\overline{a}\), \(b'\) for \(\overline{b}\), and \(c'\) for \(\overline{c}\).

(a) (4 points) Draw the logic circuit for the function \(f\) given above.

(b) (4 points) Let the cost of a logic circuit be the total number of gates plus the total number of inputs to all gates in the circuit. (See pages 49-50 in the book for examples.) What is the cost of the circuit in (a)?

(c) (4 points) Simplify \(f\) using Boolean algebra as much as possible.

(d) (4 points) Draw the logic circuit for the simplified version of \(f\) in (c).

(e) (4 points) What is the cost of the circuit in (d)?
P6. (20 points) A function \( f \) has four inputs \( x, y, z, \) and \( w \) and one output such that the output is a 1 if and only if the number of 1s in the inputs is either one or three.

(a) (4 points) Derive the truth table for \( f \).
(b) (4 points) Write the canonical sum-of-products expression for \( f \). Do not use the shorthand notation.
(c) (2 points) Write the canonical sum-of-products expression for \( f \) in shorthand notation.
(d) (2 points) Write the canonical sum-of-products expression for \( f \) in shorthand notation.
(e) (4 points) Write the canonical product-of-sums expression for \( f \). Do not use the shorthand notation.
(f) (2 points) Write the canonical product-of-sums expression for \( f \) in shorthand notation.
(g) (2 points) Write the canonical product-of-sums expression for \( f \) in shorthand notation.

P7. (10 points) Use algebraic manipulation to show that for three input variables \( x_1, x_2, \) and \( x_3 \) the following is true:

\[
\Pi M(0, 1, 2, 3, 4, 5, 6) = x_1x_2x_3
\]

Show the steps in your derivation.

P8. (10 points) Use algebraic manipulation to find the minimum sum-of-products expression for the following function:

\[
f = x_1\overline{x}_2\overline{x}_3 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4
\]