1. Concentric Circles (10 points)

Write a C program that calculates the area of the white surface shown in the figure below. The program must ask the user to enter the radius of the outer circle and also the radius of the inner circle. The output must be formatted with exactly three significant digits after the decimal point.

============= START OF SAMPLE RUN =======================
Enter the outer radius: 2.5
Enter the inner radius: 1.5

The area of the white surface is: 12.566

=============  END OF SAMPLE RUN  =======================
2. Quarter-Mile Race (15 points)

Write a complete C program that calculates the estimated time (ET) for two cars to finish a quarter-mile race. The estimated time can be calculated using the weight of the car in pounds and the cars horsepower with the formula:

\[ ET = \left(\frac{\text{Weight}}{\text{HP}}\right)^{1/3} \times \left(\frac{233}{40}\right) \]

============= START OF SAMPLE RUN ===============
Enter weight and HP of car 1: 2996 225
Enter weight and HP of car 2: 2745 190

Car 1 ET: 13.806
Car 2 ET: 14.187
============= END OF SAMPLE RUN ===============

============= START OF SAMPLE RUN ===============
Enter weight and HP of car 1: 2900 550
Enter weight and HP of car 2: 3100 650

Car 1 ET: 10.139
Car 2 ET: 9.805
============= END OF SAMPLE RUN ===============
### 3. Matrix Vector Multiplication (15 points)

Write a program that multiplies a 2x2 matrix A with a 2x1 column vector x. The result is a 2x1 column vector b.

The relevant formulas for calculating the two components of b are shown below.

\[
\begin{align*}
Ax = b & \quad \iff \quad \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\
& \quad \iff \quad b_1 = a_1x_1 + a_2x_2 \\
& \quad \iff \quad b_2 = a_3x_1 + a_4x_2
\end{align*}
\]

In other words, write a C program that takes six values from the user, calculates two values (b1 and b2) according to the above formulas, and then prints the vector b on the screen.

The result vector must be printed in a separate function. The vertical bars to the right of the result vector must be aligned.

You can assume that the user will only enter integer values in the range -99 to +99 such that the calculated values for b1 and b2 can be printed with three characters or less.

------------- START OF SAMPLE RUN -------------

Please enter the four values of matrix A (a1 a2 a3 a4):
3 5 1 -1
Please enter the two values of vector x (x1, x2):
-3 1
Vector b, the product of A and x, is therefore:
\[ \begin{bmatrix} -4 \\ -4 \end{bmatrix} \]
------------- END OF SAMPLE RUN -------------

------------- START OF SAMPLE RUN -------------

Please enter the four values of matrix A (a1 a2 a3 a4):
-12 3 3 1
Please enter the two values of vector x (x1, x2):
3 -1
Vector b, the product of A and x, is therefore:
\[ \begin{bmatrix} -39 \\ 8 \end{bmatrix} \]
------------- END OF SAMPLE RUN -------------
4. Determinant of a Matrix (15 points)

Write a C program that:

1) takes in the elements of a 3x3 matrix from the user
2) calls a function to print the matrix
3) calls another function to compute the determinant of the matrix
4) prints the determinant inside the main function.

You can assume that all elements of the matrix will be integers in the range 0-99.

The determinant of a 3x3 matrix is given by the following formula:

\[
\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = ae i + bfg + cdh - ceg - bdi - afh.
\]

============ START OF SAMPLE RUN ===============
Enter row 1 of the matrix> 1 2 3
Enter row 2 of the matrix> 4 11 6
Enter row 3 of the matrix> 10 9 8

Matrix:  | 1  2  3|
| 4 11  6|
|10  9  8|

The determinant of the matrix is -132
= &=& END OF SAMPLE RUN ===============
5. Binary Orbits (15 points)

According to Kepler's laws of planetary motion, as well as Newton's laws of universal gravitation, celestial bodies in space will orbit in elliptical paths along a common center of mass called the barycenter. For a system with only 2 stars (a binary star system), the stars will appear to move in circles around the barycenter. The orbital elements of each star can be calculated using the following equations:

\[
\begin{align*}
V_a &= \frac{\sqrt{(m_b^2 \cdot G)}}{\sqrt{(m_a + m_b) \cdot d}} \\
R_a &= \frac{d \cdot m_b}{m_a + m_b} \\

V_b &= \frac{\sqrt{(m_a^2 \cdot G)}}{\sqrt{(m_a + m_b) \cdot d}} \\
R_b &= \frac{d \cdot m_a}{m_a + m_b}
\end{align*}
\]

Where:
* \(d\) is the distance between the 2 stars (in meters),
* \(m_a\) and \(m_b\) are the masses of stars A and B (in kilograms)
* \(V_a\) and \(V_b\) are the orbital velocities of stars A and B (in m/s)
* \(R_a\) and \(R_b\) are the distances from each star to the barycenter (in m)
* \(G\) is the Universal Gravitational Constant: \(6.674 \times 10^{-11}\) (m\(^3\) / kg \(\cdot\) s\(^2\))

You must write a complete C program that reads in the masses of the 2 stars (in kilograms), as well as the distance between them (in meters). You must then calculate the orbital elements \(V_a\), \(V_b\), \(R_a\), and \(R_b\), and print these out to the screen. Since the numbers involved will be quite large, you first must convert the star velocities from m/s to km/s and the barycenter distances from m to km.

Also, since stars are VERY massive objects, with masses well into the \(10^{30}\) kg range, you may want to use scientific notation form. It works the same as any other double value, but instead of entering the number in normal form, you enter it in scientific notation (ex. double num = 1e30; or \#define myvalue 8.8e-2).

To ensure the highest degree of accuracy, make sure to use doubles, NOT floats. Print all of your results to 2 decimal places, i.e. %.2f in printf.

============= START OF SAMPLE RUN ===============
Enter the mass of star A in kg: 1.63e30
Enter the mass of star B in kg: 8.72e29
Enter the distance between the two stars in m: 1.55e10

Star A orbits the barycenter at a velocity of 36.17 km/s, and a distance of 5402078.34 km.

Star B orbits the barycenter at a velocity of 67.62 km/s, and a distance of 10097921.66 km.

============= END OF SAMPLE RUN ===============