1. Calculate the following determinants.

\[
\begin{vmatrix}
-1 & 9 \\
8 & -5
\end{vmatrix} \quad \begin{vmatrix}
3 & -7 \\
6 & 8
\end{vmatrix} \quad \begin{vmatrix}
3 & 0 & 4 \\
2 & 3 & 2 \\
0 & 5 & -1
\end{vmatrix} \quad \begin{vmatrix}
a & b & c \\
a + x & b + x & c + x \\
a + y & b + y & c + y
\end{vmatrix}
\]

2. Show that the value of the determinant does not depend on \( \theta \).

\[
\begin{vmatrix}
sin\theta & -\cos\theta & sin\theta - cos\theta \\
\cos\theta & sin\theta & sin\theta + cos\theta \\
0 & 0 & 1
\end{vmatrix}
\]

3. Calculate the inverse of the given matrix.

\[
\begin{bmatrix}
5 & 10 \\
4 & 7
\end{bmatrix} \quad \begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix} \quad \begin{bmatrix}
1 & 0 & -2 \\
-3 & 1 & 4 \\
2 & -3 & 4
\end{bmatrix} \quad \begin{bmatrix}
0 & 1 & 2 \\
1 & 0 & 3 \\
4 & -3 & 8
\end{bmatrix}
\]

4. Solve the following systems of equations. In (b) describe the set of all \( b \) for which \( Ax = b \) has a solution.

\[
\begin{align*}
5x_1 + 7x_2 &= 3 \\
2x_1 + 4x_2 &= 1
\end{align*}
\]
\[
\begin{align*}
x_1 - 3x_2 - 4x_3 &= b_1 \\
-3x_1 + 2x_2 + 6x_3 &= b_2 \\
5x_1 - x_2 - 8x_3 &= b_3
\end{align*}
\]

5. Let \( u = (3, -1, -5) \), \( v = (0, -1, -3) \), and \( w = (6, -2, 3) \). Compute:

\[
\begin{align*}
(a) \ u \cdot w & \quad (b) \|u\|^2 & \quad (c) \ v \times w & \quad (d) \ dist(u, v) = \|u - v\| & \quad (e) \ proj_v u = \frac{u \cdot v}{\|v\|^2} v
\end{align*}
\]
6. Let \( u = (3, -1, -5) \), \( v = (0, -1, -3) \), and \( w = (6, -2, 3) \). Consider whether the following problems are possible. Compute AND verify the answer if the problem is possible. Otherwise give a SHORT explanation if the problem is not possible.

   (a) Find a vector \( t \in \mathbb{R}^3 \) that is perpendicular to both \( v \) and \( w \).
   (b) Find a vector \( t \in \mathbb{R}^3 \) that is perpendicular to \( u \), \( v \) and \( w \).

7. Find an equation for the plane passing through the given points.

   (a) \( P(3, 7, 4), Q(6, 0, 1), R(1, 1, 3) \)
   (b) \( P(-1, 3, -2), Q(4, 5, 4), R(-8, 5, 0) \)

8. Find the characteristic polynomial and the eigenvalues of the following matrices.

   (a) \[
   \begin{bmatrix}
   7 & 4 \\
   -3 & -1 
   \end{bmatrix}
   \]
   (b) \[
   \begin{bmatrix}
   4 & 0 & 0 \\
   5 & 3 & 2 \\
   -2 & 0 & 2 
   \end{bmatrix}
   \]
   (c) \[
   \begin{bmatrix}
   4 & -7 & 0 & 2 \\
   0 & 3 & -4 & 6 \\
   0 & 0 & 3 & -8 \\
   0 & 0 & 0 & 1 
   \end{bmatrix}
   \]

9. In the following problems, find the \( 3 \times 3 \) matrices that produce the described composite 2D transformations, using homogenous coordinates.

   (a) Translate by \((-2, 3)\), and then scale the \( x \)-coordinate by .8 and the \( y \)-coordinate by 1.2.
   (b) Reflect points through the \( x \)-axis, and then rotate \( 30^\circ \) about the origin.

10. Given the vector \( v = [x, y]^T \), find a rotation matrix \( R \) which rotates the vector by \( 60^\circ \) counterclockwise. Give the values for the vector \( v' = [x', y']^T \) in terms of \( x \) and \( y \), where \( v' = Rv \).

11. Find the equation \( y = B_0 + B_1 x \) of the least-squares line that best fits the data points \((2,1), (5,2), (7,3), (8,3)\).

12. Given the following experimental data points: find the mean, subtract the mean from all data points, construct the sample covariance matrix, and find the principal components of the data. (Hint: \((19, 12)\) is one data point)