1. Let a random process \( X(t) \) with mean value 128 and covariance function \( C_{XX}(\tau) = 1000 \exp(-10|\tau|) \) be filtered by the low pass filter

\[
H(\omega) = \frac{1}{1 + j\omega}
\]

to produce the output process \( Y(t) \).

a) Find \( \mu_Y(t) \).

b) Find the covariance function \( C_{YY}(\tau) \).

2. Consider the system shown in the figure below. Let \( X(t) \) and \( N(t) \) be WSS and mutually uncorrelated with p.s.d’s \( S_{XX}(\omega) \) and \( S_{NN}(\omega) \) and zero means.

\[
\begin{array}{c}
N(t) \\
X(t) \quad \text{h(t)} \quad Y(t)
\end{array}
\]

Figure 1:

a) Find the psd of the output \( Y(t) \).

b) Define the error \( \epsilon(t) = Y(t) - X(t) \) and evaluate the psd of \( \epsilon(t) \).

c) Assume that \( h(t) = a\delta(t) \) and choose the value of \( a \) that minimizes the power of \( \epsilon(t) \).

3. Let the random variables, \( A_k, B_k, k \geq 0 \) be mutually independent with mean zero. Let \( A_k \) have variance \( \sigma^2_A \) and let \( B_k \) have variance \( \sigma^2_B \) for all \( k \). Define a discrete-time random process \( \{Y_k\}_{k \geq 0} \) such that \( Y_0 = 0 \) and \( Y_{k+1} = A_k Y_k + B_k \) for \( k \geq 0 \).

a) Find a recursive relation for \( E(Y_k^2) \).

b) Does \( Y \) have independent increments ?

c) Find the autocorrelation function of \( Y \) (in terms of \( E(Y_k^2) \))

4. Consider the random BPSK signal with random initial phase.

\[
X(t) = \cos(2\pi f_c t + \Theta(t) + \beta)
\]

where \( \beta \) is a random variable distributed uniformly between \([0, 2\pi]\) independent of \( \Theta(t) \). The signal \( \Theta(t) \) is such that

\[
\Theta(t) = B_n \quad \text{if} \quad nT \leq t \leq (n + 1)T
\]

and \( B_n \) is i.i.d. sequence of random variables such that \( P(B_n = \pi/2) = P(B_n = -\pi/2) = 0.5 \). The time interval \( T \) is an integral multiple of \( 1/f_c \). Show that \( X(t) \) is WSS and find its mean and autocorrelation function.
5. Consider a zero mean WSS process $X(t)$ such that $R_{XX}(\tau) = \frac{1}{\tau_0} \exp(-|\tau|/\tau_0)$. The process $X(t)$ is input to an ideal low-pass filter with frequency response

$$G(\omega) = \begin{cases} 1 & |\omega| \leq \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

to obtain a signal $Y(t)$. Show that when $|\omega_0\tau_0| << 1$ i.e. the product $\omega_0\tau_0$ is much smaller than 1, $S_{YY}(\omega)$ is approximately the psd of white noise band-limited to $[-\omega_0, \omega_0]$. 