1. Let \( X_t = A \cos(2\pi V t + \Theta) \) where \( E(A) = 2, \text{Var}(A) = 4 \), and \( V \) is distributed uniform over \([0, 5]\) and \( \Theta \) is uniform over \([0, 2\pi]\) and \( A, V \) and \( \Theta \) are independent. Find the mean function and the autocorrelation function of \( X_t \). Is it wide sense stationary?

2. Let \( \{Z_n\} \) be a sequence of uncorrelated real-valued random variables with zero mean and unit variance and define the moving average,
\[
Y_n = \sum_{i=0}^{r} \alpha_i Z_{n-i},
\]
for constants \( \alpha_0, \alpha_1, \ldots, \alpha_r \). Show that \( Y \) is wide sense stationary and find its autocovariance function.

3. **Random telegraph.** Let \( \{N(t) : t \geq 0\} \) be a Poisson process of rate \( \lambda \), and let \( T_0 \) be an independent random variable such that \( P(T_0 = \pm 1) = \frac{1}{2} \). Define \( T(t) = T_0(-1)^{N(t)} \). Find,
   a) \( E(T(s)) \) and \( E(T(s), T(s+\tau)) \).
   b) The mean and variance of \( X(t) = \int_0^t T(s)ds \).

4. Consider a Poisson process with rate \( \lambda > 0 \). Recall that \( N_t \) represents the number of arrivals in the interval \((0,t]\).
   a) Find the probability of the event \( \{N_1 = 1, N_2 - N_1 = 1, N_3 - N_2 = 1\} \).
   b) Find the probability that there are two arrivals in the interval \((0,2]\) and two arrivals in the interval \((1,3]\).
   c) Find the probability that there are two arrivals in \((1,2]\), given that there are two arrivals in \((0,2]\) and two arrivals in \((1,3]\).

5. a) Consider a stream of packets arriving at a router according to a Poisson process with rate \( \lambda \). Suppose that each packet is independently \textit{Good} with probability 0.1 and \textit{Bad} with probability 0.9. Given that 1000 good packets arrived in the time interval \((0,1]\) what is the expected number of bad packets that arrived in \((0,1]\).
   b) Good packets arrive at rate \( \lambda_G \) and bad packets arrive at a rate \( \lambda_B \) to a router. We are given that 100 packets arrived in the interval \((0,1]\). Find the probability that exactly 40 of those packets were good.
   c) Suppose that packets arrive at a router with rate \( \lambda \). You are given that in the interval \((0,40]\), 100 packets arrived. What is the probability that 20 of those packets arrived in the interval \((0,5]\).
   d) Suppose packets arrive at a router with rate \( \lambda \). Let \( W_n \) represent the time of the arrival of the \( n^{th} \) packet. Find the distribution of \( W_n \).

6. Let \( N_t \) be a Poisson process with rate \( \lambda \) and let \( X = \{X_t : t \geq 0\} \) be defined by \( X_t = N_{t+1} - N_t \). Find the mean function and covariance function for \( X \).