1. Give an example of a field of sets that is not a \( \sigma \)-field.

2. Let \( A \) and \( B \) be events with probabilities \( P(A) = 3/4 \) and \( P(B) = 1/3 \). Show that \( \frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3} \) and show that both upper and lower bounds are possible. Find corresponding bounds for \( P(A \cup B) \).

3. Let \( A_1, A_2, \ldots, A_n \) be events where \( n \geq 2 \). Show that
\[
P(\bigcup_{i=1}^{n} A_i) = \sum_{i} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \ldots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n)
\]

4. For events \( A_1, A_2, \ldots, A_n \) satisfying \( P(\bigcap_{i=1}^{n} A_i) > 0 \), prove that
\[
P(\bigcap_{i=1}^{n} A_i) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \cdots P(A_n|A_1 \cap A_2 \cdots \cap A_{n-1}).
\]

5. Show that the conditional independence of \( A \) and \( B \) given \( C \) neither implies, nor is implied by the independence of \( A \) and \( B \).

6. We roll a die \( n \) times. Let \( A_{ij} \) be the event that the \( i^{th} \) and the \( j^{th} \) rolls produce the same number. Show that the events \( \{A_{ij} : 1 \leq i < j \leq n \} \) are pairwise independent but not independent.

7. Show that the probability that exactly one of the events \( A \) and \( B \) occurs is
\[
P(A) + P(B) - 2P(A \cap B)
\]

8. Show that
\[
P(\bigcup_{r=1}^{n} A_r) \geq \sum_{r=1}^{n} P(A_r) - \sum_{r<k} P(A_r \cap A_k)
\]

9. You are given a circle that has 15% of its circumference colored red and the remaining part colored blue. Show that you can inscribe a square in the circle such that all four vertices in the square are colored blue. \textit{Hint: Consider choosing an inscribed square at random.}

10. There are two roads from city A to city B and two roads from city B to city C. Each of the four roads has probability \( p \) of being blocked by snow, independently of the others. What is the probability that there is an open road from A to C.