HCI/ComS 575X: Computational Perception
Homework 1 (part 1)
Out: Wed. Jan 16, 2008
Due: Wed. Jan 30, 2008
For extra credit submit by Fri. Jan 18 before 4pm.
(submit it to Karen in VRAC front office.)

Note: Not all assignments are created equal :(

These problems are designed to test your background knowledge in linear algebra and statistics.
Please show the intermediary steps of your calculations (not just the final answers).

1. **Calculate the following determinants.**

$$(a) \begin{vmatrix} -5 & 6 \\ -7 & -2 \end{vmatrix} \quad (b) \begin{vmatrix} 4 & 1 \\ 8 & 2 \end{vmatrix} \quad (c) \begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix} \quad (d) \begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix}$$

2. **Show that the value of the determinant does not depend on θ.**

$$\begin{vmatrix} sin\theta & cos\theta & 0 \\ -cos\theta & sin\theta & 0 \\ sin\theta - cos\theta & sin\theta + cos\theta & 1 \end{vmatrix}$$

3. **Calculate the inverse of the given matrix.**

$$(a) \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \quad (b) \begin{bmatrix} -3 & 6 \\ 4 & 5 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad (d) \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

4. **Solve the following systems of equations.**

$$(a) \begin{array}{rcr} 4x_1 & - & 3x_2 & = & -3 \\ 2x_1 & - & 5x_2 & = & 9 \end{array} \quad (b) \begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & b_1 \\ 2x_1 & + & 5x_2 & + & 5x_3 & = & b_2 \\ 3x_1 & + & 5x_2 & + & 8x_3 & = & b_3 \end{array}$$

5. **Let $u = (6, 4, -2)$, $v = (0, 4, -6)$, and $w = (4, 12, 14)$. Compute:**

$$(a)\ u \cdot v \qquad (b)\|u\| \qquad (b)\ v \times w \qquad (c)\ u \times (v \times w) \qquad (d)\ (u \times v) \times w$$

1

6. **Find a vector that is orthogonal to both u and v, where $u = (-6, 4, 2)$ and $v = (3, 1, 5)$**

7. **Find the orthogonal projection of u on a (i.e., find $proj_a u = \frac{u \cdot a}{\|a\|^2} a$)**

$$(a) \ u = (-1, -2), a = (-2, 3)$$
$$(b) \ u = (1, 0, 0), a = (4, 3, 8)$$

8. **Find the angle θ between the two vectors u and v.**

$$(a) \ u = (-4, -6), v = (5, -7)$$
$$(b) \ u = (2, -1, 1), v = (1, 1, 2)$$

9. **Find an equation for the plane passing through the given points.**

$$(a) \ \ P(-4, -1, -1), Q(-2, 0, 1), R(-1, -2, -3) \qquad (b) \ \ P(5, 4, 3), Q(4, 3, 1), R(1, 5, 4)$$

10. **Find all values of λ for which the determinant of the matrix is equal to 0.**

$$(a) \ \begin{vmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{vmatrix} \qquad (b) \ \begin{vmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{vmatrix}$$

11. **Find the eigenvalues of the following matrices.**

$$(a) \ \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \qquad (b) \ \begin{bmatrix} 3 & 0 & -5 \\ \frac{1}{5} & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$$

12. **Given the vector $v = [x, y]^T$ find a rotation matrix R which rotates the vector by $45°$ counterclockwise. Give the values for the vector $v' = [x', y']^T$ in terms of x and y, where $v' = Rv$.**

13. **Write down the analytical form of the 3D rotation matrices R_x, R_y, and R_z which rotate a vector about the X, Y, and Z axis, respectively.**

14. **By example, show that the 3D rotation matrices are not commutative.**