ABSTRACT

This dissertation proposes a continuous-time optimization dynamics approach to study the *non-convex* networked optimizations. By viewing primal and dual variables of an optimization problem as opponents playing a min-max game, the evolution of the associated optimization dynamical system can be interpreted as a competition between two players. This competition will not stop until those players achieve a balance, which turns out to be an equilibrium to the optimization dynamics and is mathematically characterized as a Karush-Kuhn-Tucker (KKT) point. Generally speaking, if the optimization under study is convex, then the KKT point is globally optimal for both primal and dual variables. Motivated by this idea, the optimization dynamics has been successfully applied to solve convex problems since 1950s.

Different from previous works, we find that if strong duality holds then it is still possible for us to seek globally optimal solutions to non-convex problems via the optimization dynamics. A convergence analysis is developed in this dissertation, showing that under certain conditions the associated optimization dynamical system is locally asymptotically stable. However, after the optimization dynamics converges to a KKT equilibrium, we have to further check whether or not the obtained KKT point is globally optimal, since KKT conditions are only necessary for the local optimality of non-convex problems.

In this dissertation, we present a global optimality condition for the general quadratically constrained quadratic programmings (QCQPs). If an isolated KKT point of a general QCQP satisfies our condition, then it is locally asymptotically stable with respect to the optimization dynamics. We next apply the optimization dynamics approach to a special class of networked non-convex QCQPs, namely, the optimal power flow (OPF) problems, and we discover that the associated optimization dynamics possesses an intrinsic distributed structure. Such a structure is an exclusive property of the continuous-time dynamics, since it will be destroyed by digital implementations. Simulations are also provided to show the effectiveness of our approach.