Virtual MIMO Channels in Cooperative Multi-hop Wireless Sensor Networks

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Abstract— **In this paper we analyze the performance of virtual multiple-input-multiple-output (MIMO) channels for multi-hop transmission in wireless sensor networks. First, we propose a clustered network topology with sensor nodes grouped in collaborative sets attending to proximity. Then, we propose that all nodes belonging to any given cluster cooperatively transmit and receive data from other clusters, exploiting the diversity advantages of cluster-to-cluster Virtual MIMO channels. In order to construct the cooperative transmission, we arrange each hop into two consecutive time slots: the *Intracluster Slot*, that accounts for data sharing within the cluster, and the *Intercluster Slot* for transmission between clusters; and we devise a cooperative reception protocol within the clusters based upon a simplified selection diversity algorithm. Optimum time assignment and power allocation for both slots are derived taking the cluster-to-cluster probability of outage as the metric. Results for one hop networks and for multi-hop networks are obtained, showing substantial diversity gains and energy savings. Furthermore, results show that the performance of the proposed virtual MIMO channels is equal to that of real MIMO channels but for a small SNR loss.**

I. Introduction

The use of distributed communications in wireless sensor networks allows for energy savings through spatial diversity gains [1]. Cooperative transmission and/or reception of data among sensors is known to diminish the per-node energy consumption (the main constraint of sensor systems), increasing network lifetime [2].

The original work of Cover and El Gamal [3] set up the relaying scenario of Gaussian channels when supported by one relay node, showing capacity gains when properly allocating power. This result was the baseline for the proposed cooperative network in [4], where the relay node is considered another network user. The necessary extension of this results to multiple relay channels has been recently carried out by [5], setting up the first information theoretic approach to cooperative multi-hop transmission. The specific relationship between spatial diversity of cooperative networks and the decrease of transmit power is studied for single relay and multirelay channels in [6] and [7], respectively. Furthermore, the importance of optimal resource allocation in the relay channel is analyzed in [8] and [9]. Therein, the impact of optimum time and power allocation in half-duplex relay networks is shown.

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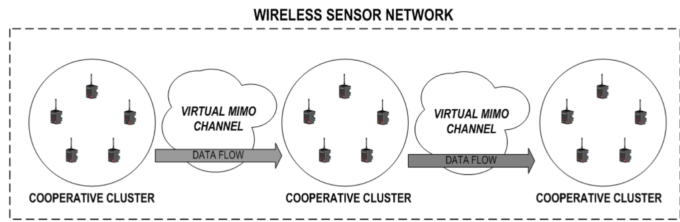

Fig. 1. Cooperative clusters in multi-hop wireless sensor network.

In this paper we propose a clustered cooperative multi-hop sensor network that implements cooperative transmission and reception of data among cluster nodes (see Figure 1), with the aim of exploiting the diversity gain of MIMO systems. Every cluster-to-cluster link (also referred to as Virtual MIMO channel) is defined as a a time division, decode-and-forward, multiple relay channel, composed of a broadcast channel within the cluster and an inter-cluster space-time coded MIMO channel (see Figure 2). The cooperative multiple antenna reception protocol, devised at the clusters, is based upon a simplified selection diversity receiver, which allows to obtain the full receiver spatial diversity. In the network layer, we assume the multi-hop routing algorithm defined at the cluster level, based upon hierarchical routing. However, the routing algorithm is out of the scope of this paper.

We define different degrees of channel state information (CSI) at the sensor nodes. We consider that every node belonging to a cluster has perfect and updated CSI of all nodes of the cluster (e.g., it can be obtained via channel reciprocity). On the contrary, we assume that the CSI among nodes of two independent clusters is not known, due to the impossibility of channel reciprocity in the proposed virtual MIMO channels. Finally, we consider all sensor nodes working on half-duplex mode. Our main contribution is the analysis and minimization of the outage probability of the clustered cooperative sensor network, assuming cooperative transmission as depicted in Figure 2. The optimum time assignment and power allocation is derived for any independent cluster-to-cluster hop.

Finally, results show that the cooperative scheme achieves full transmit-receive diversity with low SNR losses, and depict the performance degradation of the system with the cluster size.

II. Cluster-to-Cluster Transmission Model

The construction of the MIMO channel between two clusters is presented in this subsection, with independent focus on how to build it up on the transmitter side (cluster) and on the receiver side (cluster). We deal with the problem of how nodes of the transmitter cluster optimally share the data to transmit, how they encode it and how the nodes of the receiver cluster implement the reception protocol (see Figure 2).

1) Transmitter Side: We assume a transmitter cluster (TxC) be composed of a set of N_t cooperative sensor nodes, communicating with a receiver cluster (RxC) composed of a set of N_r cooperative sensor nodes. In order to transmit the data to the neighboring cluster, the TxC implements two functions: 1) broadcasting of data within the cluster, so that all active nodes can decode the data to relay during the MIMO transmission (in general, the set of active nodes n_t is a subset of the total cluster nodes N_t), and 2) the transmission of the data via a $n_t \times N_r$ MIMO channel. Due to half duplex limitations, both functionalities are carried out in two orthogonal channels, henceforth assumed as time division (TD) channels. These two TD-channels are referred to as *Intracluster* (ITA) channel and *Intercluster* (ITE) channel, used for broadcasting and MIMO transmission respectively. Thereby, every cluster-to-cluster hop is arranged into two consecutive time slots: ITA slot and ITE slot.

- *Intracluster (ITA) Slot*: During this slot, the data to transmit is broadcasted within the cluster with power p_1 during time α. The set of nodes falling into the broadcast capacity region decodes data and cooperates in the ITE slot (this set is henceforth called *decoding set*). Of course, the number of nodes belonging to the decoding set depends on the selection of α and p_1.
- *Intercluster (ITE) Slot*: In this slot, the subset of n_t nodes (consisting of the source of the broadcast plus the *decoding set*) jointly transmit data, with power p_2 during time $1-\alpha$, to the RxC by using a $n_t \times N_r$ Virtual MIMO channel. Assuming no intercluster channel knowledge, and symbol synchronization between cluster nodes, the proposed transmission scheme is based upon distributed space-time codes (DSTC) with the transmitted power per each active node equal to p_2/n_t.

2) Receiver Side: During the $1-\alpha$ time interval of the ITE slot, the RxC receives the data through the $n_t \times N_r$ MIMO channel and, with aim of obtaining full receiver diversity, it implements a multiple antenna decoding algorithm. Given the distributed nature of the multiple receiving antenna, we propose a simplified reception algorithm based upon selection diversity (SD) over the set of N_r MISO channels arriving to RxC. In other words, the sensor node of RxC with highest signal-to-noise ratio (SNR) acts as cluster coordinator and decodes data. It is straightforward to show that this scheme achieves receiver diversity order N_r [6]. Since the power budget is the main constraint within sensor networks, we

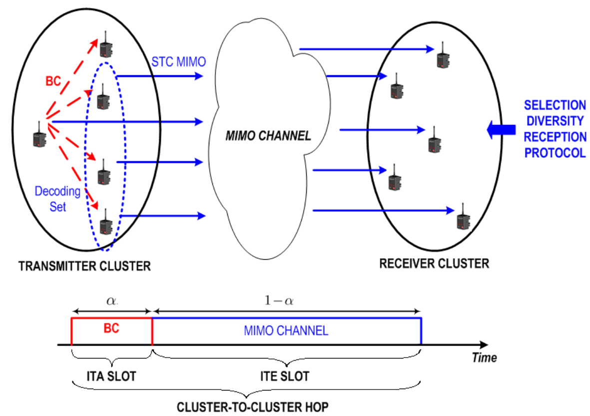

Fig. 2. Cluster-to-cluster transmission scheme.

assume that the energy consumption in the cluster-to-cluster hop is limited to E_t, thus

$$E_t = \alpha \mathrm{p}_1 + (1-\alpha)\,\mathrm{p}_2 \ . \tag{1}$$

Notice that the transmission performance can be optimized from a judicious choice of the three (dependent) terms p_1, p_2 and α.

III. Signal Model and Probability of Outage

Let a multi-hop communication be composed of $M-1$ hops that, for notation convenience, connect cluster 1 (source cluster) with cluster M (destination cluster) through clusters $2, ..., M-1$ (routing clusters). We consider each hop being split into ITA and ITE Slot, where slot duration, α, and power allocation, $(\mathrm{p}_1, \mathrm{p}_2)$, are independently optimized for each cluster-to-cluster link. In every cluster m, we assume that the total number of available nodes when acting as transmitter (N_t) equals to the total number of available nodes when cluster acts as receiver (N_r), i.e., $N_t = N_r = N_m$. Intercluster propagation is modelled as Rayleigh faded with the distance between the center of consecutive clusters set to d_{ITE} for any inter-cluster link m. Taking into account the considerations above, the received signal at sensor κ of cluster $m+1$, during the ITE slot of hop m, corresponds to the MISO channel:

$$y_{\kappa,m+1}\left(t\right) = \Gamma \cdot \mathbf{h}_{\kappa,m}^{T} \cdot \mathbf{x}_m\left(t\right) + n_{\kappa,m+1}\left(t\right) \ , \tag{2}$$

being $\Gamma = d_{ITE}^{-\delta/2}$ the intercluster path loss, $\mathbf{h}_{\kappa,m} = \left[a_{1,\kappa}^{m}, \ldots, a_{n_t,\kappa}^{m}\right]^T$ the channel vector and $\mathbf{x}_m\left(t\right) = \left[x_{1,m}\left(t\right), \ldots, x_{n_t,m}\left(t\right)\right]^T$ the transmitted vector at time $t \in (\alpha, 1]$. $a_{i,\kappa}^{m}$ is a unitary power, Rayleigh fading coefficient between node i of cluster m and node κ of cluster $m+1$, we assume invariant channels during the entire frame duration and independent, identically distributed (i.i.d) entries on the channel matrix $\mathbf{h}_{\kappa,m} \sim \mathcal{CN}\left(0, \mathbf{I}_{n_t}\right)$. Finally, $n_{\kappa,m+1}\left(t\right) \sim \mathcal{CN}\left(0, \sigma_o^2\right)$ is additive white Gaussian noise (AWGN) at sensor κ. Furthermore, considering a transmitted space-time codeword of arbitrary length $s = \frac{1-\alpha}{\Delta}$, the $(s \times 1)$ received vector signal at sensor κ is:

$$\mathbf{y}_{\kappa,m+1}^{T} = \Gamma \cdot \mathbf{h}_{\kappa,m}^{T} \cdot \mathbf{X}_m + \mathbf{n}_{\kappa,m+1}^{T} \ , \tag{3}$$

being $\mathbf{X}_m = [\mathbf{x}_m(1), \ldots, \mathbf{x}_m(s)] \in \mathcal{C}^{n_t \times s}$ the transmitted ST Code with $\mathbf{R}_{\mathbf{x}_m} = \frac{1}{s} \cdot \mathbf{X}_m \cdot \mathbf{X}_m^{\dagger} = \mathrm{p}_2/n_t \cdot \mathbf{I}_{n_t \times n_t}$; $\mathbf{y}_{\kappa,m+1} \in \mathcal{C}^{s \times 1}$ and $\mathbf{n}_{\kappa,m+1} \in \mathcal{C}^{s \times 1}$.

Decoding at the receiver cluster is based on the selection diversity receiver (SD) over the N_{m+1} MISO channels of the RxC (i.e., the sensor node with highest SNR acts as cluster coordinator and decodes data). Therefore, taking into account the model in (3), the output SNR at the SD of cluster $m+1$ is computed as:

$$\gamma_{m+1}^{SD} = \frac{\eta_2}{n_t} \max_{\kappa} |\mathbf{h}_{\kappa,m}|^2 \ , \tag{4}$$

where $\eta_2 \doteq \frac{\mathrm{p}_2}{\sigma_q^2} d_{ITE}^{-\delta}$. Thereby, the probability of outage for the hop m is given by:

$$\begin{aligned} P_{out}^{m} &= P\left[\frac{\eta_2}{n_t} \max_{\kappa} |\mathbf{h}_{\kappa,m}|^2 < 2^{\frac{C_{out}}{1-\alpha}} - 1\right] \\ &= \prod_{\kappa=1}^{N_{m+1}} P\left[\frac{\eta_2}{n_t} |\mathbf{h}_{\kappa,m}|^2 < 2^{\frac{C_{out}}{1-\alpha}} - 1\right] \end{aligned} \tag{5}$$

where C_{out} [bps/Hz] is the outage capacity (e.g., selected from network design), scaled by $1-\alpha$ according to the proposed TD system. Second equality follows from the cumulative density function (cdf) of the maximum of i.i.d. channels. The outage probability (5) can be optimized with respect to the power allocated in ITA and ITE slots and the time duration, when constraining the overall energy as in (1):

$$\begin{aligned} P_{out}^{m} = \min_{(\eta_1, \eta_2, \alpha)} & \prod_{\kappa=1}^{N_{m+1}} P\left[\frac{\eta_2}{n_t} |\mathbf{h}_{\kappa,m}|^2 < 2^{\frac{C_{out}}{1-\alpha}} - 1\right] \\ \text{s.t. } & \alpha \eta_1 + (1-\alpha)\eta_2 = \mathrm{SNR} \end{aligned} \tag{6}$$

Constraint (1) has been normalized to refer power to the receiving cluster, by defining $\mathrm{SNR} \doteq \frac{E_t}{\sigma_o^2} d_{ITE}^{-\delta}$ and $\eta_i \doteq \frac{\mathrm{p}_i}{\sigma_o^2} d_{ITE}^{-\delta}$ for $i \in \{1, 2\}$.

The optimization in (6) is not an straightforward task. It can be shown that as the power and time allocated for the ITA slot increases, the number of nodes that successfully decode and cooperate in the ITE slot also increases, improving the outage performance. Nevertheless, the power allocated for the ITE slot diminishes, increasing the outage probability. Therefore, an optimum tradeoff between the number of cooperating nodes n_t and the power to cooperate η_2 has to be found to obtain the lower outage probability. All the mathematical analysis associated to this optimization problem is fully detailed below in section IV.

However, from these optimized outage probability between cluster m and $m+1$, the overall outage probability of the $M-1$ hop communication is evaluated as:

$$P_{out} = 1 - \prod_{m=1}^{M-1} \left(1 - P_{out}^{m}\right) \ . \tag{7}$$

IV. Optimum Design of ITA and ITE Slots

In this section, we derive the per-hop optimum power allocation and time duration for the ITA and ITE Slots. We consider the outage probability P_{out} as the performance metric when optimizing each cluster-to-cluster link independently according to (6). First hop, intermediate hops, and the final hop are analyzed independently.

A. First Hop

The first hop connects cluster 1 (i.e., the cluster that contains the source node) with cluster 2 and starts the communication.

1) ITA Slot: Let node 1 of cluster 1 be the source node of the multi-hop communication; it uses the ITA slot to broadcast its data to nodes $\{2, \cdots, N_1\}$ of cluster 1. During this time slot, each node $i \in \{2, \cdots, N_1\}$ is expected to correctly decode the broadcasted data (and thus to be able to cooperate during the ITE Slot) if and only if the broadcast rate R_{BC} is below the node 1 to node i channel capacity:

$$R_{BC} \leq \alpha \log_2 (1 + \eta_1 \xi_i) \ , \tag{8}$$

being $\xi_i = |a_{1,i}|^2 \left(\frac{d_{ITE}}{d_{1,i}}\right)^{\delta}$ the source-relay path gain, with $d_{1,i}$ the distance between source and node i and $a_{1,i} \sim \mathcal{CN}(0,1)$ corresponding to the Rayleigh fading coefficient (assumed invariant during the communication). Nevertheless, in decode-and-forward degraded relay channels the source-relay rate (i.e., the broadcast rate) cannot be lower than the relay-destination rate (i.e., the MIMO rate) [3, Theorem 1] [5, Theorem 1]. Therefore, being the intercluster communication rate set to C_{out}, the capacity region of node 1 to node i is constrained to:

$$C_{out} \leq R_{BC} \ . \tag{9}$$

Thereby, node i is guaranteed to decode during the BC slot and to cooperate during the MIMO transmission if and only if:

$$\eta_1 \geq \frac{h(C_{out}, \alpha)}{\xi_i} \ , \tag{10}$$

with $h(R, \alpha) \doteq 2^{\frac{R}{\alpha}} - 1$. Furthermore, by ordering the instantaneous path gains for all receiver nodes:

$$\xi_2 \geq \ldots \geq \xi_i \geq \ldots \geq \xi_{N_1} \ , \tag{11}$$

we derive the relationship between the number n_t of active nodes during the ITE Slot and the pair (η_1, α) as:

$$\begin{aligned} n_t = 1 & \qquad \eta_1 < \tfrac{h(C_{out},\alpha)}{\xi_2} \\ n_t = n & \qquad \tfrac{h(C_{out},\alpha)}{\xi_n} \leq \eta_1 < \tfrac{h(C_{out},\alpha)}{\xi_{n+1}}, \quad 2 \leq n \leq N_1 \ . \end{aligned} \tag{12}$$

Notice that $n_t = 1$ means that only the source transmits within the ITE slot, while for $n_t = n > 1$ there are $n-1$ relays that cooperate to transmit during the ITE slot. We make use of slack variable $\xi_{N_1+1} = 0$.

2) ITE Slot: In this interval, the n_t nodes jointly transmit data with transmission rate C_{out} [bps/Hz] to destination cluster 2. The outage probability of this multiantenna link is given by (6), where the number of cooperating transmitters n_t follows (12) and $||\mathbf{h}_{\kappa,1}||^2 \sim \mathcal{X}_{2n_t}^2$ (i.e., chi-square distributed R.V. with $2n_t$ degrees of freedom). Since the cdf of the $\mathcal{X}_{2n_t}^2$

is the incomplete gamma function $\gamma(n_t, b) = 1/(n_t - 1)! \cdot \int_o^b x^{n_t-1} e^{-x} dx$, the optimization in (6) is rewritten as:

$$P_{out}^1 = \min_{(\eta_1,\eta_2,\alpha)} \left(\gamma\left(n_t, \frac{h(C_{out},(1-\alpha))}{\eta_2/n_t}\right)\right)^{N_2} \quad (13)$$
$$\text{s.t. } \alpha\eta_1 + (1-\alpha)\eta_2 = \text{SNR}$$

Moreover, since n_t in (12) is constant over N_1 regions in (η_1, η_2, α), the minimization of (13) may be carried out by first minimizing the objective function on every region and then selecting the minimum of minima. Every region is interpreted as the subset in (η_1, η_2, α) that makes the number of active sensors during the ITE slot constant and equal to n. Therefore, we may rewrite:

$$P_{out}^1 = \min_{1\leq n\leq N_1} \left\{ \min_{(\eta_1,\eta_2,\alpha)} \left(\gamma\left(n, \frac{h(C_{out},(1-\alpha))}{\eta_2/n}\right)\right)^{N_2} \right\} \quad (14)$$
$$\text{s.t. } \alpha\eta_1 + (1-\alpha)\eta_2 = \text{SNR}$$
$$\eta_1 \geq \frac{h(C_{out},\alpha)}{\xi_n}$$

The second constraint in (14) follows from (12), where it is shown that the link has n active transmitter nodes if and only if $\eta_1 \geq \frac{h(C_{out},\alpha)}{\xi_n}$ (notice that we set $\xi_1 = \infty$). The outage probability of the first link can be obtained as (see Appendix I):

$$P_{out}^1 = \min_{1\leq n\leq N_1} \left(\gamma\left(n, \frac{n\cdot h(C_{out},(1-\alpha_n))}{\eta_{2n}}\right)\right)^{N_2} \quad (15)$$
$$= \left(\gamma\left(\tau, \frac{\tau\cdot h(C_{out},(1-\alpha_\tau))}{\eta_{2\tau}}\right)\right)^{N_2}$$

being τ the optimum number of cooperating (active) nodes within the ITE slot and:

$$\alpha_n = \arg \max_{\alpha_{on}\leq\alpha\leq 1} \frac{\text{SNR}\xi_n - \alpha h(C_{out}, \alpha)}{(1-\alpha) h(C_{out}, (1-\alpha))}$$
$$\eta_{1n} = \frac{h(C_{out}, \alpha_n)}{\xi_n} \qquad \eta_{2n} = \frac{\text{SNR}}{1-\alpha_n} - \frac{\alpha_n \eta_{1n}}{1-\alpha_n} \ . \quad (16)$$

Therefore, the optimum power and time allocation for the first hop will be:

$$\alpha = \alpha_\tau \qquad \eta_1 = \eta_{1\tau} \qquad \eta_2 = \eta_{2\tau} \ . \quad (17)$$

B. Intermediate Hops

The cooperative strategy for the intermediate hops is slightly different than for the first hop. In an intermediate (routing) hop m, all nodes belonging to the transmitter cluster has previously received a copy of data through the ITE slot of hop $m-1$. Every sensor node has received the data with different instantaneous power and, according to the SD algorithm, at least the sensor with the largest SNR has fully decoded the message (otherwise the communication would be in outage). This *decoder* sensor uses the ITA Slot to broadcast the decoded data within the transmitter cluster. Nevertheless, since all cluster nodes already have a degraded copy of the data, the broadcasting should only provide the differential of mutual information that allows them to decode the codeword free of errors. We define this communication as a differential broadcast channel (DBC).

1) ITA Slot: In this differential broadcast channel, we assume node 1 of cluster m to be decoder sensor (i.e.,the coordinating node, selected to decode in hop $m-1$ according to the SD algorithm) and nodes $\{2, \cdots, N_m\}$ of cluster m to be the receiver nodes. Every node $i \in \{2, \cdots, N_m\}$ has previously received a copy of the data (during hop $m-1$) with spectral efficiency $R_i = \frac{1}{s}\mathcal{I}(\mathbf{y}_{i,m}; \mathbf{X}_{m-1})$ (being $\mathcal{I}(\cdot;\cdot)$ the mutual information).

During the ITA slot, the coordinating node, broadcasting with rate R_{DBC}, is able to increase the mutual information of node i, $\mathcal{I}(\mathbf{y}_{i,m}; \mathbf{X}_{m-1})$, if and only if:

$$R_{DBC} \leq \alpha \log_2(1 + \eta_1\xi_i) \ , \quad (18)$$

being ξ_i the path gain between coordinating node and sensor i. Nevertheless, to guarantee that node i decodes data and retransmits during the ITE Slot, its decoding rate $R_{DBC} + R_i$ cannot be lower than the relay-destination rate [3, Theorem 1] [5, Theorem 1]:

$$C_{out} \leq R_{DBC} + R_i \ . \quad (19)$$

Therefore, by defining $C_i = \max\{0, C_{out} - R_i\}$, a node i belongs to the decoding set only if:

$$\eta_1 \geq \frac{h(C_i, \alpha)}{\xi_i} \ . \quad (20)$$

Now, considering that there are $N_m - 1$ receiver nodes of the DBC channel that, without loss of generality, can be ordered as

$$\frac{h(C_2,1)}{\xi_2} \leq ... \leq \frac{h(C_i,1)}{\xi_i} \leq ... \leq \frac{h(C_{N_m},1)}{\xi_{N_m}} \ , \quad (21)$$

then, we may fairly approximate the relationship between the number of active nodes n_t during the ITE slot and η_1 and α as:

$$n_t = 1 \qquad \eta_1 < \frac{h(C_2,\alpha)}{\xi_2}$$
$$n_t = n \qquad \frac{h(C_n,\alpha)}{\xi_n} \leq \eta_1 < \frac{h(C_{n+1},\alpha)}{\xi_{n+1}}, \quad 2 \leq n \leq N_m \quad (22)$$

2) ITE Slot: The analysis of the $n_t \times N_{m+1}$ MIMO transmission between cluster m and $m+1$ is equivalent to the analysis carried out for the first hop. Nevertheless, here n_t depends upon η_1 and α according to (22). Therefore, by adapting the optimization (14), the outage probability remains:

$$P_{out}^m = \min_{1\leq n\leq N_m} \left\{ \min_{(\eta_1,\eta_2,\alpha)} \left(\gamma\left(n, \frac{h(C_{out},(1-\alpha))}{\eta_2/n}\right)\right)^{N_{m+1}} \right\} \quad (23)$$
$$\text{s.t. } \alpha\eta_1 + (1-\alpha)\eta_2 = \text{SNR}$$
$$\eta_1 \geq \frac{h(C_n,\alpha)}{\xi_n}$$

With $C_1 = 0$. Similarly to optimization for the first hop, and using results on the Appendix I, we derive the outage probability of m-th link:

$$P_{out}^m = \min_{1\leq n\leq N_m} \left(\gamma\left(n, \frac{n\cdot h(C_{out},(1-\alpha_n))}{\eta_{2n}}\right)\right)^{N_{m+1}} \quad (24)$$
$$= \left(\gamma\left(\tau, \frac{\tau\cdot h(C_{out},(1-\alpha_\tau))}{\eta_{2\tau}}\right)\right)^{N_{m+1}}$$

with τ the optimum number of active nodes in the transmitter cluster and:

$$\alpha_n = \arg \max_{\alpha_{on} \le \alpha \le 1} \frac{\mathrm{SNR}\xi_n - \alpha h(C_n, \alpha)}{(1-\alpha) h(C_{out}, (1-\alpha))}$$

$$\eta_{1n} = \frac{h(C_n, \alpha_n)}{\xi_n} \qquad \eta_{2n} = \frac{\mathrm{SNR}}{1-\alpha_n} - \frac{\alpha_n \eta_{1n}}{1-\alpha_n}\ . \tag{25}$$

Finally, the optimum time and power allocation for the intermediate hop will be:

$$\alpha = \alpha_\tau \qquad \eta_1 = \eta_{1\tau} \qquad \eta_2 = \eta_{2\tau}\ . \tag{26}$$

C. Final Hop

The optimization of the final hop is similar to the intermediate hops. Nevertheless, in this hop the receiver cluster contains the destination node, which introduces one modification on the deployed reception protocol. Basically, we assume that the destination cluster receives the data, through a $n_t \times N_M$ MIMO channel, and decodes it according to the selection diversity criteria described previously. If the link is not in outage, the sensor node with highest SNR decodes the data addressed to the destination node and, later, forwards it to the destination node through a differential broadcast channel. In our results, we assume that the power and time used in this specialized intracluster communication at the destination cluster is negligible compared with the total time and energy allocated for the cluster-to-cluster communication, and thus it is not taken into account in computation.

V. Simulation Results

The outage probability of the proposed cooperative sensor network is simulated here, following results derived in previous sections. The setup of the simulation is as follows: 1) clusters are defined as circles of radius R_{ITA}, 2) the distance between the center of two consecutive clusters is $d_{ITE} = 1$, 3) cluster nodes are randomly located within the cluster, following a uniform distribution, and 4) path loss exponent is set to $\delta = 2$.

Results for a single hop transmission are depicted in Figure 3 for different $\frac{R_{ITA}}{d_{ITE}}$ ratios, assuming a transmitter cluster with $N_1 = 3$ cooperating nodes and a receiver cluster with $N_2 = 3$ cooperating nodes. Likewise, the outage performance of a 3×3 MIMO system with SD at the receiver side is shown for comparison. The probability of outage of the cooperating scheme is computed from (15), randomizing the source-relay path gains ξ_i (random distance and Rayleigh fading), and considering SNR the energy constraint in (6). Some conclusions over the virtual MIMO scheme can be drawn: simulation validates that the proposed cooperative MIMO achieves the full spatial diversity of the system $N_1 \cdot N_2$; there is a constant SNR loss between real and virtual MIMO justified by the fraction of power used to broadcast data in the ITA Slot. Of course, virtual MIMO performance degrades when increasing the cluster radius R_{ITA}. Figure 4 shows, for different number of hops, the performance of a multi-hop communication based upon virtual MIMO channels with $N_m = 3$ and $\frac{R_{ITA}}{d_{ITE}} = 0.3$. The non-cooperate multi-hop case is also plot as reference. In the figure, the outage probability

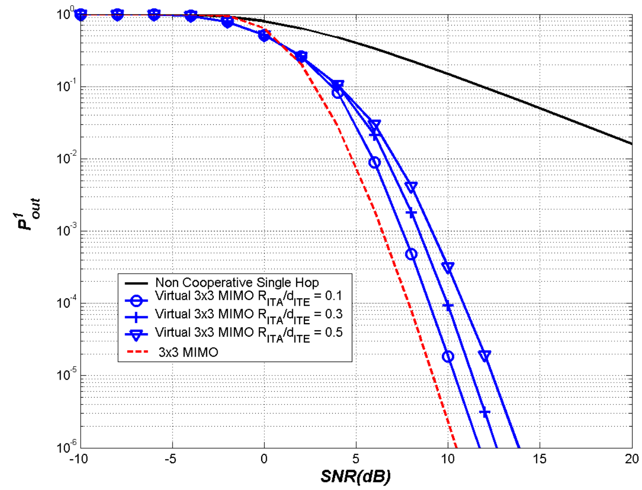

Fig. 3. Single cluster-to-cluster outage probability vs. SNR for different $\frac{R_{ITA}}{d_{ITE}}$ ratios and for $N_1 = 3$ and $N_2 = 3$.

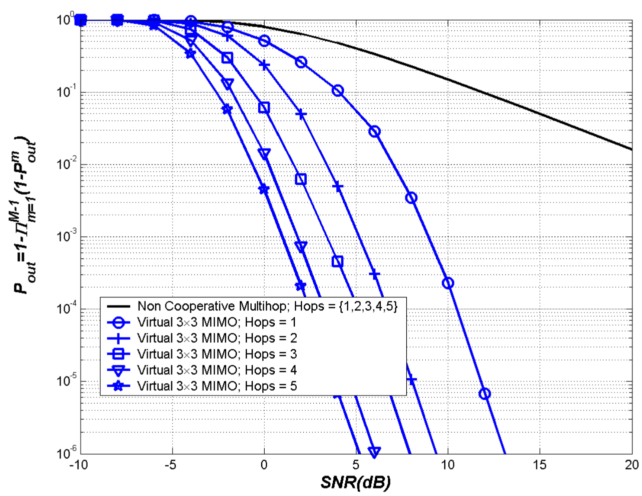

Fig. 4. multi-hop outage probability vs. SNR for different number of cluster-to-cluster hops assuming 3 nodes per cluster and $\frac{R_{ITA}}{d_{ITE}} = 0.3$.

of the non-cooperative system is kept constant for all number of hops as we consider that the network designer fixes the end-to-end outage probability independently of the number of hops in between; and for any given number of hops the outage capacity of cooperative and non-cooperative system are equal. Numerical analysis shows a definite advantage of the distributed MIMO approach with respect to the non-cooperative multi-hop case. Moreover, when increasing the number of hops (i.e., when decreasing the outage capacity) the energy savings also increase to very significant values.

VI. Conclusions

In this paper we proposed a clustered topology to introduce cooperative diversity in multi-hop wireless sensor networks. We proposed that, within every network cluster, all sensor nodes cooperate to transmit and receive data to take advantage of the diversity gain that arises when exploiting virtual MIMO communications between clusters. To build up the virtual MIMO channel, we assumed a time division, decode-and-forward multirelay channel composed of a broadcast channel and a space-time coded MIMO channel. To construct an energy aware multiantenna reception protocol we proposed a selection diversity technique. The joint optimization of the time assignment for the time-division channels and the power allocation was decoupled into the cluster-to-cluster link optimization. Numerical analysis over this cooperative scheme (for single hop and multi-hop networks) showed that: 1) full transmit-receive diversity is obtained with low SNR losses, 2) cooperation degrades when the cluster size increases with respect to the hop length, and 3) energy savings per hop increase for growing number of hops.

APPENDIX I
OPTIMIZATION PROBLEM

In this appendix, we analyze the minimization problem:

$$P = \min_{(\eta_1,\eta_2,\alpha)} \gamma\left(n, \frac{n\cdot\left(2^{\frac{C}{1-\alpha}}-1\right)}{\eta_2}\right) \quad \text{(A-1.1)}$$

$$\text{s.t.} \quad \alpha\eta_1 + (1-\alpha)\,\eta_2 = \text{SNR} \quad \text{(A-1.2)}$$

$$\eta_1 \geq \frac{2^{\frac{R}{\alpha}}-1}{\xi} \quad \text{(A-1.3)}$$

$$\alpha\eta_1 \leq \text{SNR} \quad \text{(A-1.4)}$$

$$0 \leq \alpha \leq 1 \quad \text{(A-1.5)}$$

where n, R, C and ξ are fixed constants, and $\gamma(n,b)$ the incomplete gamma function. We consider $R = C_{out}$ for the first hop and $R = C_n$ for the rest of hops.The first two constraints (A-1.2) and (A-1.3) are explicit constraints. The others, (A-1.4) and (A-1.5) are implicit constraints, forcing η_2 to be positive and α to be positive and not greater than one.

The first step in the optimization is the analysis of the feasible set: constraint (A-1.3) establishes that η_1 has to be at least $\frac{2^{\frac{R}{\alpha}}-1}{\xi}$, while constraint (A-1.4) forces the product $\alpha\eta_1$ to be lower than SNR. Therefore, all α for which:

$$\alpha \cdot \frac{2^{\frac{R}{\alpha}}-1}{\xi} > \text{SNR} \quad \text{(A-2)}$$

do not belong to the feasible set. Thereby, since:

$$\alpha \cdot \frac{2^{\frac{R}{\alpha}}-1}{\xi} \leq \text{SNR} \Leftrightarrow$$

$$\alpha \geq \alpha_o = \frac{-\ln(2)\cdot R}{\text{W}_{-1}\left(\frac{-\ln(2)\cdot R}{\text{SNR}\cdot\xi}\cdot e^{\frac{-\ln(2)\cdot R}{\text{SNR}\cdot\xi}}\right)+\frac{\ln(2)\cdot R}{\text{SNR}\cdot\xi}} \quad \text{(A-3)}$$

only $\alpha \geq \alpha_o$ must be considered in the minimization. $\text{W}_{-1}(\kappa)$ is defined as the branch -1 of the Lambert W function [10].

Second step is the concatenation of the minimization process:

$$P = \min_{\alpha_o\leq\alpha\leq 1}\left\{\min_{(\eta_1,\eta_2)} \gamma\left(n, \frac{n\cdot\left(2^{\frac{C}{1-\alpha}}-1\right)}{\eta_2}\right)\right\}$$

$$\text{s.t.} \quad \alpha\eta_1 + (1-\alpha)\,\eta_2 = \text{SNR}$$

$$\eta_1 \geq \frac{2^{\frac{R}{\alpha}}-1}{\xi} \quad \text{(A-4)}$$

From (A-4), it can be easily shown that, being (for fixed α) the goal function a decreasing function with η_2 (in the feasible set) and independent of η_1, then the minimum is given at the point where η_2 is maximum, and due to constraint (A-1.2), where η_1 is minimum:

$$\eta_1^* = \frac{2^{\frac{R}{\alpha}}-1}{\xi} \;\rightarrow\; \eta_2^* = \frac{\text{SNR}}{1-\alpha} - \frac{\alpha\cdot\eta_1^*}{1-\alpha}\,. \quad \text{(A-5)}$$

Therefore, minimization in (A-4) reduces to:

$$P = \min_{\alpha_o\leq\alpha\leq 1} \gamma\left(n, n\cdot\xi\cdot\frac{(1-\alpha)\cdot\left(2^{\frac{C}{1-\alpha}}-1\right)}{\text{SNR}\cdot\xi-\alpha\cdot\left(2^{\frac{R}{\alpha}}-1\right)}\right) \quad \text{(A-6)}$$

Moreover, taking into account that the incomplete gamma function satisfies that the minimum over b of $\gamma(n,b)$ is given for the minimum value of b, the optimum time allocation α^* will be:

$$\alpha^* = \arg\max_{\alpha_o\leq\alpha\leq 1} \frac{\text{SNR}\cdot\xi-\alpha\cdot\left(2^{\frac{R}{\alpha}}-1\right)}{(1-\alpha)\cdot\left(2^{\frac{C}{1-\alpha}}-1\right)} \quad \text{(A-7)}$$

Now, by defining $f(K,\alpha) = \alpha\cdot\left(2^{\frac{K}{\alpha}}-1\right)$ with $K \in \mathcal{R}^+$ constant, and noting that: i) $f(K,\alpha) \geq 0$ and $f''(K,\alpha) \geq 0$, ii) for the feasible set, $\text{SNR}\cdot\xi \geq f(R,\alpha)$, then (by computing the second derivative) it is readily shown that maximization in (A-7) is a convex optimization problem and therefore α^* exists and may be found.

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