Least Squares & KF

$$\underset{\sim}{y} = H\underline{x} + e$$

→ Know nothing about e.

$$\hat{x} = (H^T H)^{-1} H^T y$$

WEIGHTED L. S.

→ Know: $E[ee^T] = V$.

$$\hat{x} = \underbrace{(H^T V^{-1} H)^{-1} H^T V^{-1}}_{L} y$$

↳ Min Variance Unbiased estimator.

x. (unbiased ⟹ ~~...~~ $E[\hat{x}] = E[x]$

$E[LHx] = E[x]$
$LH = I$).

→ ~~Prev~~ Given $y = Hx$,

but H not full rank.

REGULARIZED LS.

eg
$$\left.\begin{aligned} 2x_1 + 4x_2 &= 1 \\ 4x_1 + 8x_2 &= 2 \end{aligned}\right\} \text{ 2 eq in 2 variable}$$

Sometimes for large dim data, "H" almost not full rank.

eg
$$2x_1 + 4x_2 = 2$$
$$4x_1 + 8 \cdot 2 x_2 = 2$$

$H^T H$: very large condition #.

- Need regularized LS.

$$\underline{\text{argmin}} \; (x - x_0)^T W_0 (x - x_0) + (y - Hx)^T W (y - Hx)$$

- in corporate "prior knowledge"

→ What if either too many equations -

Storage problem

or data coming in sequentially.

or dim. of $\underline{x}$ is very large. (do not want to invert) & surely not multiple time

↳ Recursive LS

$$\hat{x}_i = \hat{x}_{i-1} + \underbrace{\left(P_{i-1} h_i^T \left(R_i + h_i P_{i-1} h_i^T\right)^{-1}\right)}_{\text{for } K_i}\left(y_i - h_i \hat{x}_{i-1}\right)$$

$$P_i = P_{i-1} - P_{i-1} h_i^T h_i P_{i-1} \left(R_i + h_i P_{i-1} h_i^T\right)^{-1}$$

$$y_i = h_i x + e. \quad E[ee^T] = R_i$$

→ assumes $x_i = x_{i-1}$

If there is time dynamics that can be modeled eg motion - velocity.

= $K_o F_o$.

KF - prediction step : easy.

$\boxed{\text{update}}$

$$\cancel{\alpha} = x_i | y_{i-1} \sim N(\bar{x}_{i|i-1}, P_{i|i-1})$$

$$\cancel{\beta =} \; y_i |_{x_i, y_{i-1}} \sim N(h_i x_i, R_i)$$

$\cancel{P(\alpha | \beta) = P}$

$$P(x_i | y_i, y_{i-1}) = \frac{P(y_i, x_i | y_{i-1})}{P(y_i | y_{i-1})} = \frac{P(y_i | x_i, y_{i-1}) \cdot P(x_i | y_{i-1})}{\int_{x_i=-\infty}^{\infty} P(y_i | x_i, y_{i-1}) \cdot P(x_i | y_{i-1}) \, dx_i}$$

~~EKF - nonlinear system : linearize.~~

KF : object is moving with vel v.

$v_t - v_{t-1} = n_t$

const vel model

$$\begin{cases} P_t = P_{t-1} + V_{t-1} + \frac{1}{2} n_t . \\ V_t = V_{t-1} + n_t . \end{cases}$$

$$X_t = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} X_{t-1} + \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \underbrace{n_t}_{N(0, q^2)}$$

$$Y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} X_t + \underbrace{v_t}_{N(0, q_2^2)}$$